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Union duopoly with homogeneous labor: The effect of membership and employment constraints

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Abstract. This research analyzes the labor market outcome when there are two unions in the industry, representing homogeneous workers (hence, unions use employment strategies), in the presence of union employment (quantity) constraints. Three strategic environments are considered: Nash-Cournot duopoly, Stackelberg duopoly and efficient cooperation between the two unions. Employment constraints -ceilings and floors- originate kinks and/or discontinuities in the reaction functions, leading to corner solutions and special features of the labor market equilibrium. Two types of constraints are discussed. One is insufficient employed membership (ceiling) for the interior solution. Then it may be optimal for a Stackelberg leader to push the other to the bound. The other case considered is the legal requirement of a minimum number of employed members that the union must have to be constituted. Entry-deterrence strategies of the leading union may then emerge.
Keywords. Unions, Wage determination models, Union bargaining, Corporatism, Imperfect competition and union behavior, Union oligopoly, Union membership, Union representativeness, Occupational licensure.
JEL. J51, E24, D45, D49, C79.

1. Introduction

This paper considers a two-union closed-shop equilibrium and studies the effect of quantity or employment constraints that generate corner solutions in the labor market equilibrium. These constraints may be legally induced; they can be used as tools for employment-enhancing policies in strongly unionized (or corporatized) economies, professions and industries.

The multiple union case has been previously studied in the literature. Oswald (1979) [1] departs from unions with price competition strategies - framework also used by Gylfason & Lindbeck (1984a) and (1984b) [2] - and derives the properties of the Cournot [3]-Nash [4] equilibrium, comparing it with the Stackelberg [5] one and even describing efficient cooperation [6] between unions. He assumes heterogeneous labor with substitutability between workers. Martins & Coimbra (1997 and 1997a) analyze the equilibrium solutions for homogeneous workers in a market with two and n unions; as Hart´s (1982) syndicates, unions employ quantity rather than

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price strategies. The assumption applies whenever unions can influence lay-off regulation – as is the case for Portugal, for example.

In the standard imperfect competition framework, pure or unqualified quantity restrictions leading to corner solutions have not been assigned particular interest. However, two problems can be addressed in the union case. Both introduce kinks and one discontinuities in the affected union's reaction function.

The first case we address is insufficient number of members of one of the unions to achieve the interior solution. The effect of a membership constraint has been studied in a monopoly union framework by Carruth & Oswald (1987) in a different context - they model the restriction as embedded in the union's utility function and analyze the possible implications for the hiring of outsiders. Instead, we focus on a two union scenario, and start by general utility functions in a closed-shop. If one of the unions reaches the bound, the other may behave as a monopolist with respect to residual demand; and we suggest the possible advantage for the "larger" union to decrease her own employed members and push the other to the membership constraint.

Employment ceilings (limits of access to employment) may have an immediate application in the study of occupational licensure, the analysis reflecting Friedman's (1962,1982) historical exposition on its effects: if licensing may provide consumer protection in an imperfectly informed world, it can also be used by an incumbent group to secure monopoly (oligopoly, monopolistic) power over a specific market - the argument is mathematical, yet simply, developed below. The bound can be seen to work through the recognition of diplomas or certification requirements for, say, an immigrant group. One can offer a practical application of this environment in the Portuguese dentist market: Brasilian dentists co-existed in the market but illegaly - only recently have they achieved recognition – with recognized Portuguese dentists. The same can be said about formal and parallel medicine. In such cases, professional associations, take the role of "our" union – and usually fight for barriers to foreign diploma´s recognition.

The other case is the existence of union representative rules (laws) or minimum employment requirements. Say the union to be considered legal must exhibit a minimum number of employed members. The hypothesis, which is realistic, was introduced in the n-union framework by Martins & Coimbra (1997a) to explain, or rather, limit union formation; for example, Portuguese labor law requires a minimum of 10% of target workers or 2000 workers present for a founding assembly to function [7]. The existence of such laws, again, may lead to corner solutions; or may elicit entry-deterrence - "limit output" - practices by the incumbent [8]. The research in this respect shows analogies with the capacity constraint literature for the product market, such as Dixit's (1979) problem [9], with entry-deterrence working in the union framework through overemployment of the incumbent.

The implications of the two constraints are studied for the Cournot-Nash equilibrium, the Stackelberg solution and for cooperative unions. Some of the strategic behavior analyzed is more plausible in Stackelberg than in the Cournot-Nash framework - for example, employment-pushing and entry-deterrence practices. Others may arise in both situations.

This research is mainly theoretical; it has empirical relevance in the understanding of the behavior of industry unions, in the presence of representative requirement rules. The closed-shop environment reproduces the scenario where bargaining agreements are extended to non-unionized workers – as they usually are in Portugal: if unionisation was around 30% in the early nineties, the collective bargaining coverage rate – number of workers covered by collective agreements as a percentage of wage and salary earners - was almost 80%, according to OECD [10]; the proportion of non-self-employed covered seems to have remained stable in the second half of the decade [11]. The duopoly setting applies, for example, to Portugal, where some industries or professions are represented by two unions. At the macro or aggregate level, in the first half of the nineties, two union confederations [12] represented 88% of Portuguese unionised workers, the behavior of which this research may also address; insufficient membership, for example, may have conditioned the behavior of the newer confederation [13] in its early stages; and may have affected (formally, the corresponding restriction can become active after more or less sudden decreases in unionised affiliates) the older confederation, that lost, on average, 46% of its members from 1979-84 to 1991-95 [14].

Interior solution results and notation are summarized in section II. Insufficient membership for internal solutions is dealt with in section III. Implications of the existence of a minimum employed members requirement is advanced in section IV. The exposition ends with a brief summary in section V, which includes tables with the main analytical results along with those for an example.

2. Union duopoly: Interior solutions [15].

Consider that we have two unions, 1 and 2. Let L_i be the employed members of union i, and W the wage rate. The labor demand in the market is given by:

$$L_1 + L_2 = L(W), \quad \text{or} \tag{1}$$

$$W = W(L_1 + L_2) = P\,F_L(L_1 + L_2) \tag{2}$$

Workers are perfect substitutes and the wage set by firms will be extended to all workers - or firms will equate marginal product for the two types of workers, or will only hire workers of lower wage.

Assume

a) each union has utility function of the general form $U^i(L_i,W)$, increasing in the arguments and quasi-concave, for which U^i_L / U^i_W - the marginal rate of substitution between employment and wage - decrease with L_i and increases with W.

b) the demand function is given by (1), decreasing in W, coming from maximization (in $L = L_1 + L_2$) of the (aggregate) profit function ◎ (L,W). Therefore, (2) establishes the value of the marginal product of labor, equal for both types of workers [16].

c) a sort of closed-shop setting, i.e., the firm(s) can only hire unionized workers, either from union 1 or 2.

