Towards a Democratization of Knowledge with Topological Emphasis in Economics

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Abstract. We formulate and prove a theorem which consists in how the natural endogenous antagonist interaction of agents who look for understanding a generalizable phenomenon, results in a tendency towards chaos. This takes us to the final absolution of implementing the majority rule as the only instrument that generates socially acceptable knowledge, escaping from the chaos tendency. Finally, we extend our analysis to consider the arise of multiple simultaneous antagonist postures on the explanation of a phenomenon, and through an application of the Pythagoras theorem, we prove that it takes less effort or sacrifice for an agent to learn strategically to get an explanation, than if she was the creator of the concerning knowledge, which implies different consequences of possible topological private and public tendencies.

Keywords. Antagonist endogenous knowledge, Social entropy, Chaos theorem, Social choice.

JEL. B50, I20, O31, O35, P16, P27, Z13, Z18.

> "Pure mathematics is, in its way, the poetry of logical ideas"
> Albert Einstein

1. Introduction

This work is meant to capture a broad representation of the process which takes place on the formation of knowledge. It is based on qualitative observations more than on quantitative ones. Although this generalization can be understood as explanatory for different sciences, we shall introduce it with examples of economics that result highly illustrative, not only because we are more familiarized with them, but also because we consider that are more popular among readers.

The formation of knowledge is always preceded by complicated eternal and antagonist debates which are leaded by thinkers that truly believe they are right, as if their position was a sort of religion. For example, we can look at the well known Keynesians vs Neoclassical debate. Another more recent but not less essential in the understanding of economics which starts from questioning the very foundations of microeconomics, is the Sraffian capital debate vs the Neoclassical praxis, where according to each group of prominent thinkers, the truth lies in their position[i].

Most of these debates are developed upon wether some assumptions or properties should be treated as truth or not[ii], and the deeper the reader gets, the more likely will be for her to join a partial position.

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Among other debates we can highlight the different opinions on the future of the economic growth of the countries, where authors like Kaldor argued that the regions show convergence in real per capita GDP levels, while forexample Accinelli *et al.* (2010) and Howitt & Mayer-Foulkes (2005) explainhow the nations diverge depending on initial levels.

Independently of time, although there are specific theoretical positions based on the statisticinference that look for answering which of the previously mentioned views isright, such as the approach to assess robustness proposed by Sala-i-Martin (1997), deep thinkers do not seem to be satisfied, and keep arising with newpoints of view or proposed variables concerning the debated topics,diverging continuously.

Considering the antagonistic nature of these phenomena, we shall develop arepresentation and a theorem which allows the reader to understand thedivergent dynamic of positions over facts, also enhancing the visualizationof a chaotic future in the absence of a stopping mechanism and thus aknowledge democracy. Posteriorly we prove a sacrifice theorem which takinginto account the existence of knowledge prerequisites, stablishes that it iseasier to follow than innovating, and we do this through an application ofthe Pythagoras theorem. Furthermore, this work can also be found as anexposition of foundations behind why it is important not only for economiststo study Social Choice.

2. The model

All the agents are located in a convex closed set of $\mathbb{R}_+^n$ which contains the vector zero. The model is based on theintuitive assumption that one of the dimensions captures the level ofantagonism and capability, such that if the agents are located in a closureit means that are the creators of the deepest explanation which justifies anantagonist position. This means that the agents in this dimension's closuresgo deeper in the debated points (considering more aspects), which thus pushesthem to such position, and that they propose the more complex and ruling explanations, where the closures of such dimension thus represent a current boundary of human understanding[iii]. Furthermore this means that the intermediate point inthis dimension is occupied by the agents who are indifferent in terms of thediscussed topic.

The rest of the dimensions capture a location of the agents in terms ofother aspects, like space or a sport taste. Each agent $i \in N$ has apersonal scope to share theoretical findings representing her flexibility tointeract with the individuals who are different. We accept that the scope intolerating other positions is equal for all the agents, and that it isequally annoying and possible for them to do a personal sacrifice in anydirection despite their personal characteristics and the income distribution. *Therefore* we can represent the individual's scope with a constant distance k, such that the k radio ball surrounding an agentdenotes her effective interaction area[iv].

The first entry of the vector of coordinates of the location of an agent i at the time t is given by $l_i \in [A_t, D_t]$, where A_t and D_t represent the borders of humanknowledge.

Although looking for understanding the intertemporal metric behavior of thenew knowledge happenings could takes us to fruitful findings, in this workwe rather consider the following properties or attributes, which are inherent or present inthe process that we are dealing with.

Constant presence of leadership: $l_i = A_t \wedge l_j = D_t$ for $i, j \in N$, and thus $|N| \geq 2$. Moreover, the individuals in this closure are called knowledge authority.

