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Economic Growth and the Growth of Human Population in the Past 2,000,000 Years

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Abstract. Growth of human population in the past 2,000,000 years is analysed. It is shown that the growth was in three major stages: (1) 2,000,000 to 27,000 BC, (2) 27,000 BC to AD 510 and (3) AD 510 to present. Each stage is described by hyperbolic distribution followed by a significantly shorter, non-hyperbolic transition to a new stage. Data show also a minor disturbance in the third hyperbolic stage. Each hyperbolic stage was prompted by a single force, the biologically-controlled force of procreation expressed as the difference between the biologically-controlled force of sex drive and the biologically-controlled process of aging and dying. The fundamental parameter describing hyperbolic growth is given by the ratio of the force of growth and of the resistance to growth. It is assumed that during transitions, this fundamental force remained the same but the resistance to growth was changing. All these three stages, and the minor disturbance in the middle of the third stage, are now described mathematically and explained. The derived parameters are used to calculate the size of the world population in the past 2,000,000 years and to fill in the gaps between data. These parameters can be used to calculate the growth rate at any time in the past 2,000,000 years. Analysis of population data and the earlier analysis of the Gross Domestic Product (GDP) per capita allow also for the evaluation of the economic growth in the past 2,000,000 years. The size of the population and the GDP values are tabulated.
Keywords. Growth of human population, Economic growth, Hyperbolic growth, Mechanism of hyperbolic growth.
JEL. A10, A12, A20, C12, Y80.

1. Introduction

The aim of this publication is to analyse the growth of human population and the associated economic growth in the past 2,000,000 years. This work is an extension of our previous analysis of the growth of human population in the past 12,000 years (Nielsen, 2016a) and of the analysis of economic growth during the AD era (Nielsen, 2016b). These earlier studies demonstrated that the natural tendency for the growth of human population and for the economic growth is to follow hyperbolic distributions. Hyperbolic growth can be faster or slower but it is always prompted by the fundamentally the same mechanism.

We have shown that the mechanism of hyperbolic growth of human population can be easily explained (Nielsen, 2016c). It is a growth prompted by just one indispensable force, the biologically-controlled force of procreation expressed as a difference between the biologically-controlled force of sex drive and the biologically-controlled process of aging and dying. No other forces are needed. A change in the growth trajectory occurs only if other forces interfere substantially with this fundamental force of growth. In the past 12,000 years, there was *only one strong* interference, around AD 1, and *one minor* interference, around AD 1300. Each time, the fundamental character of the growth trajectory was not changed.

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There was only a transition from one hyperbolic trajectory to another. The first time, it was a transition from a fast to a slow hyperbolic growth, while the second time it was a transition to only a slightly faster growth. With the exception of these two, relatively brief transitions, the growth was always hyperbolic. In addition to these past transitions we now experience a new strong interference reflected in the gradual slowing down growth. The growth of population is no longer hyperbolic but it is still close to the historical hyperbolic growth. Now, we are going to demonstrate that hyperbolic growth prevailed not only in the past 12,000 years but also during the past 2,000,000 years.

2. Data for the early growth of population

If data for the BC era down to 10,000 BC are scarce, data beyond that time are even more difficult to find. However, we now have a few estimates from reputable sources and we can use them to extend the analysis of the growth of human population down to 2,000,000 million years ago.

The earliest estimates were made by Deevey (1960). He estimated that during the Lower Palaeolithic (around 1,000,000 years ago) the size of population was 0.125 million, during the Middle Palaeolithic (around 300,000 years ago) it was 1 million and 3.34 million during the Upper Palaeolithic (around 25,000 years ago). Birdsell (1972) estimated 0.4, 1 and 2.2 million for the same years, respectively, while Hassan (1981) estimated 0.6, 1.2 and 6. In 2002, he estimated 0.4, 0.8, 1.2 and 3.3 million at 1,500,000, 1,000,000, 100,000 and 14,000 years ago, respectively (Hassan, 2002). In our calculations, we shall use his updated estimates (Hassan, 2002). Incidentally, it should be noted that his two values listed in his Table 17.2 (Hassan, 2002, p. 684) are clearly misplaced. The values of 0.4 and 0.8 million should have been aligned with 1,500,000 and 1,000,000 respectively. However, his diagram presented as Figure 17.2 is correct.

All these estimates are listed in Table 1. The corresponding years are expressed as BC. The expression *years ago* or *before present* are interpreted as before 2000. The years 1,500,000, 1,000,000 and 300,000 years ago or before present are interpreted as 1,500,000 BC, 1,000,000 BC and 300,000 BC. The values for the years after100,000 BC were reduced by 2000.

Table 1. *Estimates of the size of population before 10,000BC(in million)*

Year (BC)	Deevey (1960)	Birdsell (1972)	Hassan (2002)	Average Values
1,500,000			0.4	0.4
1,000,000	0.125	0.4	0.8	0.44
300,000	1	1		1
100,000			1.2	1.2
23,000	3.34	2.2		2.77
12,000			3.3	3.3

Reciprocal values of these data are shown in Figure 1. This figure shows also data used in the earlier analysis (Nielsen, 2016a). As pointed out earlier (Nielsen, 2014), linearly decreasing reciprocal values identify hyperbolic growth, because hyperbolic growth is described by the reciprocal of a decreasing straight line:

$$S(t) = \frac{1}{a - kt} \tag{1}$$

where $S(t)$ is the size of the growing entity, in our case the size of population or the size of economic growth, t is the time and a and k are positive constants.

In Figure 1 we see two straight lines. They cross at 34,350 BC. Around that time there was a transition from a slow hyperbolic trajectory to a significantly faster growth, also described by hyperbolic trajectory. This transition was one of only two *major* transitions in the past 2,000,000 years. The later major transition was around AD 1 (Nielsen, 2016a). A closer view of this first earlier transition is shown in Figure 2.

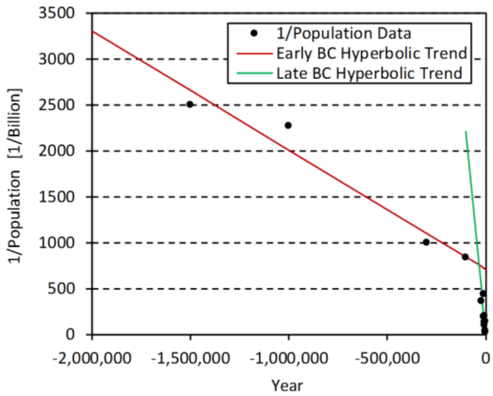

Figure 1. *Reciprocal values of the size of population between 1,500,000 BC and 1000 BC. Decreasing straight lines for the reciprocal values identify hyperbolic trajectories (Nielsen, 2014). The two trajectories cross at 34,350 BC marking a transition from a slow to a fast hyperbolic trajectory. The late BC trajectory was discussed earlier (Nielsen, 2016a). The BC years are represented by negative numbers.*