2.1. Cournot-Nash duopoly

Each union maximizes

$$\max_{L_i, W} U^i(L_i, W) \quad (3)$$

s.t.: $L_1 + L_2 = L(W)$ or $W = W(L_1 + L_2) = P F_L(L_1 + L_2)$

or, alternatively:

$$\max_{L_i} U^i[L_i, W(L_1 + L_2)] \quad , \quad i=1,2, \quad (4)$$

F.O.C. imply

$$U^i_L + U^i_W W_L = 0 \quad , \quad i=1,2 \quad (5)$$

(5) establishes the optimal policy of union i given union j's employment strategy, i.e., union i's reaction function:

$$L_i = R^i(L_j) \qquad i = 1, 2 ; \quad j = 2, 1 \quad (6)$$

The Nash-Cournot equilibrium will be described by the two equations:

$$U^1_L / U^1_W = U^2_L / U^2_W = -W_L \quad \text{or} \quad (7)$$

$$U^1_W / U^1_L = U^2_W / U^2_L = -L_W$$

and the demand function.

Alternatively, the equilibrium labor market outcome is defined by the two equations (and labor demand which defines the wage):

$$L_1 = R^1(L_2) \qquad \text{and} \qquad L_2 = R^2(L_1) \tag{8}$$

that is:

$$L_1 = R^1[R^2(L_1)] \qquad \text{and / or} \qquad L_2 = R^2[R^1(L_2)] \tag{9}$$

As in the standard imperfect competition problem, (static) stability [17] is only achieved iff - for negatively sloped reaction functions:

$$| \, dR^2 / dL_1 \, | \circledcirc 1 / | \, dR^1 / dL_2 \, | \tag{10}$$

This will be satisfied if both unions' reaction functions have slope smaller than 1 in absolute value, i.e., $| \, dR^2 / dL_1 \, | \leq 1$ and $| \, dR^1 / dL_2 \, | \leq 1$. Therefore, we will assume continuous, smooth and well-behaved reaction functions $R^i(L_j)$ - derived as in (6) -, and that:

$$-1 < dR^i(L_j) / dL_j < 0 \tag{11}$$

Given that we have only two unions, we can represent the equilibrium solution in the space (L_1, L_2) - analogous to the conventional product market solution for quantities of an homogeneous good sold by two firms. This is presented in Fig. 1 below. PN is union 1's reaction function and MQ is union 2's. Each union's indifference curve will have the general form:

$$U^i[L_i, W(L_1 + L_2)] = \bar{U}^i \quad , \quad i = 1, 2 \tag{12}$$

Once the reaction functions come from (5), each union's indifference curve has a maximum on its reaction function. The level of utility increases as we approach, on the reaction curve, the union's employment axis; that curve crosses this axis at the monopoly union solution (where the other union's employment is 0).

Both reaction functions are negatively sloped, obeying condition (10). They cross at point A, the Cournot-Nash equilibrium.

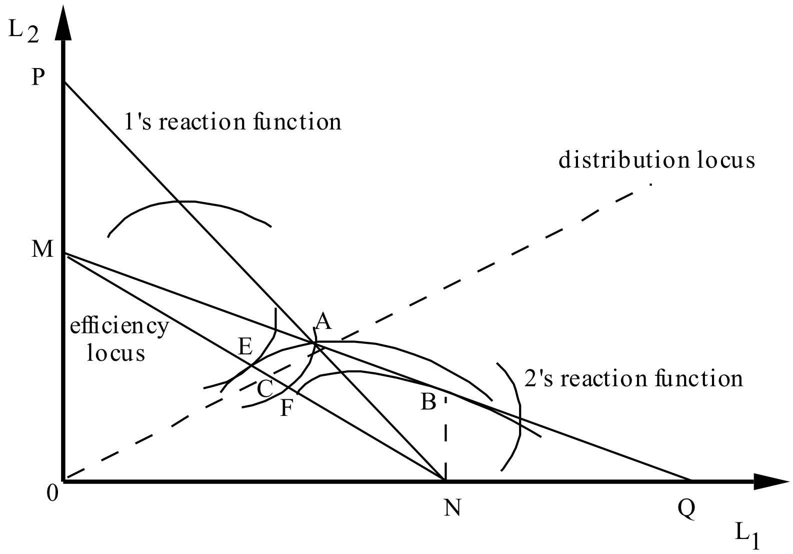

Fig. 1

2.2. Stackelberg duopoly

Take now the case where union 1 operates as a leader and union 2 responds as its follower. Then the leader solves:

$$\max_{L_1, L_2} \quad U^1[L_1, W(L_1 + L_2)] \tag{12}$$

$$\text{s.t.:} \quad U^2_L + U^2_W W_L = 0 \quad \text{or} \quad L_2 = R^2(L_1)$$

Constructing the Lagrangean, or replacing the restriction in the utility function, and considering F.O.C. we get:

$$U^1_L / U^1_W = (1 + dR^2 / dL_1)\, U^2_L / U^2_W = -(1 + dR^2 / dL_1)\, W_L \tag{14}$$

$$L_2 = R^2(L_1) \quad \text{or} \quad U^2_L / U^2_W = - W_L \tag{15}$$

We can see the Stackelberg solution in Fig. 1. Union 1 picks the point on 2's reaction function which allows her to reach the highest level of utility, i.e., the indifference curve closer to the L_1 axis that touches 2's reaction function. This is point B. Confronting with A, the Cournot-Nash equilibrium, the leader reaches now a higher employment and utility level, and the follower smaller levels, than in the Cournot-Nash solution. Total employment will be larger than in the Cournot solution: the Stackelberg equilibrium, point B, is on 2's reaction function, with slope smaller than one in absolute value, to the right of and below A.

3.3. Efficient cooperation between the unions

Assume the unions cooperate efficiently. That is, the two unions maximize the Nash-bargaining problem:

$$\underset{L_1,\, L_2,\, W}{\text{Max}}\ [U^1(L_1, W) - \bar{U}^1]^{\circledcirc}\ [U^2(L_2, W) - \bar{U}^2] \tag{16}$$

$$\text{s.t.:}\quad L_1 + L_2 = L(W) \quad \text{or} \quad W = W(L_1 + L_2)$$

$\bar{U}^1$ and $\bar{U}^2$ denote the alternative utility of each union in case of no agreement. Eventually, $\bar{U}^i$ would be the utility union i gets in the Cournot game [18]. $\circledcirc$ corresponds to the relative strength of union 1 with respect to union 2 within the coalition [19]. The optimal solution will yield:

$$U^1_W / U^1_L + U^2_W / U^2_L = -L_W \tag{17}$$

We immediately expect a higher W and a lower L when cooperation between unions is established.

If we replace the demand schedule - $W = W(L_1 + L_2)$ - in (17), this equation defines the "efficiency locus" [20] - a relation between L_1 and L_2 where the utility of one of the unions is maximized given the utility level of the other -, given that employers react on the demand.

From F.O.C., we can also derive:

$$\circledcirc = \{[U^1(L_1, W) - \bar{U}^1] / [U^2(L_2, W) - \bar{U}^2]\}\ (U^2_L / U^1_L) \tag{18}$$

Replacing, again, the demand schedule, this equation establishes the "distribution locus" - a relation between L_1 and L_2.

The intersection of the efficiency locus and the distribution locus yields the particular solution of the problem.

