Journal of Economics Bibliography
www.kspjournals.org
Volume 4 March 2017 Issue 1

Puzzling Features of the Historical Income per Capita Distributions Explained

By Ron W. NIELSEN [†]

Abstract. Distributions describing growth of the Gross Domestic Product per capita (GDP/cap) are puzzling. They show that income per capita was approximately constant over hundreds of years, maybe even over thousands of years, but then, as if suddenly it started to increase. The growth was changed apparently rapidly from approximately horizontal to approximately vertical. We shall show that these features represent nothing more than purely mathematical property of dividing two hyperbolic distributions represent nothing more than purely mathematical property of dividing two hyperbolic distributions. Historical growth of income per capita can be explained as having been controlled by the simple and familiar forces of growth.
Keywords. Income per capita, Gross Domestic Product, Growth of population, Hyperbolic growth.
JEL. A10, A12, A20, B41, C02, C12, C20, C50, Y80.

1. Introduction

Hyperbolic distributions appear to be creating significant problem with their interpretation. They are routinely seen as being made of two distinctly different components, slow and fast, jointed perhaps by a transition stage. However, these distributions are easy to understand if they are represented by their reciprocal values (Nielsen, 2014) because in this representation the confusing features disappear and hyperbolic distributions are represented by straight lines.

It is always convenient to reduce the analysis of data to a straight line, if possible, for two reasons: (1) straight lines are easy to understand and (2) any deviation from a straight line can be easily observed. For the exponential growth, the analysis can be reduced to a straight line by calculating the logarithm of data. For the hyperbolic growth, a straight line is produced by calculating the reciprocal values of data. However, for the income per capita, this simple method cannot be applied and we have to use a different approach. Furthermore, distributions describing income per capita are even more confusing than hyperbolic distributions because features, which were already difficult to understand for hyperbolic distributions, are even more confusing.

Incorrect interpretation of the historical GDP/cap data is a serious problem and the prominent example is the Unified Growth Theory (Galor, 2005a, 2011). Using the reciprocal values of the GDP data, it has been already demonstrated (Nielsen, 2014) that the fundamental postulates of this theory are contradicted by empirical evidence. We shall now demonstrate that the same conclusion can be reached by the analysis of the GDP/cap data coming from precisely the same source as used in developing this theory.

Unified Growth Theory tries to explain the apparent different stages of growth but we shall demonstrate that this explanation is grossly incorrect because the

† Griffith University, Environmental Futures Research Institute, Gold Coast Campus, Qld, 4222, Australia.
☎. +61407201175
✉. r.nielsen@griffith.edu.au or ronwnielsen@gmail.com

GDP/cap data follow a *single, monotonically increasing,* trajectory, which should be interpreted as a whole. We shall demonstrate that the three regimes of growth, postulated in the Unified Growth Theory and generally accepted in other related publications did not exist and that there was no generally claimed takeoff in the economic growth at any time.

2. Crude representation of data

The GDP/cap distributions are frequently displayed in a grossly simplified way by selecting just four strategically-located points (Ashref, 2009; Galor, 2005a, 2005b, 2007, 2008a, 2008b, 2008c, 2010, 2011, 2012a, 2012b, 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008) as shown in the top panel of Figure 1.

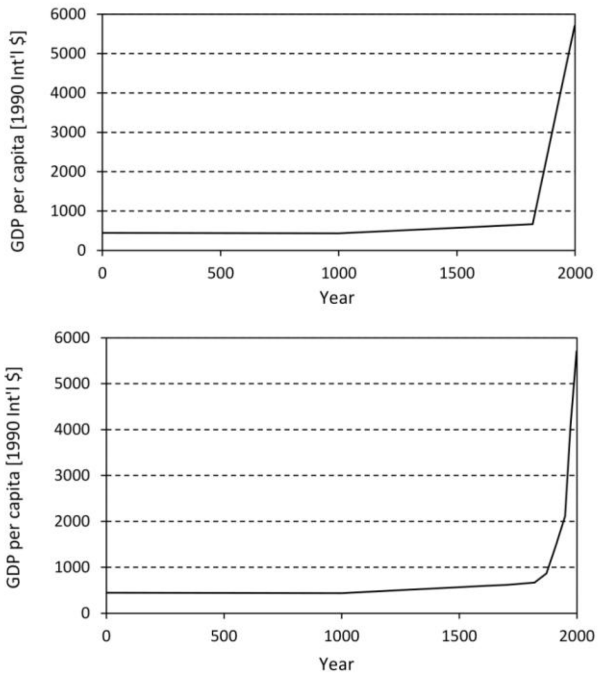


Figure 1. *Gross Domestic Product (GDP) per capita (Maddison, 2001) as frequently presented in numerous publications (Ashref, 2009; Galor, 2005a, 2005b, 2007, 2008a, 2008b, 2008c, 2010, 2011, 2012a, 2012b, 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008).Strongly misleading impressions created by such presentations of data are the basis for promoting erroneous interpretations of the mechanism of economic growth and the prominent example is the Unified Growth Theory (Galor, 2005a, 2011)*

In this figure, we show an example for the world economic growth but similar plots are also used for regional data. Such displays are strongly suggestive and misleading, and they serve as a perfect prescription for drawing incorrect conclusions. This is a good example of the unscientific approach to research and it is hardly surprising that such handling of data leads to incorrect conclusions. Galor's Unified Growth Theory and all other associated publications are not based on science. They are unreliable and strongly questionable. Indeed, when closely analysed they are found to be repeatedly contradicted by data (Nielsen, 2014, 2016a, 2016b, 2016c, 2016d, 2016e, 2016f, 2016g, 2016h), even by the same data, which in their distorted way were used to support these numerous publications.

The GDP/cap distributions are already sufficiently confusing even if all data are plotted (see the lower panel in Figure 1). They do not have to be distorted to create even greater confusion. They have to be methodically and carefully analysed. Displays such as shown in Figure 1 are not helpful because they reinforce incorrect impressions and interpretations.