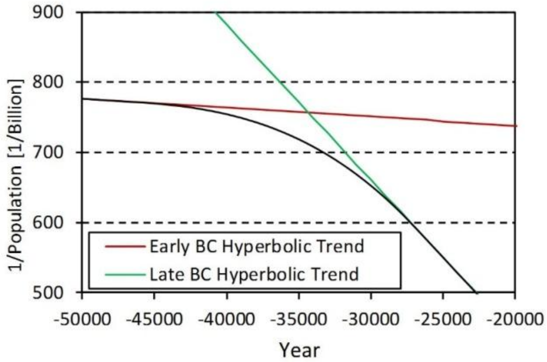

Figure 2. *The first major demographic transition. It occurred between 46,000 BC and 27,000 BC. It was a transition from a slow hyperbolic growth to a significantly faster hyperbolic growth, which prevailed until 425 BC to be replaced in AD 510 by a significantly slower hyperbolic growth during the AD era (Nielsen, 2016a).The BC years are represented by negative numbers.*

This early transition commenced around 46,000 BC and continued until around 27,000 BC. From around that year, the growth of the world population started to follow a significantly faster hyperbolic trajectory. These new data confirm that the natural tendency for the growth of population is to follow hyperbolic distributions.

3. Growth of human population in the past 2,000,000 years

3.1. Overview

In Figure 3 we show the average values of data describing the growth of the world population in the past 2,000,000 years (Biraben, 1980; Birdsell, 1972; Clark, 1968; Cook,1960; Deevey, 1960; Durand, 1974; Gallant, 1990; Hassan, 2002; Haub, 1995; Livi-Bacci, 1997; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994, United Nations, 1973; 1999; 2013; US Census Bureau, 2017). The time scale is in years before 2100. We also display the best fit to the data, which most of the time is hyperbolic. We can see these hyperbolic distributions more clearly in Figure 4. The fit presented in Figure 3, combined with the exceptionally slow growth during the first stage, allows for the extension of the growth of population to 2,000,000 years before 2100 or to approximately to 2,000,000 BC.

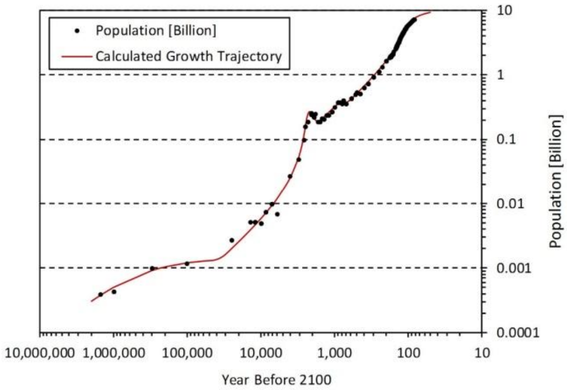

Figure 3. *Growth of human population in the past 2,000,000 years.*

Growth of human population in the past 2,000,000 was in three major stages but it was not in the stages imagined by Deevey (1960). It is remarkable that based on a strongly limited information he did realise that the growth of population was in three major stages. However, while being close to the correct interpretation of the growth of population, Deevey imagined the three stages incorrectly. He imagined that each stage was leading to an equilibrium, i.e. to a plateau in the growth of population as shown in Figure 5. This figure is based on his conceptual diagram (Deevey, 1960, p. 198). If we compare his interpretation of growth shown in Figure 5 with the growth presented in Figure 3, we can see that the growth was indeed in three stages as suggested by Deevey but the details of the growth trajectories are clearly different. Only the first stage looks similar to the stage suggested by Deevey. However, as we shall explain later, this stage also did not lead to an equilibrium.

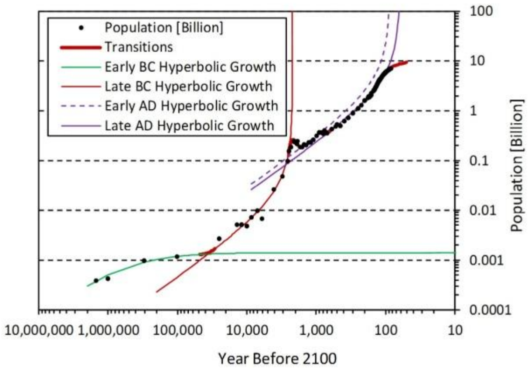

Figure 4. *The three major stages of growth of the world population in the past 2,000,000 years: (1) between 2,000,000 BC and 27,000 BC, (2) between 27,000 BC and AD 510 and (3) between AD 510 and present. The last stage experienced a minor distortion between around AD 1195 and 1470. This distortion caused a small shift in the hyperbolic growth.*

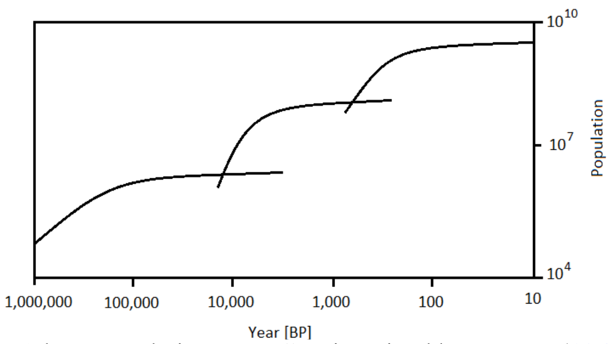

Figure 5. *The "three population surges" as imagined by Deevey (1960, p. 198).*

It is remarkable, that the currently established knowledge (Nielsen, 2016g), which is based on the doctrine of Malthusian stagnation, ignores not only results of von Foerster, Mora & Amiot (1960) but also the observation of Deevey (1960). These two early studies clearly demonstrated that there was no stagnation in the growth of population. They indicated that there was a regular and well-defined pattern of growth, which contradicts the doctrine of Malthusian stagnation.

Results of von Foerster, Mora & Amiot (1960) demonstrated that the growth of human population during the AD era was hyperbolic, and consequently not stagnant. It followed a monotonically increasing trajectory. Hyperbolic growth is in the direct contradiction of the concept of Malthusian stagnation.

Hyperbolic growth is slow over a long time (but not stagnant) and fast over a short time, so fast that it escapes to infinity at a fixed time. It is the growth, which is governed by the same mechanism when it is slow and when it is fast. If we want to interpret the slow growth as stagnant we should apply the same interpretation to the fast growth. It would be obviously ludicrous to describe the growth escaping to infinity at a fixed time as stagnant, but because it is always the same growth, then it is also ludicrous to describe it as stagnant when it is slow. The concept of Malthusian stagnation is based on the incorrect interpretation of hyperbolic growth and has no place in science.

Results of Deevey indicated that over a longer time, extending as far back as to 1,000,000 years ago, the growth was in three distinctly different stages. It is again a clearly different pattern than the pattern suggested by the concept of Malthusian stagnation. The doctrine of Malthusian stagnation claims an endless stagnant state of growth, characterised by unpredictable, random fluctuations often described as Malthusian oscillations. Hyperbolic growth is definitely predictable and consequently it suggests an entirely different interpretation of the mechanism of growth.