We can see the equilibrium, again, in Fig. 1. The efficiency locus is MN, a curve formed by the intersection of the two unions' indifference curves; it should cross the axis of L_i (i=1, 2) at the same point where i's reaction curve does, once this point determines the monopoly solution for union i. If $\bar{U}^i$ is the utility union i gets in the Cournot game, the distribution equation will cross the efficiency locus between points E and F - points on the indifference curves of each union corresponding to their utility level in the Cournot-Nash equilibrium. The efficient bargaining equilibrium is, thus, point C. In general, we expect lower employment than in the previous solutions - distribution between the unions depending on the location of the distribution locus, and, therefore, also on ◎ .

4. Insufficient membership

Assume that a particular union i has an exogenously fixed number of members M_i which is smaller than the equilibrium level of employment corresponding to the interior solution. What will this imply for the labor market outcome? Will the other (presumably larger) union have any advantage in restricting its own quantity in order to make the former reach its bound, i.e., the employment ceiling?

4.1. Cournot duopoly

1. Consider that the two unions are Cournot duopolists. Then, we know that for an interior solution:

$$L_i = R^i(L_j) \quad i=1,2, \quad j=2,1 \tag{19}$$

Alternatively, the equilibrium L_i^* satisfies:

$$L_i^* = R^i(R^j(L_i^*)) \quad i=1,2, \quad j=2,1 \tag{20}$$

2. If $R^2(R^1(L_2^*)) > M_2$, then:

$$L_2 = M_2 \quad \text{and} \quad L_1 = R^1(M_2) \tag{21}$$

This would seem to require that:

$$R^2(R^1(M_2)) > M_2 \tag{22}$$

That if $R^2(R^1(L_2^*)) > M_2$, then $R^2(R^1(M_2)) > M_2$, is proven in the Appendix.

Being the reaction functions negatively sloped, union 1 will benefit, to some extent, from the unfulfilled demand of the other union. At the "corner" equilibrium:

$$U^2_L + U^2_W W_L > 0 \quad \text{at} \quad L_2 = M_2 \tag{23}$$

and

$$U^1_L + U^1_W W_L = 0 \quad \text{at} \quad L_2 = M_2 \,, \quad \text{or} \quad L_1 = R^1(M_2) \tag{24}$$

Union 1 will have higher employment - once reaction functions are negatively sloped - and attain higher utility level than if 2's members allowed the interior solution.

We can see the new solution in Fig. 2. Graphically, 2's reaction function, MN without restriction, now becomes M_2BN, being M_2 the number of members of union 2 and smaller than employment of union 2 in the unrestricted Cournot equilibrium, point A. With insufficient membership of union 2, we go to point C, on 1's reaction function.

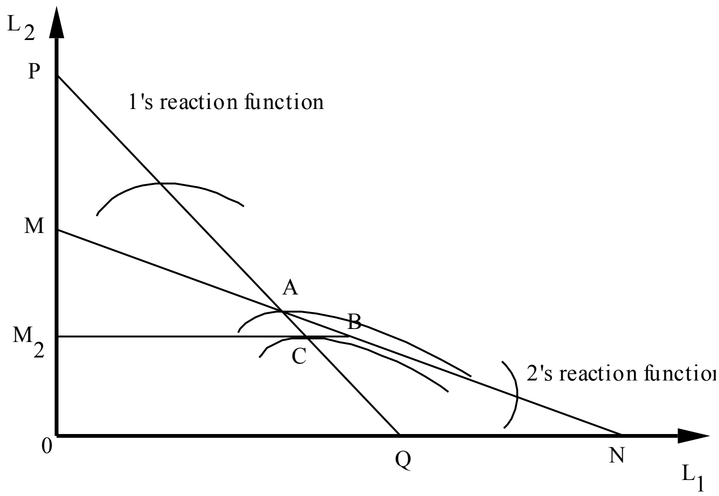

Figure 2.

3. In a corner solution for union i:

$$L = M_2 + R^1(M_2) \tag{25}$$

and

$$W = W(M_2 + R^1(M_2)) \tag{26}$$

An increase in membership of union 2, M_2, will have an impact such that:

$$dL/dM_2 = 1 + dR^1/dL_2(M_2) \tag{27}$$

Total quantity will increase (equilibrium wage will decrease) with membership of union i as long as:

$$dR^1(M_2)/dL_2 > -1 \tag{28}$$

This is included in condition (11).

As M_2 increases, we tend to the Cournot solution - as we can see in Fig. 2. Having 1's the reaction functions slope smaller than 1 in absolute value, the corner solution will - given (27)-(28) -, imply a smaller total employment and higher wage than the interior solution.

Proposition 1: If membership is not sufficient for an internal solution in a

Cournot duopoly:

1. the small union will employ all its members.
2. the large union will behave as a monopolist with respect to the residual demand. It will have higher employment and attain a higher utility level than in the interior solution.
3. Total employment will be smaller and the wage higher than in the interior solution.
4. An increase in membership of the small union will decrease the other union's employment, increase total employment and decrease the wage.

4.2. Stackelberg equilibrium

Consider now that we have a Stackelberg environment and the leader, 1, solved problem (13) which yelded an equilibrium solution such that:

$$U^1_L + U^1_W W_L + U^1_W W_L dR^2/dL_1 = 0 \quad , \tag{29}$$

$$L_2 = R^2(L_1) \quad \text{or} \quad U^2_L / U^2_W = - W_L \tag{30}$$

and also the labor demand equation.

Let us call this solution L_1^S and $L_2^S = R^2(L_1^S)$.

Assume that $R^2(L_1^S) > M_2$. Then, obviously, $L_2 = M_2$. But in that case the leader considers that the follower, at the margin, does not respond to its quantity. Therefore the leader obeys

$$U^1_L + U^1_W W_L = 0 \quad \text{and} \quad L_2 = M_2 \tag{31}$$

i.e., the leader reacts to M_2 according to its reaction function, being the equilibrium solution:

$$L_1 = R^1(M_2) \quad \text{and} \quad L_2 = M_2 \tag{32}$$

Being $R^2(L_1)$ negatively sloped, if M_2 is close to $R^2(L_1^S)$ we conclude, comparing (29) with (31), that at the new $L_1^* = R^1(M_2)$, (29) is negative. Therefore $L_1^S < L_1^*$. Also, union 1's utility level will be higher that at L_1^S.

The features of the corner equilibrium are altogether similar to the ones of the Cournot constrained solution - graphically, they coincide. Given, also, that the Stackelberg equilibrium yields higher total employment than the Cournot outcome - once it is on 2's reaction function below it -, the corner solution will imply a smaller total employment than the unconstrained Stackelberg one. If M_2 is small enough, the leader's employment may be higher than in the Stackelberg equilibrium (at the limit, if M_2 is 0, the leader chooses the monopoly union solution; this may imply a higher employment for the leader than the Stackelberg equilibrium); but if not, the leader's employment does not need to be so large as in the unconstrained case.

Proposition 2: For a Stackelberg duopoly, being the leader the large union:

1. 1., 3. and 4. of Proposition 1 hold when there is insufficient membership of the follower to ensure the interior solution.
2. the large union will behave as a monopolist with respect to the residual demand. It may have smaller employment than in the unconstrained case; and it will attain a higher utility level than in the interior solution.