Impressions can be misleading and every effort should be taken to avoid being guided by their deception. Science is not based on impressions but on a rigorous analysis of data. Unified Growth Theory (Galor, 2005a, 2011) describes ideas based on impressions created by such displays as shown in Figure 1 or by quoting certain data without making any effort to analyse them scientifically. In this theory, many complicated but rather primitive mathematical formulations are presented, but incorrect concepts remain incorrect even if translated into mathematical formulae.

3. Explaining the GDP/cap ratio

The GDP/cap ratio combines two time-dependent distributions: (1) the time-dependent GDP growth and (2) the time-dependent population growth. In order to understand the GDP/cap distributions we have to understand their two components: the growth of the GDP and the growth of population.

Over 50 years ago, von Foerster, Mora and Amiot (1960) demonstrated that the world population was increasing hyperbolically during the AD era. Recent analysis shows that that the world population was increasing hyperbolically for thousands of years not only during the AD era but also during the BC era (Nielsen, 2016i). Hyperbolic growth of population applies not only to the global but also to regional populations (Nielsen, 2016d). Contrary to the expectation of Malthus (1798), when unchecked, population does not increase exponentially but hyperbolically. Furthermore, the growth of population was hardly ever checked. Historical GDP values, global and regional, were also following hyperbolic distributions (Nielsen, 2014, 2016a, 2016h).

Even though hyperbolic distribution appears to be made of two different components, slow and fast, joined by a transition component, it has been shown (Nielsen, 2014) that such interpretation is based on strongly misleading impressions. Reciprocal values of a hyperbolic distribution describing growth follow a decreasing straight line and it is then obvious that it makes no sense to divide a straight line into arbitrarily selected sections and claim different mechanisms of growth for each section. It also makes no sense to look for a point marking a takeoff on such a monotonically decreasing straight line because a monotonically decreasing straight line remains a monotonically decreasing straight line and there is no justification in selecting a certain point on such a line and claim that there is a change of direction at this point because there is no change of direction.

In order to understand the GDP/cap distributions, the first and essential step in the past studies should have been to understand mathematical properties of their two components (GDP and population).Now we know that that they follow hyperbolic distributions. Consequently, in order to understand the historical GDP/cap data we have to understand the *mathematical process* of dividing two hyperbolic distributions.

We are going to demonstrate that the characteristic features of the GDP/cap distributions, which were used in the formulation of the Unified Growth Theory (Galor, 2005a, 2011), represent purely *mathematical properties* of dividing two hyperbolic distributions. They do not represent different socio-economic conditions describing different mechanisms of growth for different perceived sections of these distributions as claimed erroneously in the Unified Growth Theory.

Hyperbolic distribution describing *growth* is represented by a *reciprocal* of a linear function:

$$f(t) = (a - kt)^{-1}, \qquad (1)$$

where $f(t)$ is the size of the growing entity, t is the time, and a and k are *positive* constants.

A reciprocal of hyperbolic distribution, $[f(t)]^{-1}$, is represented by a *decreasing* straight line:

$$[f(t)]^{-1} \equiv \frac{1}{f(t)} = a - kt \,. \tag{2}$$

Hyperbolic *distributions* should not be confused with hyperbolic *functions* ($\sinh(t)$, $\cosh(t)$, etc). Furthermore, *reciprocal* distribution or functions, $[f(t)]^{-1}$, should not be confused with *inverse* functions, $f^{-1}(t)$. Mathematical symbol for the inverse function, $f^{-1}(t)$, is similar to the mathematical symbol for the reciprocal function, $[f(t)]^{-1}$, but the concepts are different.

In the inverse functions, the roles of variables are inversed. In the reciprocal functions, they remain the same. Thus, for instance, for the distribution given by the equation (1), the aim of using its inverse function would be to calculate how the *time* depends on the size of the growing entity. The inverse function of the eqn (1) is

$$f^{-1}(t) = \frac{a}{k} - \frac{1}{kt}, \tag{3}$$

where t is now the size of the growing entity and $f^{-1}(t)$ is the time. For the reciprocal function given by the eqn (2), t is still the time as in the eqn (1). From the eqn (3) we can see that when the size of the growing entity, t, increases to infinity, the time, $f^{-1}(t)$, reaches its terminal value of a/k.

The characteristic feature of hyperbolic distributions is that they increase slowly over a long time and fast over a short time, escaping to infinity at a certain fixed time $t_s = a/k$, i.e. when the denominator in the eqn (1) approaches its zero value. However, as we have already pointed out and as discussed earlier (Nielsen, 2014), it is a mistake to interpret such distributions as being made of two distinctly different components joined by a transition component. It is one and continuous distribution, which has to be interpreted as a whole. If such a distribution represents a certain mechanism of growth, it is the same mechanism for the whole distribution.

Let us now take two, purely mathematical, hyperbolic distributions, $f(t)$ and $g(t)$, and let us divide them. Results are presented in Figure 2.

Parameters describing hyperbolic distributions displayed in Figure 2 are: $a = 4.5$ and $k = 2.2 \times 10^{-3}$ for $f(t)$ and $a = 7$ and $k = 3.35 \times 10^{-3}$ for $g(t)$. *These distributions are purely mathematical entities. They have nothing to do with the growth of the population or with the economic growth.* However, they satisfy a simple condition: the singularity of the $f(t)$ distribution occurs earlier than the singularity of the $g(t)$ distribution. For the curves displayed in Figure 2 singularities are at $t_s \approx 2045$ for $f(t)$ and $t_s \approx 2090$ for $g(t)$. The point of singularity for the $f(t)/g(t)$ ratio is, of course, at $t_s \approx 2045$.

When the distribution $f(t)$ is divided by $g(t)$ they produce a distribution, which resembles closely a typical GDP/cap distribution (see the lower panel of Figure 1). The characteristic features of this distribution are a long stage of nearly constant values of the $f(t)/g(t)$ ratio followed by a nearly vertical increase.