This property considers how it is important to notice that the closuresexist for all $i, j \in N$ because there is at least one authority in the already mentioned frontier of human knowledge. Further more, by common sense we know that all the inventions or new theories are built or not, constantly using previous ones as abase or influence[v], and to represent this we define the following inter temporal property.

Constant scientific progress: (i) Strict; $A_t < A_{t-x} \wedge D_t > D_{t-x}$. (ii) Weak; $A_t \leq A_{t-x} - \frac{1}{\int_{t-x}^{t} du} \wedge D_t \geq D_{t-x} + \frac{1}{\int_{t-x}^{t} du}$.

2.1. Local consensus

As we have previously mentioned, the agents are able to come up with anexplanation about the discussed topic, however, the communication among agents can take them to understand and/or support deeper and *thus* ruling theories which are proposed by others. This means that if an agent $i$is located inside of the closed k radio ball surrounding j, the deeperexplanation which is accessed by these two agents can be supported by bothas a kind of local consensus.

As we have previously mentioned, a location l_i does not only capture alevel of capability, but a posture as well, such that an individual in the middle of the interval $[A_t, D_t]$, is located there because is not capable of taking a posture on her own. Furthermore, the possible local consensus also imply that this "indifferent" individual could support a partial position under certain conditions, which we will later explain.

A set of interaction is a coalition $S \in 2^N$ which satisfies $|S| > 1$, formed by the individuals who are located suchthat, for any agent $i \in S$ there is at least another individual $j \in S$ with the k radio ball surrounding her having a non empty intersection withthe k radio ball surrounding i.

To represent how the agents have defined a posture on the theoretical debate, after interacting, the individuals will always support the deepest thesis which they can access and agree with. In other words, they will support the position of the agent who belongs to their set of interaction, and that is closer to the closure in the dimension of positions to whichthey are closer. Moreover, an "indifferent" agent will thus support the position of the individual within her interaction set, that is more distant from hers[vi].

As we can see, this means that determinedly among the agents forming a coalition $S \in 2^N$ there are one or two consensuses to which we shall refer to as local consensus.

2.2. Debate and social problems

As it can be verified, these debates can originate problems which affectnegatively the life style of the population showing violent consequences like for example the witch hunting leaded by the spanish inquisition.

This is, although it may be "normal" for many regions to presence such kind of "generalizable" happenings, the produced effects can reach devastating outcomes.

Wanting to consider, how the local consensus are already in favor of one of thetheoretical positions, and that the capable individuals look for "approaching" and sharing the "true" explanation, imposing their visible localized views,we shall define a measure of social stabilitylikelihood based on howdivergent the positions are.

The antagonism which derives naturally from the knowledge formation is as we have mentioned "normally" problematic, and both postures or thoughts have usually followers, which iswhy independently of the formed intermediate local consensus and the numberof followers, we shall base our measure on how difficult it becomes to reacha global consensus that avoids social problems.

In this way, for a given period t we can just take the distance $l_t = D_t - A_t$ as a therefore absolute global indicator of social instability, due to the impossibility to reach agreements, such that a higher l_t indicatesa thus more chaotic social situation.

H is the set of coalitions of elements of $\mathbb{R}$ withcardinality two. The function $f: H \longrightarrow H$ takes the pair of leading positions $\{A_{t-1}, D_{t-x}\}$ and gives back the knowledgefrontiers of the next period whose standardspace $\mathbb{R}_+^n$ fitting thus has had then ever contradicting properties of "Constant scientific progress" and "Constant presence of leadership".

This in turns means that $f(t)$ is a function that focuses only on the evolution ofthe frontiers of knowledge.

Theorem (Chaos): $\lim_{t \to \infty} I_t \longrightarrow \infty$.

Proof: To avoid limiting our analysis i.e. get a more strict proof, we shall focus only on the weak "constant scientific progress" property being present such that $t > x \longrightarrow A_t < A_x \wedge D_t > D_x$ .From this we deduce that $D_t - A_t > D_x - A_x \longleftrightarrow t > x$. Since we only have interest in the standard proof that considers the strict increase of the distances in the middle we get that $t - x \longrightarrow \infty \longleftrightarrow (D_t - D_x) - (A_t - A_x) \longrightarrow \infty$. Finally we can justconsider the particular case of $x = 0$, and any posterior t tending toinfinite illustrates the divergence growth (of I_t) and chaos.Q.E.D.

As we can see this theorem means that with the pass of time, the understanding of the agents on an issue gets deeper, and that it also beyond limits becomes more difficult for them to reach a global consensus, or agreement, thus tending to more chaotic social dynamics at least due to the need foradopting a posture when general decisions are taken[vii]. Further more, for some reason, as a sort of established rivalry of novelties, due to a continuous application of the mean value theorem, we know that the indifferent agent(s) shall constantly occupy the mean point between the determined closures.