If $R^2(L_1^S) < M_2$, we can have the standard interior Stackelberg solution. But an interesting possibility may occur: it may be worthwhile for the leader to decrease its employment and force the other union to employ all its members. This is the issue taken below.

4.3. "Employment-pushing"

Consider that we have the Stackelberg scenario above but that $R^2(L_1^S) < M_2$. It may be the case that it is worthwhile for union 1 to establish an $L_1^{**} < L_1^S$ and make union 2 reach its bound. This allows union 1 to behave as a monopolist with respect to the residual demand.

This strategy is worthwhile iff:

$$U^1\{L_1^S, W[L_1^S+R^2(L_1^S)]\} < U^1\{R^1(M_2), W[R^1(M_2)+M_2]\} \quad (33)$$

Clearly, this can happen iff there is equilibrium stability. Let us see Fig. 3. The unrestricted Stackelberg equilibrium yielded point A, which was possible with union's members, M_2. If $M_2 < M$ - where M is the employment of union 2 that corresponds to the point where 1's indifference curve attained in the Stackelberg solution crosses 1's reaction function -, union 1 may decrease its quantity and still benefit from the fact that 2 can no longer expand its own employment. 1 chooses the point more to the south that touches the new reaction function M_2BN, i.e., the new solution is point C.

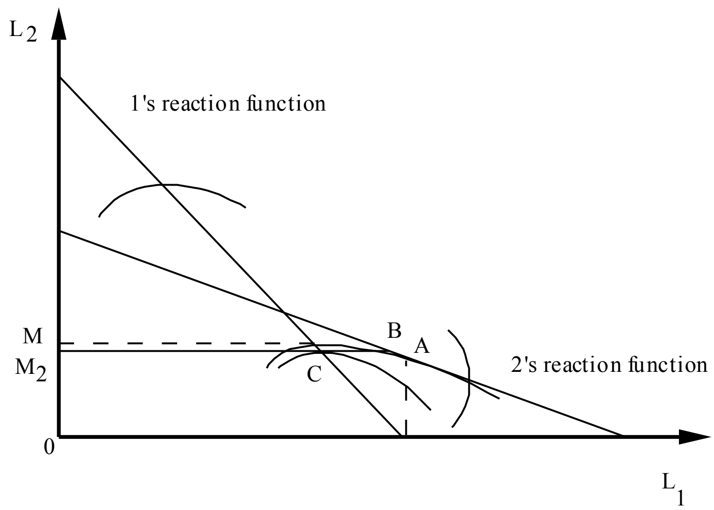

Figure 3.

From the figure we can also conclude that such possibility did not exist if 1 was a Cournot follower...

M solves:

$$U^1\{L_1^S, W[L_1^S+R^2(L_1^S)]\} = U^1\{R^1(M), W[R^1(M)+M]\} \quad (34)$$

Then, if $M_2 > M > R^2(L_1^S)$, we have the Stackelberg solution. Being $M > M_2 > R^2(L_1^S)$, union 1 decreases its quantity in order to restrict reaction of the opponent. It chooses an L_1^{**} such that:

$$L_1^{**} = R^1(M_2) \quad (35)$$

In this solution,

$$L = R^1(M_2) + M_2 \tag{36}$$

and therefore, an increase in membership of union 2 will lead to:

$$dL/dM_2 = dR^1/dL_2(M_2) + 1 \tag{37}$$

This will be positive iff:

$$dR^1/dL_2(M_2) > -1 \tag{38}$$

which is guaranteed by (11). Then, an increase of M_2 around this equilibrium will lead to an increase in total employment and a decrease in the equilibrium wage. Total employment will be smaller than in the interior solution: the Cournot outcome originates a smaller total employment than the Stackelberg equilibrium; and point C is on 1's reaction function - with slope smaller than 1 in absolute value - to the right of the Cournot solution and therefore with lower employment than the latter.

Proposition 3: In a Stackelberg duopoly, when membership of the follower, even if sufficient for the interior solution, is near the latter:

1. it may be "profitable" for the union leader to decrease its own employment relative to the Stackelberg outcome in order to be able to behave as a monopolist with respect to residual demand.
2. If the behavior of the leader is the one described in 1. of this proposition, 3. and 4. of Proposition 1. hold.

Notice that whenever one of the unions is pushed to the employment ceiling, a change in the bound will have the same effect on the equilibrium level of total employment. The constrained solution will always imply a smaller total employment and higher wage than the corresponding interior solution.

Also, the large union always benefits from restricting membership of the other union – that is, in professional markets, the recognition of foreign certificates by national institutions may be seen as a means of constraining membership of the “fringe” union. Alternatively, immigration constraints may have the same effect.

4.4. Efficient bargaining between unions

Consider the efficient bargaining solution when there are membership bounds. It is clear that if union members could be switched from one union to the other, we would eventually arrive at the interior efficient bargaining solution of section II.3.

Let us instead consider the case in which such behavior is not possible. Assume the small union is 2 - the only union that will be affected by a ceiling (we assume the other is a "large" union). Then the efficient bargaining problem can be stated for the membership ceilings as:

$$\max_{L_1,\, L_2,\, W} \; [U^1(L_1, W) - \bar{U}^1]^{\circledcirc} \; [U^2(L_2, W) - \bar{U}^2] \tag{39}$$

$$\text{s.t.:} \quad L_1 + L_2 = L(W) \quad \text{or} \quad W = W(L_1 + L_2) \quad \text{and}$$

$$L_2 \circledcirc M_2$$

The lagrangean of the problem can be stated as:

$$\max_{L_1,\, L_2} \; \{U^1[L_1,W(L_1+L_2)]-\bar{U}^1\}^{\circledcirc}\{U^2[L_2,W(L_1+L_2)]- \bar{U}^2\}+\circledcirc \; (M_2 - L_2) \tag{40}$$

If the interior solution of the efficient bargaining problem without the restriction originates an $L_2^* < M_2$, we will stay in the interior solution. If not, the corner solution, from first derivative with respect to L_1 - which will hold both in restricted or unrestricted solutions - will yield that, at $L_2 = M_2$, it must be the case that:

$$\circledcirc \;\; (U^1_L + U^1_W \; W_L) / [U^1 - \bar{U}^1] + \; W_L \; U^2_W / [U^2 - \bar{U}^2] = 0 \tag{41}$$

Then this equality determines L_1.

The new solution does not obey either the previously defined efficiency or distribution locus. But we conclude that, if (41) (at least near the relevant range) increases with L_2 (then, at an M_2 smaller than the unrestricted efficient bargaining solution, (41) is negative and the maximand (40) is already decreasing with L_1), L_1 will be smaller than if the restriction was not binding, i.e., than its unrestricted equilibrium level. The opposite will occur if (41) decreases with L_2 - we could not rule this out, once it depends on second derivatives also of labor demand.