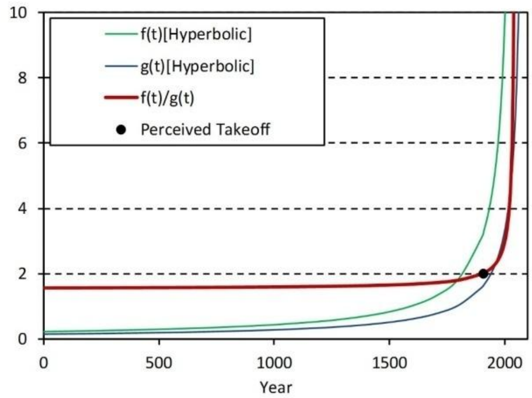


Figure 2. *Two, mathematically-defined, hyperbolic distributions,* $f(t)$ *and* $g(t)$*, and their ratio* $f(t)/g(t)$*. The time of the perceived, but non-existent, takeoff is indicated.*

It is important to notice that for the ratio of two hyperbolic distributions, the difference between slow and fast growth is much more clearly pronounced than for the corresponding hyperbolic distributions. The nearly horizontal part is flatter and the nearly vertical part is even more vertical. That is why, if the hyperbolic distributions are already so confusing, the distributions representing the ratio of two hyperbolic distributions are even more confusing and their interpretation is even more difficult. They have to be analysed with extra care and their analysis cannot be simplified by using their reciprocal values because the reciprocal of the ratio of two hyperbolic distributions is also a ratio of two hyperbolic distributions. Their analysis is significantly more difficult than the analysis of hyperbolic distributions. They represent a well-concealed trap suggesting strongly the existence of two or even three different components and even the most experienced researcher, who is not familiar with hyperbolic distributions or who is reluctant to accept them because of their singularity, can be easily misguided.

So we can see now that by dividing two, mathematically defined and monotonically increasing hyperbolic distributions, which have nothing to do with the economic growth, we have generated the fundamental features, which inspired the creation of the grossly incorrect Unified Growth Theory (Galor, 2005a, 2011) propagating such erroneous concepts as "the Malthusian Regime" represented by the flat "part," "Sustained-Growth Regime" represented by the steep "part," "the Post-Malthusian Regime," represented by the middle "part" and a "takeoff," represented by the apparent but non-existent fast transition from the flat to the steep growth. All these “parts” and the takeoffs do not exist because distributions representing the ratios of monotonically increasing hyperbolic distributions increase also monotonically. We could devote volumes on discussing the mechanism of growth of these imagined “parts” and trying to explain the triggering mechanism of the non-existent takeoffs but our discussions would have no scientific merit. Unified Growth Theory is made of such unscientific explanations but we can find them in numerous other publications, all creating the undesirable confusion and all of them diverting attentions from the correct interpretation of the mechanism of economic and population growth.

The puzzling and apparently peculiar features observed in the GDP/cap distributions can be reproduced using purely mathematical, monotonically-increasing, hyperbolic distributions. These features reflect purely mathematical properties of a *single* distribution representing the $f(t)/g(t)$ ratio. *They do not describe different stages of growth.* Furthermore, it is clear that these features cannot be attributed uniquely to the GDP/cap distributions. The division of two hyperbolic distributions may represent a certain mechanism of growth but it is still a *single* mechanism.

We have created an unusual and perhaps puzzling distribution but it would be incorrect to be so mesmerised by this simple mathematical operation as to propose different regimes of growth for different perceived parts of the $f(t)/g(t)$ ratio. We can see that the features observed for the GDP/cap distributions can be easily replicated by dividing two mathematically-defined hyperbolic distributions. It is, therefore, clear that hasty assumptions about different socio-economic conditions for the different perceived "parts" of the GDP/cap distributions can be questioned, which means that the whole Unified Growth Theory based on such assumptions can be not only questioned but indeed shown to be grossly incorrect and scientifically unacceptable (Nielsen, 2014, 2016a, 2016b, 2016c, 2016d, 2016e, 2016f, 2016g, 2016h). There is no point in presenting elaborate descriptions of different socio-economic conditions if these descriptions are contradicted by data. Even if the described socio-economic conditions did exist, they obviously had no impact on shaping economic growth trajectories, at least as expressed by the GDP or by the GDP/cap values. Such theories, as the Unified Growth Theory, could be regarded as interesting collections of stories but these stories do not assist in understanding the mechanism of economic growth.

The next step in explaining the GDP/cap distributions is now to explain why the division of two hyperbolic distributions generates such a puzzling trajectory, which appears to be made of two distinctly different components and why these apparently different components are so strongly pronounced.

4. Explaining the ratio of hyperbolic distributions

Using the eqns (1) and (2) we can see that the ratio of two hyperbolic distributions can be represented also in two other ways:

$$\frac{f(t)[Hyperbolic]}{g(t)[Hyperbolic]} = [g(t)]^{-1}[Linear]\cdot f(t)[Hyperbolic] = \frac{[g(t)]^{-1}[Linear]}{[f(t)]^{-1}[Linear]}. \qquad (3)$$

These operations are represented graphically in Figure 3. We can see that all these mathematical operations create the same distribution representing the ratio $f(t)/g(t)$. It does not matter which pathway we take – results are the same.

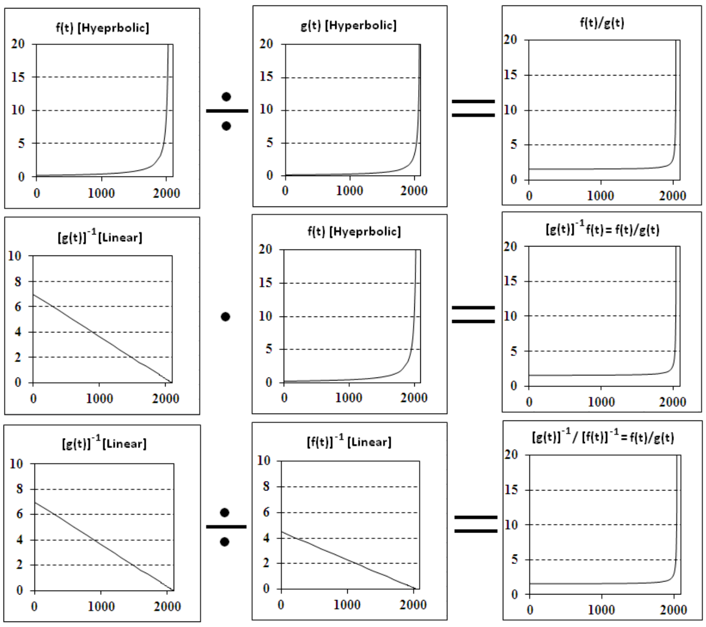


Figure 3. *Graphic representation of the eqns (3).*

Dividing two monotonically-increasing hyperbolic distributions is the same as multiplying hyperbolic distribution by a decreasing linear function and the same as dividing two decreasing linear functions. It is all just as simple as that. There are no hidden mysteries that need to be explained by some kind of complicated theories and mechanisms, but we still want to understand why these simple operations generate such a peculiar distribution, which appears to be made of two distinctly different components: horizontal and vertical.