2.3. Escaping from chaos

Now that we have identified the open tendency towards chaos, what could be donein order to avoid such a cruel destiny for the human kind?

Although dictatorship of knowledge seems to be the easiest alternative in terms of implementation by countries, because it does not require the approval of eachindividual, we recognize the right of the agents to participate in theformation of the information which shall be considered as true in thefunctioning of their world. Therefore, based on that *the equality for all $x, \in N$ cannot be* but with the majorityrule as the mechanism through which a final more desired consensus and thus,a socially acceptable knowledge could be adopted, it remainsto naturally highlight how theindividuals may increase their participation and promotion of localconsensus to impact their daily living.

Raza *et. al*. (2007) demonstrate that the social democratization of knowledgefor e-learning (without confusing it with the free access concept) can leadto the establishment of viable global civil society, helping millions inAsia, Africa and South America to contribute and share the fruits ofknowledge explosion in a just, equitable and honorable fashion. On the otherhand, some works that study problems which can arise when the majority ruleis used are Condorcet (1785), Plata (1999), and the *manipulation possibility* of strategic voting studied by Salvador Barbera (Jackson & Sonnenschein, 2011).

Assuming the democracy mechanism to start being implemented in the period t' does not mean that the evaluations of the function $f_{t'}$ start showing convergence [viii], but instead itmeans that the index $I_{t'}$ becomes useless due to theintellectual exercise subordination to the will of the majority. Therefore, the

chosen postures on the matters would depend on the distribution ofindividuals over the different local consensus[ix], which implies avoidingsocial problems at a possible cost of exercising a *minor* to the frontier local consensus[x].

2.4. Social stability and multiple antagonist postures

As we have previously mentioned, because of the antagonist nature ofknowledge there are polar disagreements between thinkers, however, in theadvance of a phenomenon understanding we can find the formation of newdebates and antagonist positions within a single posture.Considering howthis could take place, we get that the local consensus could now be locatedin different points.

The interpretation of this is given by the arise of an extra dimension whichindicates the position of the agents in terms of the new debate, where a newdirection can be taken at certain point of the previous postures. Moreover, notice that considering more than one debate allows thepossibility of theagents occupying more than one position at the same time, because a leaderin a position could also be a leader in another debate within that position.

The formation of multiple debates is of our concern because of the socialinstability that derives from it. In order to understand the socialinstability it is fundamental to understand when there are knowledgerequirements to be able to get a posture in a new debate. To illustrate thiswe consider the following case:

Case 1 (Strict requirements debate): $l_i(j) \neq a_j \leftrightarrow l_i(j-1) \geq r_j \, \forall i, j \in N$ where $l_i(j)$ is the location of i, a_j is the indifference level and r_j is the origin level in the debate $j-1$all with respect to the jth debate.

In this way if there were not requirements in a debate, the agents could belocated in any planed coordinate within the newly considered closed intervals. Moreover, given that to get a position in a debate, the requirements increase depending on how deep into the new posture an agent can be, the agents can only be located in terms of a new debate as itfollows.

Theindividuals can only be located in terms of their posture on a phenomenon j, within the triangle that has $r_j - a_{j-1}$ as base, and height $D_t(j) - a_j$ or $a_j - A_t(j)$, where $D_t(j)$ and $A_t(j)$ are the frontiers of human knowledge interms of the jth debate. _

Moreover, this means that when the debate fromwhich the new one arose gets deeper, then the agents could also be locatedin the area of the rectangle[xi] that has such new boundary $\mp$ the level r_j asa base, and $D_t(j) - a_j$ or $a_j - A_t(j)$ as height.__

This also means that if more than one debate arises at a single level ofknowledge, then the individuals could be located in terms of position withinthe triangle in the O dimensions, where there were $O - 1$ arising debates. ___t

The interpretation of this visualization is that an agent located in thehypotenuse of the triangle or "below" does not really get the new debate, but instead it has some of the requiredknowledge to get it.

Considering how each of these dimensions keeps being associated with thecapability and effort of the agents, we formulate the following theorem.

Theorem (Sacrifice): It takes less effort, sacrifice or capability to learn or copy requirementsthan to innovate, propose, or discover to get to the knowledge of thefrontiers of an arisen debate.

Proof: For $0 - 1$ arising debates we can write this argument as $h < a + b$, where h is the distance between the previous indifference point and the newfrontiers' coordinates, a is the base of the triangle in the 0 dimensions, and b is its height.Since $h^2 = a^2 + b^2$ and $(a + b)^2 > a^2 + b^2$. Q.E.D.

An example of this theorem is given by how it takes less effort to learnmath and economic intuition simultaneously, than to first understand math [xii] and then economics to be able to get the explanation of aneconomic rule like monopoly pricing. In others word, this theoremmeans that given the rational agents, it is easier to get somewhere when thefinal destination is known, or that it is easier to get something whichrequires knowledge that is already possessed by the individual.