Studies of von Foerster, Mora and Amiot (1960) and of Deevey (1960) indicated that there was nothing chaotic about the growth of population. They indicated that there was a certain regular pattern. Such a regular pattern can hardly be expected to be produced by random forces of growth.

In order to understand the growth of population in the past 2,000,000 years, it is useful to discuss separately its three stages of growth as presented in Figure 4: (1) between 2,000,000 BC and 27,000 BC, (2) between 27,000 BC and AD 510, and (3) between AD 510 and present. Each of these stages is described by hyperbolic growth followed by a transition to the next stage. However, the last stage contains a fine structure expressed as a slight shift in the hyperbolic distribution.

3.2. Mathematics of growth

Parameters describing the growth of population in the past 2,000,000 years are listed in Table 2. They are: a and k, for the hyperbolic growth and a_i ($i = 0$ to n) and b_i ($i = 0$ to $n+1$) for transitions. They can be used to calculate the size of population $S(t)$ and the growth rate $R(t)$ at any given time. For these parameters, the size of population is in billions. The time is in years and it is positive for the AD era and negative for the BC era.

Table 2 presents also the range of $S(t)$ and $R(t)$ values for hyperbolic distributions, which are also the range of values for transitions, because the end of a given hyperbolic growth is the beginning of a transition while the beginning of a hyperbolic growth is the end of a preceding transition.

Mathematics of growth of population is exceptionally simple. As discussed earlier (Nielsen, 2016a, 2016c) and as shown in Figures 3 and 4, the growth was hyperbolic, except when there was a relatively brief transition. Hyperbolic growth is described by a very simple mathematical expression, presented as eqn (1), which is a solution of a very simple differential equation:

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = kS(t). \qquad (2)$$

Transitions are described by a similar differential equation:

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = k(t)S(t). \qquad (3)$$

Table 2. *Parameters describing the growth of population in the past 2,000,000 years.*

Hyperbolic Growth		Transitions	
$S(t)=(a-kt)^{-1}$; $R(t)=kS(t)$		$S(t)=\left[\sum_{i=0}^{n+1} b_i t^i\right]^{-1}$; $k(t)=\sum_{i=0}^{n} a_i t^i$; $R(t)=k(t)S(t)$	
Years	Parameters	Years	Parameters
2.000,000 –	$a=7.120\times10^{2}$	46,000 – 27,000 BC	$b_0=-9.247\times10^{2}$ $b_1=-9.990\times10^{-2}$
46,000 BC	$k=1.296\times10^{-3}$		$b_2=-1.966\times10^{-6}$ $b_3=-1.295\times10^{-11}$
2,000,000 BC	$S(t)=3.027\times10^{5}$		$a_0=9.990\times10^{-2}$ $a_1=-1.808\times10^{-1}$
	$R(t)=3.923\times10^{-5}\%$		$a_2=3.885\times10^{-11}$
46,000 BC	$S(t)=1.296\times10^{6}$		
	$R(t)=1.680\times10^{-4}\%$		
27,000 – 425 BC	$a=2.282\times10^{0}$	425 BC – AD 510	$b_0=3.834\times10^{0}$ $b_1=2.347\times10^{-3}$
	$k=2.210\times10^{-2}$		$b_2=1.330\times10^{-5}$ $b_3=-2.493\times10^{-8}$
27,000 BC	$S(t)=1.682\times10^{6}$		$a_0=-2.347\times10^{-3}$ $a_1=-2.659\times10^{-5}$
	$R(t)=3.718\times10^{-3}\%$		$a_2=7.479\times10^{-8}$
425 BC	$S(t)=1.406\times10^{8}$		
	$R(t)=3.108\times10^{-1}\%$		
AD 510 – 1195	$a=6.940\times10^{0}$	AD 1195 – 1470	$b_0=-2.903\times10^{2}$ $b_1=1.022\times10^{0}$
	$k=3.448\times10^{-3}$		$b_2=-1.309\times10^{-3}$ $b_3=7.326\times10^{-7}$
AD 510	$S(t)=1.930\times10^{8}$		$b_4=-1.517\times10^{-10}$ $a_0=-1.022\times10^{0}$
	$R(t)=6.654\times10^{-2}\%$		$a_1=2.618\times10^{-3}$ $a_2=-2.198\times10^{-6}$
	$S(t)=3.546\times10^{8}$		$a_3=6.068\times10^{-10}$
	$R(t)=1.223\times10^{-1}\%$		
AD 1470 – 1950	$a=9.123\times10^{0}$	AD 1950 – 2016	$b_0=2.001\times10^{3}$ $b_1=-2.928\times10^{0}$
	$k=4.478\times10^{-3}$		$b_2=1.428\times10^{-3}$ $b_3=-2.323\times10^{-7}$
AD 1470	$S(t)=3.935\times10^{8}$		$a_0=2.928\times10^{0}$ $a_1=-2.856\times10^{-3}$
	$R(t)=1.762\times10^{-1}\%$		$a_2=6.968\times10^{-7}$
AD 1950	$S(t)=2.550\times10^{9}$		
	$R(t)=1.142\times10^{0}\%$		

$S(t)$ - the size of population. $R(t)$ - the growth rate. In mathematical formulae, time is in years and it has positive values for the AD era and negative for the BC era. Furthermore, for the listed parameters, the size of population is in billions. The growth rate given by the mathematical formulae is not expressed in per cent.

Parameter *k*, whether constant or dependent on time, is the driving force divided by the resistance to growth (Nielsen, 2016c). For the growth of population, the driving force is the force of procreation given by the difference between the *biologically controlled* force of sex drive and the *biologically controlled* aging and dying. It is a spontaneous, unrestrained and fundamental force of growth, which has to be considered in any attempt to explain the mechanism of growth of human population. Other forces may be added but only if necessary, i.e. if this fundamental force is unable to explain the mechanism of growth. The study presented here and in earlier publications demonstrates that this force alone explains why the growth of population was, most of the time, hyperbolic (Nielsen, 2016a; 2016c; 2016d; von Foerster, Mora & Amiot, 1960).

During transitions, the fundamental force of procreation does not change. There is no need to assume that it does. Only the resistance to growth is changing and this change is described by $k(t)$.

In the past, every change in the resistance to growth was leading to a new, constant resistance and consequently to a new hyperbolic growth. The current transition, which commenced around 1950, also describes a change in the resistance to growth but the future trajectory is unknown.

The solution of the eqn (3) is given by the following expression:

$$S(t) = -\frac{1}{\int k(t)dt} . \qquad (4)$$

In the simplest case, when $k(t) = k = const$, the eqn (3) is the same as eqn (2) and the solution (4) is the same as eqn (1). It is the reciprocal of a linear function.