The easiest way to understand the division of hyperbolic distributions is probably by looking at the middle section of Figure 3. The effect of the multiplication of hyperbolic distribution by the decreasing linear function is to lift up the left-hand part of the slowly increasing section of hyperbolic distribution and suppress the right-hand part. However, if $f(t)$ escapes to infinity earlier than $g(t)$, $f(t)$ will be escaping to infinity when $[g(t)]^{-1}$ is still positive. The values of $[g(t)]^{-1}$ will be small but the multiplication of the rapidly increasing values of $f(t)$ by small values of $[g(t)]^{-1}$ will have no effect on the escape to infinity. The product of such numbers will be also rapidly escaping to infinity. The combined effect of such a multiplication of a decreasing straight line by the increasing hyperbolic distribution is to flatten the slowly increasing section of the hyperbolic distribution without significantly changing the large values. The initial slow increase is made even slower and the *perceived* transition to the steep part is even more pronounced. However, *there is no mathematically-defined transition at any time* between these two perceived components.

The ratio of two hyperbolic distributions can be described simply as *the linearly-modulated hyperbolic distribution*. Thus, in our example the ratio of $f(t)/g(t)$ can be described as *the linearly-modulated hyperbolic $f(t)$ distribution*. The linear modulation is done by the linear function $[g(t)]^{-1}$ representing the reciprocal values of the hyperbolic $g(t)$ distribution.

Likewise, the distribution representing the historical GDP/cap growth can be described as *the linearly-modulated hyperbolic GDP distribution*. The linear modulation is done by the linear distribution representing the reciprocal values of the hyperbolic distribution describing the growth of human population.

The ratio of two hyperbolic distributions looks as if being made of two different components, slow and fast, but it is still the same, uninterrupted, monotonically increasing distribution. It is still a *single* mathematical distribution. It is the distribution, which is not made of two different sections. It is the distribution that it is *impossible* to divide into two distinctly different parts represented by two different functions. This distribution increases slowly over a long time and fast over a short time but the transition from the perceived slow to the perceived fast growth occurs *over the entire range of time*. It is *impossible* to determine the time of this perceived transition. It is impossible to determine the time of the perceived takeoff because *the takeoff does not exist* even if it appears to exist. The perceived takeoff is an illusion. There *is* a slow growth over a long time and a fast growth over a short time but there is no transition at any time between the slow and the fast growth. The slow and the fast growth are represented by the same, monotonically increasing distribution, which is not made of distinctly different components.

Even though the ratio of hyperbolic distributions, $f(t)/g(t)$, looks as if being made of two or three components (see Figures 2 and 3), even though the distribution represented by this ratio increases slowly over a long time and fast over a short time, even though it increases to infinity at a fixed time and even though it appears to be characterised by a takeoff at a certain time, it is still just a single, monotonically-increasing distribution, which is *impossible* to divide into different components. We have to accept it and learn to live with it.

Perhaps the easiest way to dispel the strong illusion of the distinctly different components of growth is to examinethe lowest part of Figure 3. It would be obviously unreasonable to claim that each of these straight lines is made of two or three distinctly different components, because these straight lines are obviously not made of different components. It would be unreasonable to claim different mechanisms of growth for various, arbitrarily-selected parts of these straight lines. At which point located on a straight line one mechanism of growth is supposed to end and a new mechanism to begin? It is *impossible* to claim two or three distinctly different sections on the monotonically decreasing straight lines. There is also obviously no feature on such straight lines that could be claimed as marking a takeoff.

We can also take a different approach and demonstrate again that the ratio $f(t)/g(t)$ represents a single, monotonically-increasing distribution and that there is no takeoff at any time. This different approach consists in calculating the gradient and the growth rate of the $f(t)/g(t)$ ratio. Results are presented in Figure 4 around the time of the perceived takeoff, i.e. when the $f(t)/g(t)$ reaches the value of 2 (see Figure 2). For better clarity, results are plotted as a function of the size of the $f(t)/g(t)$ ratio.

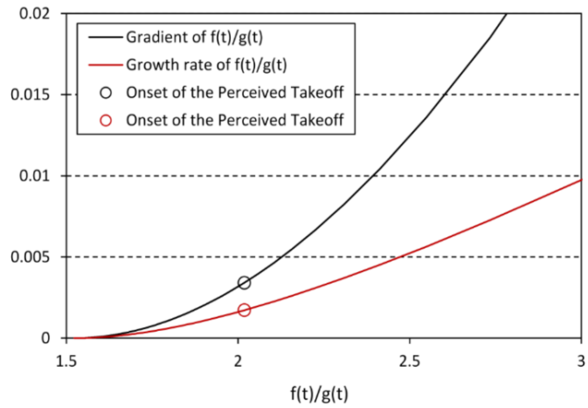


Figure 4. *The gradient and growth rate of the ratio of hyperbolic distributions* $f(t)/g(t)$. *The onset of the perceived takeoff shown in Figure 2 is indicated. This figure shows that the takeoff never happened and that the distribution representing the ratio* $f(t)/g(t)$ *is not made of different components. It is a single, monotonically-increasing distribution.*