It seems more plausible that L_1 should be now smaller than when the bound was not imposed. The intuition for this is that, with efficient bargaining, as union 2 cannot benefit from additional employment, it will try, within the coalition, to compensate by asking a rise in the wage - hence, a decrease in employment of the other union.

On the other hand, we have seen that in most of the Cournot and Stackelberg cases, in the corner solution where one union employs all its members, the other reacts according to its reaction function - behaving as a monopolist with respect to residual demand; then, in the equilibrium:

$$U^1_L + U^1_W\ W_L = 0 \quad \text{at/and} \quad L_2 = M_2 \tag{42}$$

Looking at (41) - which has the sign of $\partial \circledcirc / \partial L_1$, where $\circledcirc$ denotes the lagrangean (40) -, at M_2 and the L_1 of the solution of (42), the left hand-side - equal in that case to $W_L\ U^2_W / [U^2 - \bar{U}^2]$ - is negative; this implies that the maximand (40) is already decreasing: the efficient bargaining solution when the membership restriction is active will yield a lower L_1 than the solution of (42).

Proposition 4: With efficient bargaining between the unions and insufficient membership of one of the unions:

1. employment of the union not affected by the ceiling may be lower or higher than in the interior solution.
2. employment of the union not affected by the ceiling is lower than if she reacted as a monopolist with respect to residual demand.

5. Employed membership requirements

Suppose now that there are "minimum employment" laws: for a union to be legally constituted it must have at least a minimum of $\bar{L}$ employed members. Alternatively, we could interpret such bound as the minimum level of employment the union is willing to accept - as in the general Stone-Geary function. We inquire, below:

- what is the labor market outcome.
- in which conditions will the incumbent(s) engage in entry-deterrence practices.

5.1. Cournot duopoly

1. Clearly, if

$$L_2{}^* = R^2[R^1(L_2{}^*)] < \bar{L} \tag{43}$$

the best union 2 will be able to do is to set

$$L_2 = \bar{L} \tag{44}$$

Now, we will have that

$$L_1^* = R^1(\bar{L}) \tag{45}$$

Because reaction functions are negatively sloped, 1's employment will decrease, relative to the case with no constraint and both unions will be worse-off in terms of utility.

Let us see the equilibrium solution in Fig. 4., where we consider that union 2 is the one affected by the constraint. 2's reaction function is equal to the old one from the L_2 axis, i.e., from M to B (till $L_2 = \bar{L}$) - where the curve shows a kink - and an horizontal line afterwards till point C. At point C, union 2 is indifferent between employing $\bar{L}$ and closing - point C is in the same indifference curve that crosses the old reaction function at the point where $L_2 = 0$; after C, i.e., if $L_1 > D$, union 2 gives up the market and its reaction function continues in DN.

In sum, the effect of the minimum employment restriction on union 2 - assume the other is a "large union" - is to switch its reaction function from MN to MBC - with a kink at B - and with a discontinuity after C, continuing in DN.

The intersection of the two reaction functions switches from A, the interior Cournot equilibrium, to E when the employment restriction is imposed. At E, 1's reaction function crosses the other's new reaction function.

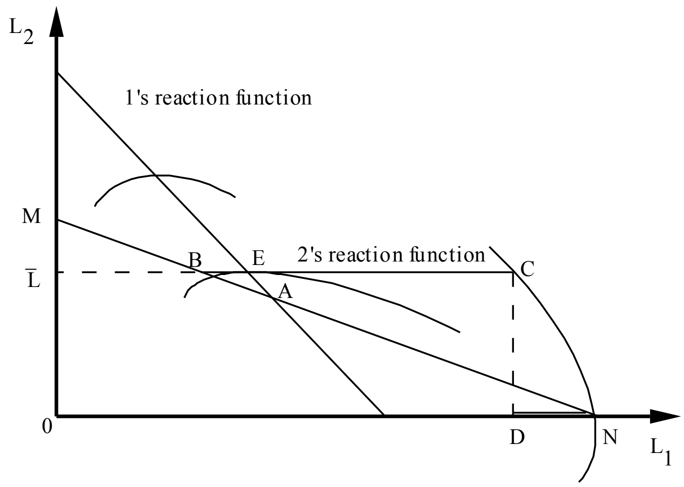

Figure 4.

Notice that in point E, union 2 may be better-off than in point A: as long as $\bar{L}$ is smaller than the level at which union 2's indifference curve that crosses A touches 1's reaction function. Union 1 will always loose utility with the bound.

Total employment will be larger than if the bound was not imposed, once E is to the the North-West of A on 1's reaction function, with slope smaller than 1 in absolute value.

Total employment will be:

$$L = \bar{L} + R^1(\bar{L}) \qquad (46)$$

An increase in $\bar{L}$ will increase total employment (decrease the wage rate) iff:

$$1 + dR^1/dL_2(\bar{L}) > 0 \quad , \qquad (47)$$

i.e., if,

$$|\, dR^1/dL_2(\bar{L}) \,| < 1 \qquad (48)$$

It is easy to show that with stability and around L_2^*,

$$R^2(R^1(L_2^*)) = L_2^* \; < \; R^2(R^1(\bar{L})) \; < \; \bar{L} \qquad (49)$$

and therefore, no problem will occur for the solution in the corner.

Proposition 5: If Cournot duopoly implies for a particular union a solution such that its employment is smaller than the membership floor:

1. The union's employment equals the floor (provided it is smaller than its membership). It may attain a higher utility level than in the interior solution.

2. The other union will behave as a monopolist with respect to the residual demand, but will have a lower utility level and smaller employment than in the unconstrained equilibrium.

3. An increase in the membership floor will decrease the other union's employment, increase total employment and decrease the wage.

4. Employment is larger and the wage lower than in the interior solution.

2. If $L_2^* = R^2[R^1(L_2^*)] > \bar{L}$, we expect the Cournot solution to hold.

5.2. Stackelberg equilibrium

Consider now that we have a Stackelberg equilibrium. Suppose the leader is union 1, and the constraint is binding for union 2.

1. Let us see the following Fig. 5 below.

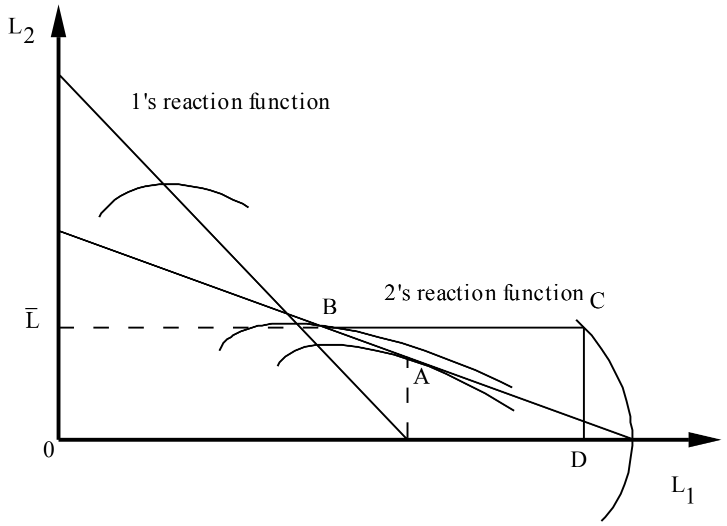

Figure 5.