If we assume that $k(t)$ is represented by the *n*-order polynomial, if

$$k(t) = \sum_{i=0}^{n} a_i t^i \qquad (5)$$

then

$$S(t) = \left[\sum_{i=0}^{n+1} b_i t^i \right]^{-1} . \qquad (6)$$

We should also notice that eqns (2) and (3) describe the growth rate $R(t)$. Thus, if we know the size of the population and k or $k(t)$, we can also calculate the corresponding growth rate at a given time:

$$R(t) = k(t)S(t) . \qquad (7)$$

For the hyperbolic growth [i.e. for the first-order hyperbolic growth given by the eqn (1)] $k(t) = k = const$ and the growth rate is directly-proportional to the size of population.

3.3. Stage 1: 2,000,000-27,000 BC

This stage is made of a hyperbolic growth between 2,000,000 BC and 46,000 BC followed by a transition to the next stage. The transition was between 46,000 BC and 27,000 BC (see Figures 2 and 4).

In Figures 3 and 4, this stage looks different than the other two stages and it resembles the distribution outlined by Deevey (see Figure5). However, it is just an illusion created by using logarithmic scales of reference and Deevey appears to have been misguided by this illusion. He imagined that it was a fast growth followed by an equilibrium, or a plateau. However, the calculated curve shown in Figures 3 and 4 is hyperbolic. It was not a fast growth followed by equilibrium but a *monotonically increasing* growth.

We know that the growth in the first stage was hyperbolic because we have shown earlier (see Figure 1) that the reciprocal values of the size of the population during that time were following closely a straight line, which identifies hyperbolic growth (Nielsen, 2014). Why then does this stage look so much different? How to explain the peculiar shape presented in Figures 3 and 4?

First, it is important to notice that hyperbolic growth during this first stage was exceptionally slow, so slow that if continued it would not escape to infinity until around AD 549,391. *Second*, we have to remember that logarithmic scales, while being useful in displaying a wide range of data, they also introduce unavoidable distortions. In Figure 3 and 4 we have *double distortion* because we are using *two* logarithmic scales. The further we go back in time to stronger is the compression of the displayed data, but there is also an increasing compression of the displayed size of the population as we move up along the vertical scale.

Every marked section of the first (left-most) cycle of the horizontal scale represents a compression of 1,000,000 years. The vertical scale introduces similar distortion but in reverse order. Here the first cycle represents an exceptionally stretched scale. This compressing and stretching, combined with the exceptionally slow growth during the first stage creates an illusion of a fast growth followed by an equilibrium, illusion so strong that it caused Deevey not only to see an incorrect pattern but also to try to explain its mechanism.

A simple way to dispel this illusion is to use linear scales as shown in Figure 6. In this figure, we present precisely the same data (for the Stage 1) and the same hyperbolic distribution as shown in Figures 3 and 4 but now we use linear scales for the time and for the size of the population.

We can now see clearly that the growth of population was *increasing monotonically*. There is obviously no sign of any plateau or equilibrium and no hope of having such a plateau in the future because the growth was hyperbolic, escaping to infinity at a fixed time. Deevey's claim of plateaus and his attempts to explain their mechanism was based on illusion.

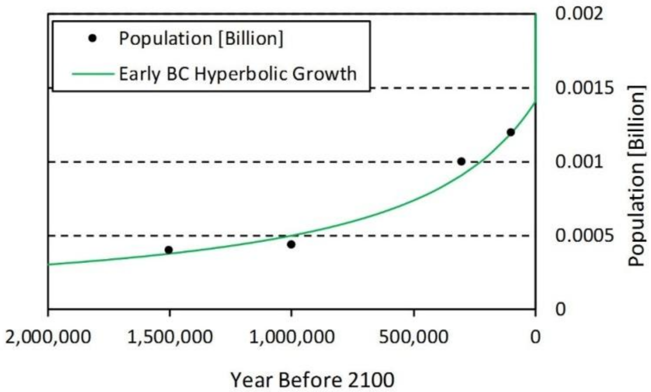

Figure 6. *The first stage of hyperbolic growth between 2,000,000 BC and 46,000 BC displayed using linear scales for the time and for the size of population. The illusion of a fast growth followed by an equilibrium created by the double-logarithmic scales used by Deevey (1960, p.198) and in Figures 3 and 4 has now disappeared. It is now clear that the growth is increasing monotonically and that it does not lead to an equilibrium.*

During this early stage of the BC growth, the size of population increased from the estimated 0.4 million in 1,500,000 BC to 1.3 million in 46,000 BC. The calculated value in 1,500,000 BC is 0.38 million. If we extend the fitted hyperbolic distribution to 2,000,000 BC, then the calculated size of population in that year is

0.3 million. If continued, the size of population would increase to one billion in AD 548,620.

Reciprocal values of data shown in Figure 1 demonstrates that this exceedingly slow hyperbolic growth was replaced by a much faster growth. The transition occurred between 46,000 BC and 27,000 BC (see Figure 2).

The size of the population during this transition increased from 1.3 million in 46,000 BC to 1.7 million in 27,000 BC. This transition converted the exceedingly slow hyperbolic growth during the first stage to a 17 times faster hyperbolic growth (as measured by the parameter k) during the second stage, the difference in the intensity of growth reflected in the distinctly different values of the gradient of the reciprocal values of the size of the population shown in Figure 1. During that time, the resistance to growth *decreased* by a huge factor of around 17 and starting from around 27,000 BC the growth of population was much faster than before 46,000 BC.

The timing of this transition agrees well with archaeological and anthropological data. Even though the emergence of modern humans is claimed to have been between 150,000 and 200,000 years ago (Mellars, *et al.,* 2007) the progress in their development was slow until around 50,000 BC, as demonstrated by archaeological evidence (Klein, 1989; 1995; Mellars, 1989; Stringer & Gamble, 1993). Human evolution appears to have experienced a great leap forward around that time.

For a long time since their emergence, modern humans were not much different than other hominins "and it was only around 50,000-40,000 years ago that a major behavioral difference developed" (Klein, 1995, p. 167). This first transition in the hyperbolic growth appears to coincide also with the extinction of Neanderthals, first in Europe and later in and later in other parts of the world, marking the beginning of the undisputed domination of *Homo sapiens* (Higham, *et al.*, 2014).

Forces operating during the first transition between 46,000 BC and 27,000 BC from an earlier large resistance to growth before around 46,000 BC to significantly smaller resistance after around 27,000 BC were of a social and intellectual nature. The long race between different representatives of hominins was over. One by one they were left behind and became extinct. Finally, the last two remaining were *Homo floresiensis* and *Homo neanderthalensis* but they also were eliminated or virtually eliminated around the time of the beginning of the first transition, i.e. around 50,000 BC. Now, only modern humans, represented by *Homo sapiens*, remained. The first transition, between 46,000 BC and 27,000 BC was a transition to a new era of the exceptionally fast and long-lasting hyperbolic growth, the unique growth which was never to be repeated.

This complete freedom of growth was eventually restricted, not by forces of nature and not by the competition with other representatives of the genus *Homo* because they were extinct for a long time but by the strong competition between humans. However, in 27,000 BC, at the end of the first transition, this change was still long time into the future. The gained momentum of the free growth was to propel the growth of human population for many thousands of years.