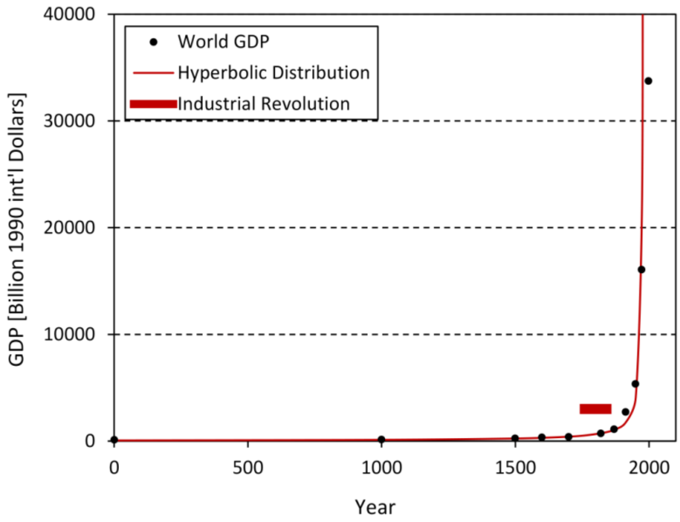
These calculations show clearly that both the gradient and the growth rate of the hyperbolic ratio $f(t)/g(t)$ increase monotonically. The perceived takeoff never happened. What looks like a takeoff in Figure 2 is in fact just the continuation of the undisturbed and monotonically-increasing distribution representing the $f(t)/g(t)$ ratio. It is impossible to claim different components for any of the distributions displayed in Figure 4, representing the $f(t)/g(t)$ distribution, which in Figure 2 looks very deceptively as being made of two different components. It is impossible to claim a takeoff for any of these two distributions. The two components simply do not exist and the takeoff is just an illusion.

5. Analysis of the historical GDP/cap data

The GDP and population data (Maddison, 2001) [the same data as used but not analysed during the formulation of the Unified Growth Theory (Galor, 2005a, 2011)] together with their fitted hyperbolic distributions are shown in Figure 5. Indicated in the figure is the time of the Industrial Revolution 1760-1840 (Floud & McCloskey, 1994), which is generally claimed as the time of the alleged takeoff in the economic growth (Galor, 2005a, 2008a, 2011, 2012). Parameters fitting the GDP data are: $a = 1.716 \times 10^{-2}$ and $k = 8.671 \times 10^{-6}$ while parameters fitting the population data are $a = 8.724$ and $k = 4.267 \times 10^{-3}$.

Points of singularity are: $t_s \approx 1979$ for the world GDP and $t_s \approx 2045$ for the population data. The point of singularity for the world GDP is before the point of singularity for the growth of the world population. Consequently, the GDP/cap ratio should display the same features as shown in Figure 2 for the $f(t)/g(t)$ ratio and indeed it does.

In Figure 6 we present the data for the GDP/cap and the corresponding fit to the data calculated by dividing the corresponding hyperbolic distributions shown in Figure 5. The calculated curve and the data shown in Figure 6 follow a similar distribution as displayed in Figure 2. The characteristic features of the nearly horizontal growth over a long time and the nearly vertical growth over a short time of the GDP/cap distribution are nothing more than the mathematical property of dividing two hyperbolic distributions.



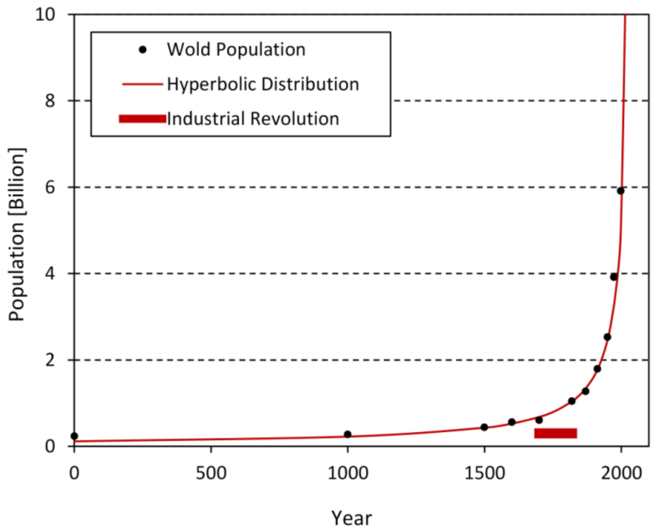


Figure 5. *Hyperbolic distributions are compared with the world GDP and population data (Maddison, 2001). The GDP is expressed in billions of 1990 International Geary-Khamis dollars and the population in billions.*

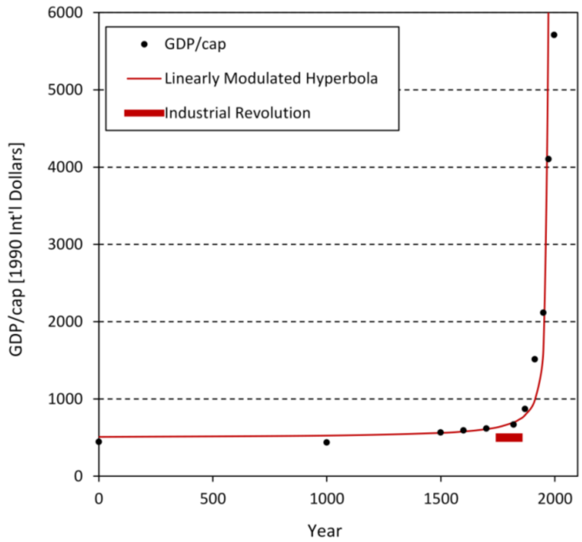


Figure 6. *Calculated, linearly-modulated hyperbolic GDP distribution, representing the GDP/cap ratio, is compared with the world GDP/cap data (Maddison, 2001). The GDP/cap is expressed in the 1990 International Geary-Khamis dollars.*

If the point of singularity for the GDP trajectory was located higher than the point of singularity for the population trajectory, the growth of the GDP/cap would also have remained nearly constant over a long time but it would eventually decrease to zero at the time of the singularity for the growth of population. Income per capita would not have been increasing with the size of the population. On the contrary, it would have been decreasing. For hyperbolic distributions, the growth of income per capita depends on the relative positions of singularities of the two components.

According to Galor (2008a, 2012a), the so called Malthusian Regime, represented allegedly by the nearly constant income per capita, commenced around 100,000 BC. There is, of course, no justification for this date because the Malthusian Regime did not exist (Nielsen, 2014, 2016a, 2016b, 2016c, 2016d, 2016e, 2016f, 2016g, 2016h; von Foerster, Mora & Amiot, 1960). However, if we wanted to claim a certain date for this mythical regime, one would imagine that the usually claimed date of 200,000 BC for the onset of the existence of *Homo Sapiens* would have been more suitable.