In the picture, $\bar{L}$ is smaller than the value of L_2 in the Cournot game. Then, the best the Stackelberg leader can now do is to choose the corner B, where 1's employment is L_1^{**} such that:

$$\bar{L} = R^2(L_1^{**}) \tag{50}$$

That is,

$$L_1^{**} = R^2(\bar{L})^{-1} \tag{51}$$

Then total employment is equal to:

$$L = R^2(\bar{L})^{-1} + \bar{L} \tag{52}$$

In this case:

$$dL/d\bar{L} = 1/[dR^2/dL_1(\bar{L})] + 1 \tag{53}$$

This will be negative, if, with 2's reaction function is negatively sloped, 2's reaction function has slope smaller than 1 in absolute value.

From Fig. 5, we also conclude that total employment will be smaller than if the bound was not imposed, once B is to the the North-West of A on 2's reaction function, with slope smaller than 1 in absolute value.

Therefore:

Proposition 6: For a Stackelberg duopoly, if the floor is smaller than the follower solution in a Cournot environment, but larger than the follower's employment under the Stackelberg equilibrium:

1. The leader's solution corresponds to the inverse of the followers reaction function evaluated at the bound $\bar{L}$. The leader will loose utility and lower its employment relative to the unconstrained Stackelberg equilibrium.

2. The follower will employ the amount $\bar{L}$ and attain a higher utility than in the interior solution.

3. An increase in the floor will decrease the leader's employment; it will decrease total employment and increase the wage (till the floor reaches the follower's Cournot solution).

4. Employment is smaller and wage higher than in the interior solution.

2. If $\bar{L}$ is larger than the value of L_2 in the Cournot game, the best the Stackelberg leader can now do is to react according to its reaction function, i.e., to behave as a monopolist with respect to the residual demand, and we have the same equilibrium properties as in the constrained Cournot equilibrium.

Proposition 7: For a Stackelberg duopoly, if the floor is larger than the follower solution in a Cournot environment, the equilibrium will have the same features as the two-follower (constrained) case:

1. The leader reacts according to its reaction function. It will show a lower utility than in the unconstrained maximum.

2. The follower will employ the amount $\bar{L}$ and may attain a higher utility level than in the interior solution.

3. An increase in the floor will decrease the leader's employment; it will increase total employment and decrease the wage.

4. Employment may be larger or smaller and wage lower or higher than in the interior solution.

3. Finally:

Proposition 8: If $\bar{L}$ is lower than the value of L_2 of the Stackelberg game, the membership restriction will be inactive.

5.3. Entry-deterrence behavior

Consider now the possibility of union 1 deterring entrance. Let us look at Fig. 5. Union 1 indifference curve that touches point B, crosses the L_1 axis to the left of D. If union 1 chooses L_1 = D union 2 will go out of the market, say, it drives wages to zero. In that case it was not worthwhile. But it can happen something like what is depicted in Fig. 6. In this case, clearly union

1 prefers to deter entrance, once the indiference curve that touches point B is now associated with a lower utility than point D.

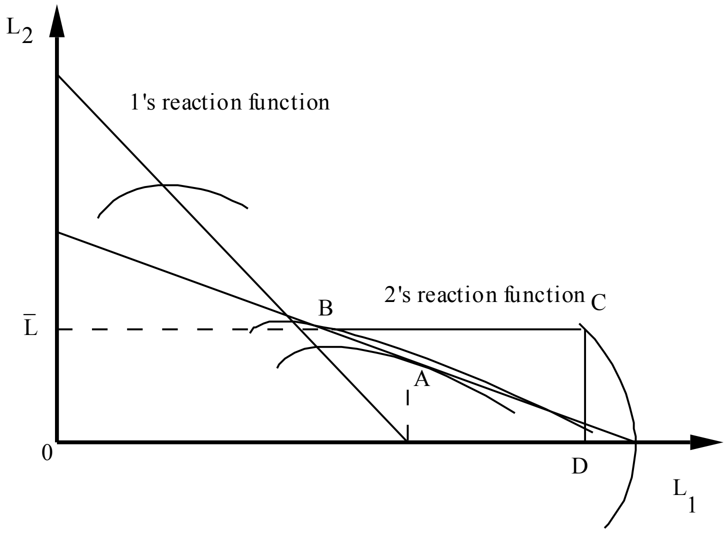

Figure 6.

She will achieve it by setting an L_1 of point D, which is the same as of point C. Point C is, at $L_2 = \bar{L}$, on 2's indifference curve that touches its reaction function at the inactivity level. Then we have that L_1 solves:

$$U^2[\bar{L}, W(L_1 + \bar{L})] = U^2\{0, W[R^2(0)^{-1}]\} \tag{54}$$

If we assume that unions' utility will only be equal to the utility level at 0 employment if (either own employment or) wage is 0, then L_1 must be such that if the potential entrant enters at the required $\bar{L}$, it drives wages to zero [21]. That is, to deter entrance, union 1 has to set :

$$L_1 = L(0) - \bar{L} = L \tag{55}$$

Then, employment decreases with the membership floor. Notice, however, that if both unions are affected by the floor, employment may actually increase when the floor increases. That is, if $L(0) - \bar{L} > \bar{L}$, or $\bar{L} < L(0) / 2$ - otherwise the incumbent will also hit the bound with this policy, and the best it can do is set $L_1 = \bar{L}$...

With (55):

$$W = W[L(0) - \bar{L}] \tag{56}$$

Clearly, this will be profitable iff:

Case A: For $R^2(L_1^S) < \bar{L} < R^2(R^1(L_2^C)) = L_2^C$

This corresponds to the situation of Proposition 6. It is depicted in Fig. 6.

$$U^1\{L(0) - \bar{L}, W[L(0) - \bar{L}]\} > U^1\{R^2(\bar{L})^{-1}, W[R^2(\bar{L})^{-1} + \bar{L}]\} \tag{57}$$

That is, it must yield higher utility than allowing the other union to enter at level $\bar{L}$.

Case B: For $R^2(L_1^S) < R^2(R^1(L_2^C)) = L_2^C < \bar{L}$

This corresponds to the situation of Proposition 7. In this case, the best union 1 could do was to react according to its reaction function. Well, this will yield a lower utility to union 1 than deterring entrance if:

$$U^1\{L(0) - \bar{L}, W[L(0) - \bar{L}]\} > U^1\{R^1(\bar{L}), W[R^1(\bar{L}) + \bar{L}]\} \tag{58}$$

This case is shown in Fig. 7.