3.4. Stage 2: 27,000 BC - AD 510

Stage 2 is made of a fast hyperbolic growth between 27,000 BC and 425 BC, followed by a transition to a slower hyperbolic trajectory during the AD era. The transition took place between 425 BC and AD 510. (In our earlier publications, we have labelled this transition as being roughly between 500 BC and AD 500.)

Hyperbolic growth between 27,000 BC and 425 BC was the fastest growth (as defined by the parameter k) in the past 2,000,000 years. During that time, the size of population increased from 1.7 in 27,000 BC million to 140 million in 425 BC, representing a nearly 82-fold increase. In contrast, there was only around 38-fold increase between AD 510 and present. If continued, this fast BC growth would escape to infinity at the end of 104 BC. We have come very close to experiencing the so-called population explosion at the end of the BC era.

In deciding which hyperbolic growth is fast we should not be confused by the growth during the AD era. It reached a higher size of population in a shorter time but we should remember that it also started with a significantly larger size of population, around 190 million, compared with only 1.7 million for the hyperbolic growth between 27,000 BC and 425 BC.

The transition between 425 BC and AD 510 can be described by the reciprocal of the third-order polynomial. During this transition, the resistance to growth *increased* by a factor of 6.4. As discussed earlier (Nielsen, 2016c), forces shaping this transition appear to have been of political nature. This transition coincides with the domination of Roman Empire over large areas surrounding the Mediterranean Sea. It also coincides with the accelerated process of the formation of countries in various parts of the world and with the rapidly changing political landscape (Teeple, 2002). From the complete freedom in around 27,000 BC, humans became slaves of their own design. They have invented many ways of self-destruction, bondage and oppression, which eventually led to a new hyperbolic growth characterised now by a larger resistance to growth. Humans appear to be their own best enemies and they might eventually cause their own extermination.

3.5. Stage 3: AD 510 - present

This stage is also made of a hyperbolic growth followed by a transition, which commenced around 1950. We have shown earlier (Nielsen, 2016a) that the growth of population between AD 510 and 1950 can be well described using a single hyperbolic distribution. However, we have also pointed out that there was a minor disturbance in this hyperbolic growth between around AD 1200 and 1400. This disturbance caused only a small shift in the hyperbolic growth (see Figure 4).

The best description of data between AD 510 and 1950 is given by two, approximately parallel hyperbolic trajectories separated by a small transition between around AD 1195 and 1470. The two hyperbolic trajectories, before AD 1195 and after AD 1470 are virtually identical. Measured by the parameter k, hyperbolic growth after AD 1470 was only 30% faster than the hyperbolic growth before AD 1195.

As discussed earlier (Nielsen, 2016a), this minor transition between AD 1195 and 1470 coincides with a unique event of a convergence of *five* demographic catastrophes. This is the only example showing a correlation between demographic catastrophes and the growth of population. However, the combined impact was small.

From around 1950, there was at first a small surge in the growth of population followed soon by a consistently slowing down growth. The data for the world population from that year are well documented by the US Bureau of Census (2017) but they can be also described using third-order polynomial with parameters listed in Table 2 (see Figure 7).

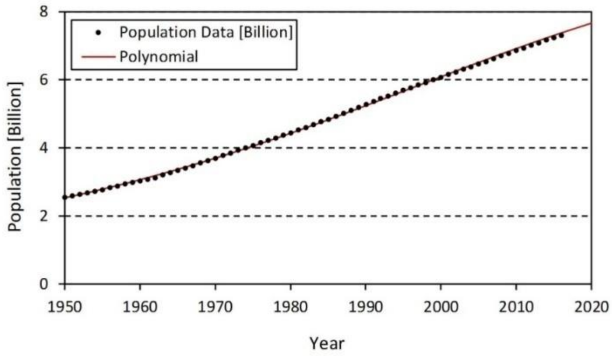

Figure 7. *Population data (US Census Bureau, 2017) are compared with the third-order polynomial distribution. Its parameters are listed in Table 2. This is just a mathematical description of data.*

This current transition appears to be associated with the increasing impact of Malthusian preventative checks (Malthus, 1798). The outcome of this transition is unknown. If the past pattern of growth is repeated, it could be a transition to a new hyperbolic trajectory. However, hyperbolic growth of population is possible only if the dominating force of growth is the biologically-controlled force of procreation. It is unlikely that this force alone will control the future growth of population. Under new conditions, with the increasing awareness of the need to control growth, the future growth of population could follow an entirely different trajectory. For the first time in human existence it will probably not be a hyperbolic growth.

The fitted distribution shown in Figure 3 with parameters listed in Table 2 can be now used to calculate the size of population at any time in the past 2,000,000 years. The calculated values are listed in Tables A1-A3 in the Appendix.

4. Economic growth in the past 2,000,000 years

De Long (1998) pointed out that income per capita (GDP/cap) can be used to estimate the past economic growth expressed in terms of the Gross Domestic Product (GDP). It is because the GDP/cap values quickly converge to an approximately constant value when we move back in time (see Figure 8). This property is nothing more than the mathematical property of dividing two hyperbolic distributions (Nielsen, 2017a) but it is useful for calculating the GDP values from the population data. What it simply means is that as we move back in time, the size of the GDP becomes approximately directly proportional to the size of the population. They follow virtually the same trajectories but displaced by an approximately constant factor.

Parameters describing the fitted GDP/cap distribution shown in Figure 8 are $a = 1.684 \times 10^{-2}$ and $k = 8.539 \times 10^{-6}$ for the GDP expressed in billions of the 1990 international Geary-Khamis dollars and $a = 7.739 \times 10^{0}$ and $k = 3.765 \times 10^{-3}$ for the Maddison's population data expressed in billions.

The fitted curve is a linearly modulated hyperbolic distribution (Nielsen, 2017a), which increases to infinity at a fixed time. For the distribution displayed in Figure 8, the point of singularity is in 1971. The growth of income per capita came close to this critical point but it bypassed it by a small margin of about 20 years. Income per capita continues now to increase along a new trajectory.

We can see that the calculated curve and the data representing the GDP/cap values quickly converge to a constant value when we move back in time. We can use this property to estimate the size of the GDP down to 2,000,000 BC. Results are presented in Tables A4-A6 in the Appendix.

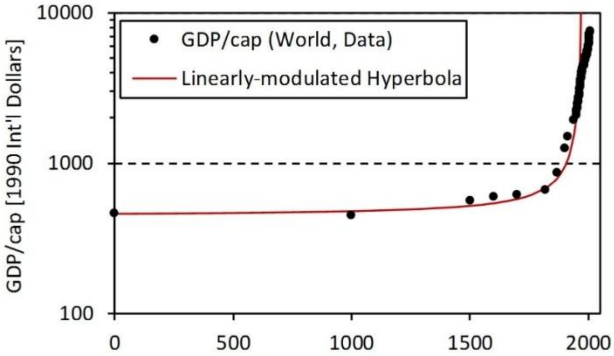

Figure 8. *Growth of the Gross Domestic Product per capita (GDP/cap) during the AD era. The full circles represent Maddison's data (Maddison, 2010) and the line is the best fit to the data (Nielsen, 2016e). The calculated curve is the linearly modulated hyperbolic distribution (Nielsen, 2017a).*

De Long (1998) carried out similar calculations. However, he used population data listed by Kremer (1993), which were taken from two sources: McEvedy & Jones (1978) and Deevey (1960). Our results are based on the analysis of all available data.