Further study can be done concerning the *rate* of growth of $r_j's$ base on the interest-need and capacity of the interagents population N, where extending our analysis to the behavior of $k's$comparisons would present an evident degree of importance, relevant to the "constant presence of leadership" property[xiii].

In terms of *stability* this means that although the *topologically* defined debates within a debatecan bring problematic social dynamics, the added instability is even higherbecause there can be more*distinct*local consensus[xiv]. Therefore, to consider this in a simple way we redefine theinstability indicator for O related debates, as if there were not, knowledge requisites in the following

$$I_t = \sum_{i=1} \prod (D_t(i) - A_t(i))$$

Up to this point we may want to ask to ourselves: What went wrong? Or, what could go wrong? Given every distinct secure agent, it is obvious how for different of these *very well topologically* defined arising debates that are in thesame dimension [xv], only the moreadvanced boundaries should be considered to understand and thus measure instability. Moreover, based on thesatisfaction of the "constant scientificprogress"property, we can verify and prove how theprevious unlimited chaos theorem is true for all the newly defined agreements$I_t's$!

3. Conclusions

From the representation we get a clear explanation and intuition behind theformation of local consensus that look for explaining certain phenomenon.

We introduced a measure of social instability which is based on howdifficult it becomes for the individuals to agree and form a globalconsensus. Moreover, our theorem contributed in showing how the evolution ofthe boundaries of knowledge leads a population to increasingly chaoticsocial dynamics, which took us to the unavoidable proposal of a mechanism,to reach social agreements on which knowledge shall be considered as true ormainstream, in this way facilitating the taking of general decisions.

As we can see, our work contributed not only to address questions abouttendencies over time in terms of social problems and stability, but also toestablish a solution to the incoming chaos named the democratization of knowledge, which has the advantage of allowing the individuals toparticipate in the formation of scientific information. In this way, thewinning postures on the matters would depend on the distribution ofindividuals among the different local consensus, which implies avoidingsocial problems at a possible cost of exercising an inferior to the frontierlocal consensus. Furthermore, we could remark how it may already beobservable "normal" in many regions to observe theemployment of this kind of mechanisms, which so far can be interpreted as ajustification not only for economists to get deeper inthe study of Social Choice.

Finally, the sacrifice theorem allows us to get the importance of knowledge requirements for the understanding of a scientific posture, and to remarkthe efforts of the individuals who although chaotically, build informationat the frontiers of

human knowledge, implying different consequences of possible private and public mechanisms tendencies for unexpected welfare levels, and thus highlighting the growing importanceof composed human capital.

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Notes

[i] e.g. Petri (2009; 2013), Garegnani (2003; 2005), Mandler (2002; 2005), Lazzarini (2011).

[ii] For example, the treatment of the capital as an homogeneous good of the models of economic growth, discussed in the mentioned Sraffian capital debate, which leads to wondering about the correct properties of an aggregate production.

[iii] A unidimensional representation of the location of agents can be found in Hotelling (1929).

[iv] We could reject the assumption about the interdimensional equally annoying and possible sacrifices, and represent the maximum sacrifice of the jth dimension by k_j, which *means* that the effective interaction area of an individual would be given by an n dimensional ellipse. Moreover, depending on different hypothesis of behavior, we could represent an effective interaction area with a not necessarily convex closed set, surrounding an agent without altering our qualitative results.

[v] This means that the leaders are always able to pass to more capable individuals, the interest for continuing the search for deeper and better explanations.

[vi] The deepest explanation that she has received, where being part of a local consensus does not necesarily mean that the explanation has been truly understood.

[vii] It can be verified that in the equality case of the "constant scientific progress" property $\frac{\Delta D_t}{\Delta D_{t+1}}, \frac{\Delta D_t}{\Delta D_{t+1}} = \frac{t+\Delta t}{t} > 1$.

[viii] Which could be true if the mechanism included the dictation or prohibition to stop the study of the bound of the loosing position.

[ix] Santerre (2008) focuses on how the scientific and technical culture has become an interface, stimulating exchanges between scientists and other social actors, resulting in research being more attuned to community needs.

[x] This is because as we have previously mentioned, the agents who are part of different local consensus are not necessarily able to effectively interact with some one who explains them at least convincingly enough a deeper consensus, or the frontier of their position.

[xi] Or square.

[xii] Or Phisics.

[xiii] The corresponding extension should consider or obey the catastrophic dynamic regimechange mathematic schema studied in Brida *et al.* (2011).

[xiv] The set of possible local positions including consensus is $2^N \backslash \emptyset$, and for enough existent knowledge, the possible knowledge tendencies are *at least* as many as possible set of disjoint non empty coalitions.

[xv] i.e. that "normally" require the same kind of knowledge requisites.

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