He also claims that Malthusian Regime was terminated in AD 1750 for developed countries and in 1900 for less-developed countries. The Post-Malthusian Regime was supposed to have existed between 1750 and 1870 for developed countries and from 1900 for less-developed countries. The Sustained-Growth Regime was supposed to have commenced in 1870 and is supposed to continue until the present time. It is impossible to determine such specific landmarks for the monotonically increasing distributions. These imagined dates are contradicted by data. There were no takeoffs in the growth of the GDP, and the historical GDP trajectory cannot be divided into two or three different regimes (Nielsen, 2014, 2016b, 2016c). We also know that the growth of human population was hyperbolic and that it was never characterised by a sudden takeoff (Nielsen, 2016d, 2016i). Consequently, even though the GDP/cap data might be suggesting the existence of different stages of growth governed by different mechanisms of growth, their scientific analysis clearly demonstrates that different regimes of growth did not exist. Each historical GDP/cap distribution, global or regional, has to be interpreted as a whole and the same mechanism has to be applied to the slow and fast growth. Under these conditions, the interpretation of the mechanism of growth appears to be complicated because we have to use the same mechanism to explain the slow and fast growth. However, the explanation turns out to be exceptionally simple (Nielsen, 2016j).

By following our earlier approach, which we used for the division of arbitrary hyperbolic distributions, we can demonstrate that there was no takeoff in the GDP/cap distribution and that the three regimes of growth did not exist. We shall do this by calculating the gradient and the growth rate for the calculated GDP/cap trajectory. These calculations are presented in Figures 7 and 8.

A takeoff in the GDP/cap trajectory would be marked by a clear change in the gradient and in the growth rate around the time of the Industrial Revolution when a transition to a new economic growth regime was supposed to have happened (Galor, 2005a, 2008a, 2011, 2012a). The shape of the trajectories describing the gradient and growth rate would have to be distinctly different before and after the Industrial Revolution. There should be a certain clear discontinuity.

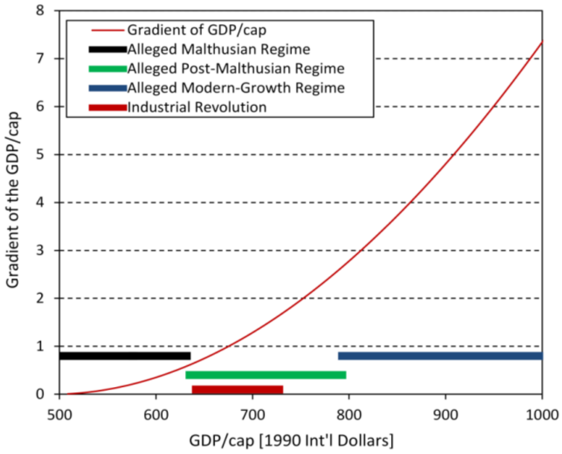


Figure 7. *Gradient of the world GDP/cap calculated using the fitted, linearly-modulated hyperbolic distribution shown in Figure 6. The GDP/cap is expressed in the 1990 International Geary-Khamis dollars. There was no takeoff at any time and the three regimes of growth postulated by Galor (2005a, 2011) did not exist.*

The gradient and the growth rate of the fitted curve increase monotonically confirming that the fitted, linearly-modulated hyperbolic distribution increases also monotonically. The calculated curve gives excellent fit to the GDP/cap data and consequently the gradient and the growth rate of the fitted curve represent also the gradient and the growth rate of the data.

Figures 7 and 8 clearly demonstrate that there is no reason for terminating the alleged Malthusian Regime around AD 1750 and for starting a new regime because there was no unusual change in the gradient and in the growth rate of the GDP/cap around that time, but there was also no scientifically-justified reason for assuming the existence of the Malthusian Regime. There is no reason for terminating the equally imaginary Post-Malthusian Regime around 1870 and starting the alleged Sustained-Growth Regime. There is no reason for slicing the monotonically-increasing distributions into three arbitrarily-selected sections. There is no reason for proposing three regimes of growth governed by distinctly different mechanism. There is no reason for claiming a takeoff at any time. There has been no scientifically justified reason for creating the Unified Growth Theory and three is no scientifically justified reason for adopting such concepts in the interpretations of economic growth and of the growth of population.

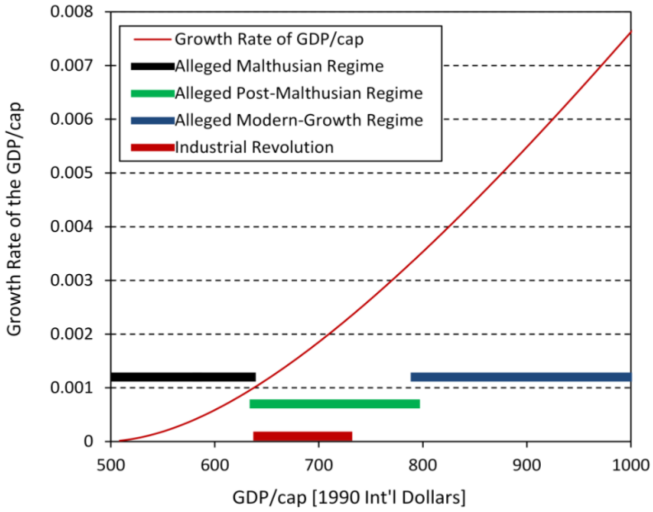


Figure 8. *Growth rate of the GDP/cap calculated using the fitted, linearly-modulated hyperbolic distribution shown in Figure 6. The GDP/cap is expressed in the 1990 International Geary-Khamis dollars. There was no takeoff at any time and the three regimes of growth postulated by Galor (2005a, 2011) did not exist.*

These calculations, supported by data, clearly demonstrate that the Industrial Revolution had no impact on the economic growth trajectory. Impacts were of different kind but the data show that the Industrial Revolution did not boost the global economic growth. It did not even boost the economic growth in Western Europe (Nielsen, 2014), or in any other region (Nielsen, 2016a, 2016g) or even in the United Kingdom (Nielsen, 2016h), the very centre of this revolutions, where its effects on the growth trajectory should have been most clearly pronounced. There was no impact whatever on the growth trajectories. Economic growth must have been prompted and controlled by some other force, which was much stronger than any other forces, including the force of the Industrial Revolution and this force is discussed in a separate publication (Nielsen, 2016j). Furthermore, Galor's three regimes of growth did not exist.