Furthermore, De Long assumed a constant GDP/cap value below AD 1500. This is good approximation below AD 1000 but not between AD 1000 and 1500. We shall use consistently the fitted trajectories but only below 1950 for two reasons: (1) good year-by-year data for the GDP starting from AD 1950 are already available (Maddison, 2010; GGDC, 2013) and (2) the calculated distribution of income per capita reproduces the data only up to 1950. From 1950, the GDP/cap values do not follow the linearly modulated hyperbolic distribution.

The GDP values presented in Tables A4-A6 are based on the best fits to the population data and to the GDP/cap data up to 1950. From 1950, the GDP values are as listed by Maddison (2010) for the years of up to 2008 and as calculated from the GDP/cap data listed by GGDC (2013) for 2009 and 2010 by using population data of the US Census Bureau (2017).

5. Summary and conclusions

We have carried out analysis of the growth of population in the past 2,000,000 years using data from a variety of sources (Biraben, 1980; Birdsell, 1972; Clark, 1968; Cook,1960; Deevey, 1960; Durand, 1974; Gallant, 1990; Hassan, 2002; Haub, 1995; Livi-Bacci, 1997; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994, United Nations, 1973; 1999; 2013; US Census Bureau, 2017). We have confirmed the earlier observation of Deevey (1960) that the growth of the world population was in three major stages. However, our analysis reveals that Deevey made a mistake by imagining that each stage was at first fast but then was reaching a certain equilibrium. Our analysis shows that each stage was *hyperbolic*. Each stage was *increasing monotonically* and was never levelling off to any form of equilibrium. On the contrary, if not terminated, hyperbolic distributions increase to infinity at a fixed time.

Nothing can increase to infinity. Consequently, any hyperbolic growth has to be, at a certain stage, terminated, which is not unusual because many other types of growth not only can but also are at a certain stage terminated. For instance, the better known exponential growth does not increase to infinity at a fixed time but if continued over a sufficiently long enough time, it leads to such large values that it becomes unsustainable.

The three stages of growth are: (1) 2,000,000 BC to 27,000 BC; (2) 27,000 BC to AD 510, and (3) AD 510 to present. Each of the listed stages includes a transition to a new growth. The transitions, as revealed by the analysis of data, are: (1) 46,000 BC to 27,000 BC, (2) 425 BC to AD 510, and (3) AD 1950 to present. During the third stage of growth, there was a minor transition between AD 1195 and 1470 but it only produced a slight shift in the hyperbolic trajectory.

Hyperbolic growth of population is generated by only one predominant force, the force of procreation, which is expressed as the difference between the ever-present, *biologically-controlled* force of sex drive and the *biologically-controlled* force of aging and dying (Nielsen, 2016c). This essential force has to be included in any explanation of the mechanism of growth of human population and it turns out that this force alone generates hyperbolic growth. As long as the growth remains hyperbolic, there is no need to include any other force. When a hyperbolic growth is being terminated or strongly disturbed, as between AD 1195 and 1470, other forces are strong enough to interfere with the usually dominant, biologically controlled, force of procreation.

Hyperbolic growth is characterised uniquely by parameter k [see eqn (1)]. This parameter is the ratio of the force of growth and of the resistance to growth. Working on the fundamental scientific principle of parsimony we can assume that during each hyperbolic growth the fundamental force of procreation per person remains unchanged and only resistance to growth is different. Transitions are associated with changing the resistance to growth. This change is described by the time-dependent parameter $k(t)$ [see eqn (3) and Table 2].

Each, of the first two stages of growth of human population in the past 2,000,000 years was terminated by a transition to a new hyperbolic growth. The third stage is now also being terminated. This transition commenced around 1950 but its outcome is unknown.

The first hyperbolic stage of growth was slow but during the first transition the resistance to growth decreased by a factor of around 17. The second stage was characterised by a fast hyperbolic growth, so fast that if continued it would have escaped to infinity around 104 BC. Fortunately, this fast hyperbolic growth was terminated. During the second transition, between 425 BC and AD 510, the resistance to growth increased by an approximate factor of 6.4. The new hyperbolic trajectory was significantly slower than the immediately preceding BC trajectory.

Each of the past two major transitions, as well as the current transition, appears to be associated with significant changes in the style of living. The first transition between 46,000 BC and 27,000 BC appears to have been associated with the surge in the evolution of *Homo Sapiens* (Klein, 1989; 1995; Mellars, 1989; Stringer & Gamble, 1993). Forces, which eventually reduced substantially the resistance to growth appear to have been of social and intellectual character. The second major transition between 425 BC and AD 510 appears to have been of political nature as reflected in the apparently intensified changes in the political landscape (Teeple, 2002). The current third major transition appears to be moulded predominantly, if not exclusively, by the Malthusian preventative checks (Malthus, 1798).

The minor transition between AD 1195 and 1470 appears to have been of an entirely different nature. It was not associated with the change in the style of living but rather with the one and only example of a strong impact of demographic catastrophes caused by an unusual convergence of five major catastrophic events (Nielsen, 2016a; 2017b). This transition caused a 30% *decrease* in the resistance to growth, reflecting the efficient action of the regeneration process triggered by the Malthusian positive checks (Malthus, 1798; Nielsen, 2016f).

Using the best fit to the data we have calculated the size of human population in the past 2,000,000 years. These values are listed in Tables A1-A3. Using results of our earlier analysis (Nielsen, 2016e) of the Gross Domestic Product per capita (GDP/cap) and the current analysis of population data, we have also listed the estimated values of the GDP in the past 2,000,000 years until 1950. The GDP values from 1950 to 2008 were taken directly from the publication of Maddison (2010). The last two values, for 2009 and 2010 were calculated using the GDP/cap values listed by GGDC (2013) and the population data of the US Census Bureau (2017). All these values, expressed in billions of 1990 international Geary-Khamis dollars are listed in Tables A4-A6.