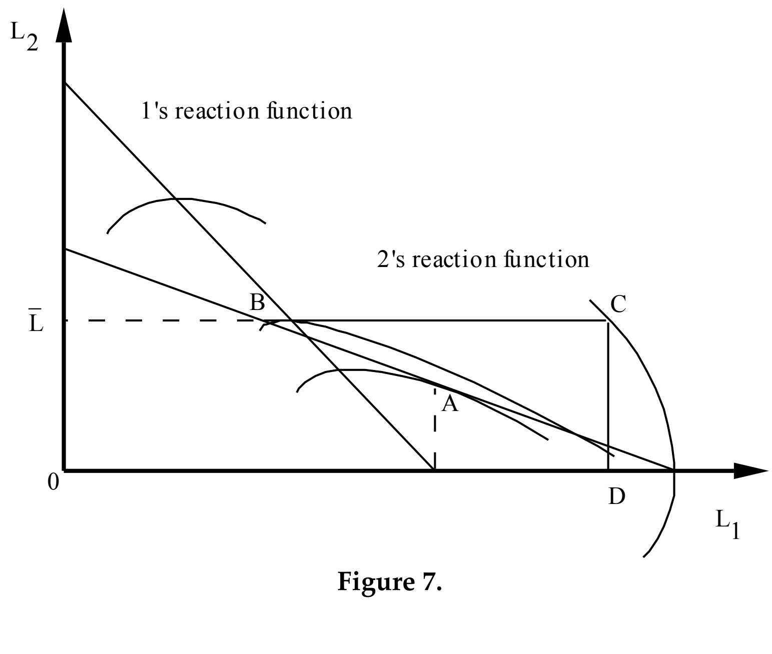

Figure 7.

Case C: For $R^2(L_1^S) > \bar{L}$

This corresponds to the situation of Proposition 8. Then, it may occur that it is more profitable to deter entrance than to allow the other union to enter at the Stackelberg level. It occurs if:

$$U^1\{L(0) - \bar{L}, W[L(0) - \bar{L}]\} > U^1\{L_1^S, W[L_1^S + R^2(L_1^S)]\} \tag{59}$$

This is depicted in Fig. 8.

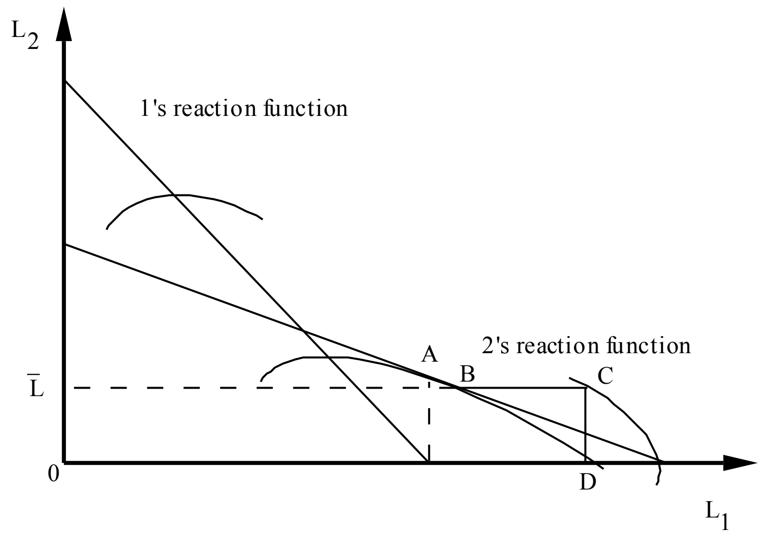

Figure 8.

Entry-deterrence may be more likely when 1 is a leader and 2 a follower. However, it can happen that 1 is a follower - or that it will end up by sharing the market in a Cournot game. Then we have similar conclusions as above, with L_1^S replaced by $L_1^* = R^1(R^2(L_1^*))$ in (59) of case C.

Notice that if we have a Cournot outcome, it may be worthwhile for the incumbent to engage in entry-deterrence even if $\bar{L} = 0$ (this will not occur for a Stackelberg equilibrium).

Proposition 9: 1. Entry deterrence by employment expansion may be profitable for a union leader when there are membership floors.

2. If it is, an increase in the membership floor will decrease total employment (equal to the leader's employment) and increase the wage - unless the leader has reached the floor itself.

3. Propositions 6, 7 and 8 hold with an addition: "provided entry-deterrence is not more profitable".

5.4. Efficient bargaining between unions

If only the second union is affected by the minimum employment requirement, the Nash-maximand becomes:

$$\max_{L_1, L_2, W} \; [U^1(L_1, W) - \bar{U}^1]^{\circledcirc} \; [U^2(L_2, W) - \bar{U}^2] \tag{60}$$

$$\text{s.t.:} \quad L_1 + L_2 = L(W) \quad \text{or} \quad W = W(L_1 + L_2)$$

$$L_2 \circledcirc \bar{L}$$

The lagrangean of the problem can be stated as:

$$\max_{L_1, L_2} \; \{U^1[L_1, W(L_1+L_2)] - \bar{U}^1\}^{\delta} \{U^2[L_2, W(L_1+L_2)] - \bar{U}^2\} + \circledcirc \; (L_2 - \bar{L}) \tag{61}$$

If the interior solution of the efficient bargaining problem without the restriction originates an $L_2^* > \bar{L}$, we will stay in the interior solution. If not, the corner solution, from first derivative with respect to L_1 - which will hold both in restricted or unrestricted solutions - will yield that, at $L_2 = \bar{L}$:

$$\delta \; (U^1_L + U^1_W \; W_L) / [U^1 - \bar{U}^1] + \; W_L \; U^2_W / [U^2 - \bar{U}^2] = 0 \tag{62}$$

and labor demand applies. (62) determines L_1. Notice that whenever we had that 1 reacted according to its reaction function:

$$U^1_L + U^1_W \; W_L = 0 \tag{63}$$

Therefore at that solution the maximand (61) is already decreasing - the efficient bargaining solution will yield a lower L_1 than if 1 reacted as a monopolist with respect to residual demand.

Proposition 10: With efficient bargaining between the unions, employment of the union not affected by the floor is lower than if the other union behaved as a monopolist towards residual demand - i.e., according to its reaction function with respect to $\bar{L}$.

6. Summary and conclusions

The paper extends the framework to model union competition behavior for employment in the presence of employment restrictions that prevent interior solutions. In particular, we analyzed the effects of insufficient membership and of the existence of minimum union employment (or employed membership) rules.

We focussed on the case with two unions and homogeneous labor, and investigate the features of the labor market outcomes when the unions behave as Cournot, Stackelberg or cooperate efficiently.

The main results can be summarized as follows:

1. An employment ceiling affecting one of the unions (or insufficient membership of one of the unions to attain the interior solution) will always benefit the other - no matter if the latter acts as a leader or as a Cournot follower -, which will be able to behave as a monopolist with respect to residual demand and lower its own employment.

If one of the unions is a Stackelberg leader and membership of the other is sufficient for the interior solution, the leader may find it worthwhile to decrease his employment pushing the other's to the bound.

In any case, when one of the union employs all its membership (i.e., is or is forced to the bound), total employment will be lower (the wage higher) than in the interior solution. An increase in that union's members will decrease the other's employment and raise total employment.

With a Stone-Geary utility function ($U^i(L_i,W) = W^{\theta_i} L_i^{(1-\theta_i)}$, $0<\theta_i<1$) and linear demand schedule ($W = a - b(L_1 + L_2)$) – results summarized in Tables 3 and 4 -, we concluded that insufficient membership is more likely to affect unions with higher preference for employment relative to wage. The cooperative solution will imply a larger employment of the unaffected union, and a smaller total employment (higher equilibrium wage) than in the unconstrained maximum.