Fundamental postulates of the Unified Growth Theory (Galor, 2005a, 2011) are contradicted by the analysis of data, the same data as used but not analysed during the formulation of his theory. Unified Growth Theory describes and explains phenomena that did not exist and consequently it does not explain the historical economic growth. It is an incorrect and misleading theory.

The discussion of socio-economic conditions presented by Galor might be interesting for some other reason but there is a clear evidence in the GDP and GDP/cap data that his discussion has no relevance to explaining the mechanism of economic growth. His discussed associations and correlations are not just questionable but plainly incorrect because they are contradicted by data he used but never analysed.

Economic growth was indeed slow over a long time and fast over a short time but it is incorrect to divide this monotonically increasing distribution into three regimes and claim distinctly different mechanisms for the arbitrarily selected sections. It is also incorrect to claim that there was a takeoff at a certain time. The data and their analysis give no scientific basis for such claims.

Historical economic growth has to be explained using a *single mechanism*. Such a mechanism should describe the slow and fast growth including the apparent transition. All these "parts" should be treated as one. Only then we could claim that we have explained the mechanism of the historical economic growth.

Dividing the past growth into three different regimes and claiming three different mechanisms is unsupported by data and it does not explain the mechanism of the historical economic growth. A truly unified growth theory will have to be based on a *single mechanism*. Such an explanation is proposed in a separate publication (Nielsen, 2016j).

6. Summary and conclusions

The aim of our discussion was to explain the puzzling features of the GDP/cap distributions. They show a slow growth over a long time, followed by a rapid increase. These features create a significant problem with their interpretations, and the outstanding example of the created confusion is the Unified Growth Theory (Galor, 2005a, 2011). Our discussion was based on precisely the same data which were used, but not analysed, during the formulation of this theory. It is both surprising and disappointing that while using excellent sets of data published by the world-renown economist (Maddison, 2001), Galor made no attempt to adopt scientific approach to developing his theory.

Historical economic and population growth, global and regional, show a clear preference for increasing along hyperbolic trajectories (Nielsen, 2014, 2016a, 2016d, 2016g, 2016h; 2016i). Hyperbolic growth contains singularity, when a growing entity escapes to infinity at a fixed time. We might think that such a growth is impossible but we have to accept the evidence in data. The past growth of the GDP and of population were hyperbolic. There is absolutely no problem with accepting hyperbolic growth for two reasons: (1) hyperbolic growth is obviously possible because it is demonstrated convincingly by data and (2) growth trajectories can change and there is nothing strange or unusual about it. Indeed, recently, hyperbolic growth was diverted to slower trajectories (Nielsen, 2016a, 2016d, 2016i).

It is remarkable, that this apparently impossible (because of its singularity) hyperbolic growth was possible for the most part of the past 12,000 years (Nielsen, 2016i). Every time it was interrupted, and it happened only twice in the past, it was converted again to a hyperbolic growth. Now, it is interrupted again but the future trajectory is yet unknown.

We have discussed mathematical properties of the historical GDP/cap distributions. We have explained how they should be analysed and interpreted.

If both components of the GDP/cap indicator increase hyperbolically, then the GDP/cap distributions represent a ratio of hyperbolic trajectories. We have a consistent evidence in data that the economic growth and the growth of population were hyperbolic (Nielsen, 2016a, 2016d, 2016i; von Foerster, Mora & Amiot, 1960). The characteristic features created by the division of hyperbolic distributions may be confusing but they can be easily explained. Data have to be analysed. Presenting them in a grossly distorted way is self-defeating and it leads to incorrect conclusions (Ashref, 2009; Galor, 2005a, 2005b, 2007, 2008a, 2008b, 2008c, 2010, 2011, 2012a, 2012b, 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008, Snowdon & Galor, 2008).

We have explained how to understand the confusing features of the historical GDP/cap distributions. They can be interpreted simply as the linearly-modulated hyperbolic GDP distributions. Linear modulation is by the reciprocal values of population data. We have discussed how these distributions can be analysed, how their features can be explored and explained.

As an illustration of our discussion, we have investigated the data (Maddison, 2001) used in developing the Unified Growth Theory (Galor, 2005a, 2011). In his theory, Galor discusses various socio-economic concepts of growth but his theory does not explain the mechanism of economic growth because it is based firmly on the misinterpretation of the purely mathematical features of hyperbolic distributions. His discussion of socio-economic issues might be interesting, for various reasons, but it has no relevance to explaining the mechanism of the economic growth because changes in socio-economic conditions had no effect on the economic growth trajectory as manifested by the available data (Maddison, 2001), the same data, which were used, but not analysed, during the formulation of the Unified Growth Theory.

Galor's speculations about socio-economic processes are strongly guided by phantom features created by the deliberately distorted presentations of data (Ashref, 2009; Galor, 2005a, 2005b, 2007, 2008a, 2008b, 2008c, 2010, 2011, 2012a, 2012b, 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008). Different stages of growth claimed in this theory did not exist. Their claimed presence is contradicted by the same data, which were used during the development of the Unified Growth Theory and in all other related publications, as listed above.

In general, the GDP and population, global, regional and even in individual countries, were increasing monotonically and consequently the GDP/cap ratios are also represented by monotonically increasing distributions governed by a single mechanism of growth.

Unified Growth Theory does not explain the historical economic growth because it is critically and inflexibly based on the deliberately constructed phantom features, which are contradicted by data. In particular, the three regimes of growth claimed by this theory did not exist and there were no takeoffs in the economic growth or in the growth of population. This theory describes a phantom world but presents it as real. Stories and explanations presented in the Unified Growth Theory might sound plausible but they are contradicted by data.