Appendices

Table A1. *Growth of human population from 2,000,000 BC to 1BC*

Year [BC]	Population [Million]	Year [BC]	Population [Million]	Year [BC]	Population [Million]
2,000,000	0.30	7000	6.56	380	160.53
1,500,000	0.38	6000	7.67	370	165.35
1,000,000	0.50	5500	8.38	360	170.22
800,000	0.57	5000	9.24	350	175.15
600,000	0.67	4500	10.29	340	180.11
400,000	0.81	4000	11.61	330	185.09
200,000	1.03	3500	13.32	320	190.09
100,000	1.19	3000	15.62	310	195.07
80,000	1.23	2800	16.78	300	200.04
60,000	1.27	2600	18.12	290	204.96
50,000	1.29	2400	19.70	280	209.82
46,000	1.30	2200	21.58	270	214.61
42,000	1.31	2000	23.85	260	219.29
40,000	1.32	1900	25.18	250	223.85
38,000	1.35	1800	26.66	240	228.28
36,000	1.37	1700	28.33	230	232.54
34,000	1.41	1600	30.23	220	236.63
32,000	1.46	1500	32.39	210	240.51
30,000	1.53	1400	34.89	200	244.18
28,000	1.62	1300	37.80	190	247.62
27,000	1.68	1200	41.25	180	250.80
26,000	1.75	1100	45.39	170	253.73
25,000	1.82	1000	50.45	160	256.38
24,000	1.89	900	56.78	150	258.75
23,000	1.98	800	64.93	140	260.83
22,000	2.07	700	75.81	130	262.61
21,000	2.17	600	91.08	120	264.10
20,000	2.27	500	114.03	110	265.29
19,000	2.39	490	116.98	100	266.19
18,000	2.53	480	120.08	90	266.80
17,000	2.68	470	123.36	80	267.12
16,000	2.85	460	126.82	70	267.17
15,000	3.04	450	130.47	60	266.95
14,000	3.26	440	134.35	50	266.49
13,000	3.51	430	138.46	40	265.78
12,000	3.80	425	140.61	30	264.85
11,000	4.15	420	142.05	20	263.71
10,000	4.57	410	146.54	10	262.37
9000	5.09	400	151.12	1	261.01
8000	5.73	390	155.78		

Table A2. *Growth of human population from AD 1 to 1330*

Year [AD]	Population [Million]	Year [AD]	Population [Million]	Year [AD]	Population [Million]
1	260.69	450	188.30	900	260.63
10	259.18	460	188.67	910	262.99
20	257.36	470	189.19	920	265.40
30	255.41	480	189.87	930	267.85
40	253.35	490	190.72	940	270.34
50	251.19	500	191.75	950	272.89
60	248.95	510	192.99	960	275.48
70	246.64	520	194.28	970	278.12
80	244.28	530	195.59	980	280.82
90	241.88	540	196.92	990	283.56
100	239.45	550	198.27	1000	286.36
110	237.00	560	199.63	1010	289.22
120	234.54	570	201.02	1020	292.13
130	232.09	580	202.42	1030	295.10
140	229.66	590	203.84	1040	298.14
150	227.24	600	205.28	1050	301.23
160	224.85	610	206.75	1060	304.39
170	222.50	620	208.23	1070	307.62
180	220.19	630	209.74	1080	310.92
190	217.94	640	211.27	1090	314.29
200	215.73	650	212.82	1100	317.73
210	213.59	660	214.39	1110	321.25
220	211.51	670	215.99	1120	324.85
230	209.49	680	217.61	1130	328.53
240	207.55	690	219.25	1140	332.30
250	205.68	700	220.92	1150	336.15
260	203.90	710	222.62	1160	340.09
270	202.19	720	224.34	1170	344.12
280	200.56	730	226.09	1180	348.26
290	199.03	740	227.86	1190	352.49
300	197.58	750	229.67	1195	354.64
310	196.22	760	231.50	1200	355.85
320	194.96	770	233.36	1210	359.73
330	193.79	780	235.26	1220	363.23
340	192.72	790	237.18	1230	366.33
350	191.75	800	239.14	1240	369.01
360	190.89	810	241.13	1250	371.27
370	190.13	820	243.15	1260	373.11
380	189.48	830	245.20	1270	374.55
390	188.95	840	247.29	1280	375.59
400	188.52	850	249.42	1290	376.28
410	188.22	860	251.58	1300	376.64
420	188.05	870	253.79	1310	376.73
430	188.00	880	256.03	1320	376.58
440	188.08	890	258.31	1330	376.26

Table A3. *Growth of human population from AD 1340 to 2016*

Year [AD]	Population [Million]	Year [AD]	Population [Million]	Year [AD]	Population [Million]
1340	375.83	1770	834.64	1974	3,984.30
1350	375.35	1780	867.04	1975	4,057.11
1360	374.90	1790	902.06	1976	4,130.85
1370	374.54	1800	940.03	1977	4,205.51
1380	374.36	1810	981.33	1978	4,281.06
1390	374.45	1820	1,026.44	1979	4,357.47
1400	374.90	1830	1,075.88	1980	4,434.73
1410	375.80	1840	1,130.33	1981	4,512.79
1420	377.26	1850	1,190.59	1982	4,591.63
1430	379.41	1860	1,257.63	1983	4,671.22
1440	382.38	1870	1,332.68	1984	4,751.52
1450	386.34	1880	1,417.25	1985	4,832.48
1460	391.48	1890	1,513.28	1986	4,914.08
1470	393.49	1900	1,623.26	1987	4,996.26
1480	400.54	1910	1,750.49	1988	5,078.98
1490	407.86	1920	1,899.36	1989	5,162.20
1500	415.45	1930	2,075.91	1990	5,245.86
1510	423.32	1940	2,288.63	1991	5,329.92
1520	431.50	1945	2,412.23	1992	5,414.31
1530	440.00	1950	2,538.51	1993	5,498.99
1540	448.84	1951	2,587.24	1994	5,583.89
1550	458.05	1952	2,636.93	1995	5,668.96
1560	467.64	1953	2,687.57	1996	5,754.14
1570	477.64	1954	2,739.19	1997	5,839.36
1580	488.08	1955	2,791.79	1998	5,924.57
1590	498.98	1956	2,845.38	1999	6,009.70
1600	510.39	1957	2,899.97	2000	6,094.69
1610	522.32	1958	2,955.57	2001	6,179.48
1620	534.83	1959	3,012.18	2002	6,263.99
1630	547.95	1960	3,069.82	2003	6,348.16
1640	561.73	1961	3,128.47	2004	6,431.94
1650	576.23	1962	3,188.16	2005	6,515.25
1660	591.49	1963	3,248.87	2006	6,598.04
1670	607.58	1964	3,310.62	2007	6,680.24
1680	624.57	1965	3,373.41	2008	6,761.79
1690	642.54	1966	3,437.22	2009	6,842.64
1700	661.57	1967	3,502.07	2010	6,922.73
1710	681.77	1968	3,567.94	2011	7,002.00
1720	703.24	1969	3,634.83	2012	7,080.40
1730	726.10	1970	3,702.74	2013	7,157.89
1740	750.50	1971	3,771.65	2014	7,234.42
1750	776.60	1972	3,841.55	2015	7,309.94
1760	804.57	1973	3,912.44	2016	7,384.42