Historical GDP/cap distributions might look puzzling and complicated but they are in fact simple distributions. Their puzzling features are nothing more than just the mathematical features created by dividing two hyperbolic distributions. Their mechanism might also look complicated but hyperbolic distributions are described by exceptionally simple mathematical formula and the mechanism of these distributions, as representing the historical economic growth and the historical growth of population, is also simple (Nielsen, 2016j).

Historical hyperbolic economic growth can be explained as having been propelled by the simplest possible market force where the growth of the GDP (or the on average growth of the common wealth) is prompted by the force directly proportional to the already existing size of the GDP. On average, wealth was generating wealth directly proportionally to the existing wealth. Historical hyperbolic growth of the population can be explained as having been propelled by the simplest force of procreation (the combination of the natural sex drive combined with the natural process of aging and dying), which on average was constant per person. Historical growth of income per capita, expressed as the GDP/cap, can be explained as having been prompted by the combination of these two forces, and the puzzling features of the GDP/cap distributions turn out to be nothing more than the mathematical properties of dividing two hyperbolic distributions. If these simplest forces of growth are combined with some other strong forces, as it is now, the economic growth and the growth of population are no longer hyperbolic.

The current GDP/cap values are still increasing but the shapes of their distributions cannot be explained by the mathematical properties of diving two hyperbolic distributions because we are no longer dealing with hyperbolic distributions. However, in principle, their shapes could be reproduced by dividing mathematical distributions describing the current growth of the GDP and population. However, the underlying mechanism of any of them is now no longer simple and the mechanism of the *current* growth of income per capita is also no longer simple.

References

Ashraf, Q.H. (2009). *Essays on Deep Determinants of Comparative Economic Development*. Ph.D. Thesis, Department of Economics, Brown University, Providence.

Floud, D. & McCloskey, D.N. (1994). *The Economic History of Britain since 1700*. Cambridge: Cambridge University Press.

Galor, O. (2005a). From stagnation to growth: Unified Growth Theory. In P. Aghion & S. Durlauf (Eds.), *Handbook of Economic Growth* (pp. 171-293). Amsterdam: Elsevier.

Galor, O. (2005b). The Demographic Transition and the Emergence of Sustained Economic Growth. *Journal of the European Economic Association*, *3*, 494-504. doi. 10.1162/jeea.2005.3.2-3.494

Galor, O. (2007). Multiple Growth Regimes-Insights from Unified Growth Theory. *Journal of Macroeconomics*, 29(3), 470-475. doi. 10.1016/j.jmacro.2007.06.007

Galor, O. (2008a). Comparative Economic Development: Insight from Unified Growth Theory. [Retrieved from].

Galor, O. (2008b). Economic Growth in the Very Long Run. In: Durlauf, S.N. & Blume, L.E., Eds., *The New Palgrave Dictionary of Economics*, Palgrave Macmillan, New York. doi. 10.1057/9780230226203.0434

Galor, O. (2008c). Comparative Economic Development: Insight from Unified Growth Theory. [Retrieved from].

Galor, O. (2010). The 2008 Lawrence R. Klein Lecture—Comparative Economic Development: Insights from Unified Growth Theory. *International Economic Review*, 51(1), 1-44. doi. 10.1111/j.1468-2354.2009.00569.x

Galor, O. (2011). *Unified Growth Theory*. Princeton, New Jersey: Princeton University Press.

Galor, O. (2012a). Unified Growth Theory and Comparative Economic Development. [Retrieved from].

Galor, O. (2012b). The Demographic Transition: Causes and Consequences. *Cliometrica*, 6(1), 1-28. doi. 10.1007/s11698-011-0062-7

Galor, O. (2012c). Unified Growth Theory and Comparative Economic Development. [Retrieved from].

Galor, O. & Moav, O. (2002). Natural Selection and the Origin of Economic Growth. *The Quarterly Journal of Economics*, 117(4), 1133-1191. doi. 10.1162/003355302320935007

Maddison, A. (2001). *The World Economy: A Millennial Perspective*. Paris: OECD.

Maddison, A. (2010). Historical Statistics of the World Economy: 1-2008 AD. [Retrieved from].

Malthus, T. R. (1798). *An Essay on the Principle of Population*. London: J. Johnson.

Nielsen, R.W. (2014). Changing the paradigm. *Applied Mathematics*, 5, 1950-1963. doi. 10.4236/am.2014.513188

Nielsen, R.W. (2016a). Mathematical analysis of the historical economic growth with a search for takeoffs from stagnation to growth. *Journal of Economic Library,* 3(1), 1-23.

Nielsen, R.W. (2016b). Unified Growth Theory contradicted by the absence of takeoffs in the Gross Domestic Product. *Economic Review,* 3(1), 16-27.

Nielsen, R. W. (2016c).The postulate of the three regimes of economic growth contradicted by data. *Journal of Economic and Social Thought,* 3(1), 1-34.

Nielsen, R.W. (2016d).Unified Growth Theory contradicted by the mathematical analysis of the historical growth of human population. *Journal of Economics and Political Economy,* 3(2), 242-263.

Nielsen, R.W. (2016e). Puzzling properties of the historical growth rate of income per capita explained. *Journal of Economics Library,* 3(2), 241-256.

Nielsen, R.W. (2016f). The unresolved mystery of the great divergence is solved. *Journal of Economic and Social Thought,* 3(2), 196-219.

Nielsen, R.W. (2016g). Mathematical analysis of the historical income per capita distributions. *Economic Review*, 3(2), 300-319.

Nielsen, R.W. (2016h). Industrial Revolution did not boost the economic growth or the growth of population even in the United Kingdom. *Journal of Economic Bibliography,* 3(4), 434-446.

Nielsen, R.W. (2016i). Growth of the world population in the past 12,000 years and its link to the economic growth. *Journal of Economics Bibliography,* 3(1), 1-12.

Nielsen, R. W. (2016j). Mechanism of Hyperbolic Growth Explained. *Journal of Economic Library,* 3(4), 411-428.

Snowdon, B. & Galor, O. (2008). Towards a Unified Theory of Economic Growth. *World Economics*, 9, 97-151.

von Foerster, H., Mora, P., & Amiot, L. (1960). Doomsday: Friday, 13 November, A.D. 2026. *Science,* 132, 1291-1295.