Table A4. *Economic growth from 2,000,000 BC to 1BC*

Year	GDP	Year	GDP	Year	GDP
2,000,000	0.13	7000	2.92	380	73.30
1,500,000	0.17	6000	3.42	370	75.51
1,000,000	0.22	5500	3.74	360	77.75
800,000	0.25	5000	4.12	350	80.01
600,000	0.30	4500	4.60	340	82.29
400,000	0.36	4000	5.19	330	84.58
200,000	0.45	3500	5.96	320	86.87
100,000	0.52	3000	7.00	310	89.17
80,000	0.54	2800	7.53	300	91.45
60,000	0.56	2600	8.14	290	93.71
50,000	0.57	2400	8.85	280	95.95
46,000	0.57	2200	9.70	270	98.16
42,000	0.58	2000	10.74	260	100.31
40,000	0.59	1900	11.34	250	102.42
38,000	0.59	1800	12.02	240	104.46
36,000	0.61	1700	12.78	230	106.43
34,000	0.62	1600	13.64	220	108.31
32,000	0.65	1500	14.63	210	110.11
30,000	0.68	1400	15.76	200	111.81
28,000	0.72	1300	17.09	190	113.40
27,000	0.74	1200	18.67	180	114.88
26,000	0.77	1100	20.56	170	116.24
25,000	0.80	1000	22.87	160	117.48
24,000	0.84	900	25.77	150	118.59
23,000	0.87	800	29.49	140	119.56
22,000	0.91	700	34.47	130	120.40
21,000	0.96	600	41.46	120	121.11
20,000	1.01	500	51.98	110	121.67
19,000	1.06	490	53.33	100	122.11
18,000	1.12	480	54.75	90	122.41
17,000	1.19	470	56.25	80	122.58
16,000	1.26	460	57.84	70	122.63
15,000	1.35	450	59.52	60	122.55
14,000	1.44	440	61.29	50	122.36
13,000	1.56	430	63.18	40	122.06
12,000	1.69	425	64.16	30	121.66
11,000	1.84	420	64.82	20	121.16
10,000	2.03	410	66.88	10	120.57
9000	2.26	400	68.98	1	119.97
8000	2.55	390	71.12		

Year: BC; GDP: Gross Domestic Product, billion 1990 international Geary-Khamis dollars.

Table A5. *Economic growth from AD 1 to 1330*

Year	GDP	Year	GDP	Year	GDP
1	119.83	450	87.59	900	123.89
10	119.15	460	87.79	910	125.10
20	118.34	470	88.07	920	126.33
30	117.47	480	88.42	930	127.59
40	116.55	490	88.84	940	128.87
50	115.58	500	89.36	950	130.18
60	114.57	510	89.96	960	131.52
70	113.53	520	90.60	970	132.88
80	112.47	530	91.25	980	134.27
90	111.39	540	91.90	990	135.69
100	110.30	550	92.57	1000	137.14
110	109.19	560	93.24	1010	138.62
120	108.09	570	93.92	1020	140.14
130	106.98	580	94.62	1030	141.68
140	105.89	590	95.32	1040	143.27
150	104.80	600	96.04	1050	144.88
160	103.72	610	96.76	1060	146.54
170	102.66	620	97.50	1070	148.23
180	101.62	630	98.25	1080	149.96
190	100.60	640	99.01	1090	151.73
200	99.61	650	99.78	1100	153.55
210	98.65	660	100.56	1110	155.41
220	97.71	670	101.36	1120	157.31
230	96.81	680	102.16	1130	159.26
240	95.94	690	102.99	1140	161.26
250	95.10	700	103.82	1150	163.31
260	94.30	710	104.67	1160	165.42
270	93.53	720	105.53	1170	167.58
280	92.81	730	106.41	1180	169.79
290	92.12	740	107.30	1190	172.06
300	91.48	750	108.20	1195	173.22
310	90.88	760	109.12	1200	173.92
320	90.32	770	110.06	1210	176.04
330	89.80	780	111.02	1220	177.99
340	89.33	790	111.99	1230	179.75
350	88.91	800	112.97	1240	181.32
360	88.54	810	113.98	1250	182.69
370	88.21	820	115.00	1260	183.87
380	87.94	830	116.04	1270	184.85
390	87.72	840	117.10	1280	185.65
400	87.55	850	118.18	1290	186.29
410	87.44	860	119.28	1300	186.77
420	87.38	870	120.40	1310	187.12
430	87.39	880	121.54	1320	187.37
440	87.46	890	122.71	1330	187.55

Year: AD; GDP: Gross Domestic Product, billion 1990 international Geary-Khamis dollars.

Table A6. *Economic growth from AD 1340 to 2010*

Year	GDP	Year	GDP	Year	GDP
1340	187.67	1750	471.71	1970	13,765.94
1350	187.79	1760	495.02	1971	14,336.49
1360	187.92	1770	520.73	1972	15,018.42
1370	188.12	1780	549.22	1973	16,015.15
1380	188.42	1790	580.95	1974	16,388.00
1390	188.87	1800	616.51	1975	16,637.92
1400	189.51	1810	656.64	1976	17,449.53
1410	190.39	1820	702.24	1977	18,157.09
1420	191.59	1830	754.52	1978	18,955.43
1430	193.14	1840	815.03	1979	19,633.16
1440	195.15	1850	885.85	1980	20,029.99
1450	197.68	1860	969.81	1981	20,422.61
1460	200.85	1870	1,070.89	1982	20,648.35
1470	202.45	1880	1,194.80	1983	21,235.64
1480	206.68	1890	1,350.09	1984	22,204.27
1490	211.09	1900	1,550.16	1985	22,969.60
1500	215.69	1910	1,817.13	1986	23,781.92
1510	220.50	1920	2,190.37	1987	24,693.77
1520	225.52	1930	2,747.20	1988	25,753.18
1530	230.78	1940	3,662.62	1989	26,576.36
1540	236.29	1945	4,381.03	1990	27,134.08
1550	242.06	1950	5,335.86	1991	27,494.23
1560	248.12	1951	5,649.96	1992	28,077.30
1570	254.50	1952	5,911.28	1993	28,693.57
1580	261.20	1953	6,208.99	1994	29,697.95
1590	268.27	1954	6,421.22	1995	30,942.24
1600	275.73	1955	6,830.52	1996	31,990.50
1610	283.61	1956	7,151.72	1997	33,241.79
1620	291.96	1957	7,423.90	1998	33,803.49
1630	300.80	1958	7,662.29	1999	34,997.33
1640	310.20	1959	8,013.45	2000	36,688.28
1650	320.19	1960	8,432.82	2001	37,739.37
1660	330.85	1961	8,725.32	2002	39,021.27
1670	342.23	1962	9,136.47	2003	40,809.56
1680	354.42	1963	9,533.55	2004	42,950.18
1690	367.51	1964	10,224.89	2005	44,982.59
1700	381.58	1965	10,760.25	2006	47,340.58
1710	396.77	1966	11,346.93	2007	49,411.11
1720	413.20	1967	11,769.15	2008	50,973.94
1730	431.04	1968	12,416.76	2009	50,762.92
1740	450.47	1969	13,101.91	2010	53,650.54

Year: AD; GDP: Gross Domestic Product, billion 1990 international Geary-Khamis dollars. From 1950, the data are as listed by Maddison (2010) up to 2008. The two values for 2009 and 2010 were calculated using the GDP/cap values listed by GGDC (2013) and the population data of the US Census Bureau (2017).

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