2. Minimum employed members´ rules (: a minimum employed membership is required for a union to be considered legal) produce kinks and discontinuities in the unions' reaction functions. In general, these rules may benefit the union that faces the constraint directly; they will decrease the other union's utility relative to the interior solution. If the equilibrium - Cournot, Stackelberg or cooperative between unions - implies that a follower's interior solution is lower than the floor, his employment is pushed to this floor. An increase of the legal minimum will always increase total employment and decrease the equilibrium wage in a Cournot game; total employment is smaller and wage higher than in the interior solution. However, a change in the floor will decrease total employment (wage) in a Stackelberg game if the floor would allow the Cournot outcome but not the Stackelberg interior solution for the follower; then, total employment is higher and wage lower than in the interior solution.

With homogeneous workers, the existence of high employment floors may induce entry deterrence behavior of the incumbent(s). When entry deterrence is being practiced, an increase in the floor decreases total employment - once it makes the incumbent's behavior less "costly".

With a Stone-Geary utility function and linear demand schedule, we concluded that minimum employment rules are more likely to affect - directly - unions with higher preference for wage relative to employment. Entry deterrence practices seem more likely - "profitable" for the leader - when the follower has low preference for wage relative to employment; in that case, we expect larger total employment and lower wage than if entry deterrence was not engaged. The cooperative solution will imply a smaller employment of the unaffected union, a larger total employment and lower equilibrium wage than in the unconstrained maximum.

Table 1. *Insufficient Membership of Union 2.*

	Conditions	Equilibrium Solution
Cournot	$R^2(R^1(L_2^*)) = L_2^* > M_2$	$L_2 = M_2$ $L_1 = R^1(M_2)$
	$R^2(R^1(L_2^*)) = L_2^* < M_2$	$R^2(R^1(L_2^*)) = L_2^*$ $R^1(R^2(L_1^*)) = L_1^*$
	$R^2(L_1^S) = L_2^S > M_2$	$L_2 = M_2$ $L_1 = R^1(M_2)$
Stackelberg	$R^2(L_1^S) = L_2^S < M_2$ and $U^1\{L_1^S, \quad W[L_1^S + R^2(L_1^S)]\} >$ $U^1\{R^1(M_2), W[R^1(M_2) + M_2]\}$	$R^2(L_1^S) = L_2^S$ $U^1_L / U^1_W = -(1 + dR^2 / dL_1) W_L$
Employment - Pushing	$R^2(L_1^S) = L_2^S < M_2$ and $U^1\{L_1^S, \quad W[L_1^S + R^2(L_1^S)]\} <$ $U^1\{R^1(M_2), W[R^1(M_2) + M_2]\}$	$L_2 = M_2$ $L_1 = R^1(M_2)$
Efficient Bargaining	$L_2^* > M_2$	$L_2 = M_2$ $\delta \ (U^1_L + U^1_W \ W_L) / [U^1 - \bar{U}^1] +$ $+ W_L \ U^2_W / [U^2 - \bar{U}^2] = 0$
	$L_2^* < M_2$	$U^1_W / U^1_L + U^2_W / U^2_L = - L_W$ $\delta = \{[U^1(L_1, W) - \bar{U}^1] /$ $/ [U^2(L_2, W) - \bar{U}^2]\} \ (U^2_L / U^1_L)$

Table 2. *Insufficient Employment of Union 2.*

	Conditions	Equilibrium Solution
Cournot	$R^2(R^1(L_2^*)) = L_2^* < \bar{L}$	$L_2 = \bar{L}$ $L_1 = R^1(\bar{L})$
	$R^2(R^1(L_2^*)) = L_2^* > \bar{L}$	$R^2(R^1(L_2^*)) = L_2^*$ $R^1(R^2(L_1^*)) = L_1^*$
Stackelberg	$R^2(R^1(L_2^*)) = L_2^* > \bar{L} > L_2^S$	$L_2 = \bar{L}$ L_1 such that $\bar{L} = R^2(L_1)$
	$R^2(R^1(L_2^*)) = L_2^* < \bar{L}$	$L_2 = \bar{L}$ $L_1 = R^1(\bar{L})$
	$\bar{L} < L_2^S$	$R^2(L_1^S) = L_2^S$ $U^1_L / U^1_W = -(1 + dR^2 / dL_1)\, W_L$
Entry Deterrence	(One for each Stackelberg case)	$L_2 = 0$ $L_1 = L(0) - \bar{L}$
Efficient Bargaining	$L_2^* < \bar{L}$	$L_2 = \bar{L}$ $\delta\ (U^1_L + U^1_W\ W_L) / [U^1 - \bar{U}^1] +$ $+ W_L\ U^2_W / [U^2 - \bar{U}^2] = 0$
	$L_2^* > \bar{L}$	$U^1_W / U^1_L + U^2_W / U^2_L = - L_W$ $\delta = \{[U^1(L_1, W) - \bar{U}^1] /$ $/ [U^2(L_2, W) - \bar{U}^2]\}\ (U^2_L / U^1_L)$

Table 3. *Insufficient Membership of Union 2 - Stone-Geary Utility and Linear Demand*

	Conditions	Equilibrium Solution
Cournot	$(a/b)\ (1-\theta_2)\ \theta_1\ /$ $/\ [1-(1-\theta_1)\ (1-\theta_2)] > M_2$	$L_2 = M_2$ $L_1 = (1-\theta_1)\ (a/b - M_2)$
	$(a/b)\ (1-\theta_2)\ \theta_1\ /$ $/\ [1-(1-\theta_1)\ (1-\theta_2)] < M_2$	$L_2^* = (a/b)\ (1-\theta_2)\ \theta_1\ /$ $/\ [1-(1-\theta_1)\ (1-\theta_2)]$ $L_1^* = (a/b)\ (1-\theta_1)\ \theta_2\ /$ $/\ [1-(1-\theta_1)\ (1-\theta_2)]$
Stackelberg	$(a/b)\ (1-\theta_2)\ \theta_1 > M_2$	$L_2 = M_2$ $L_1 = (1-\theta_1)\ (a/b - M_2)$
	$M_2 > (a/b)\ (1-\theta_2\theta^1)$	$L_2^S = \theta_1\ a\ (1-\theta_2)\ /\ b$ $L_1^S = a\ (1-\theta_1)\ /\ b$
Employment - Pushing	$(a/b)\ (1-\theta_2)\ \theta_1 < M_2$ and $M_2 < (a/b)\ (1-\theta_2\theta^1)$	$L_2 = M_2$ $L_1 = (1-\theta_1)\ (a/b - M_2)$
Efficient Bargaining	$a\ (1-\theta_2)\ /\ [\ b\ (\theta+1)] > M_2$	$L_2 = M_2$ $L_1 = (1-\theta_1)\ (a/b - M_2)\ /\ (1+\theta_2\ /\ \circledcirc)$
	$a\ (1-\theta_2)\ /\ [\ b\ (\circledcirc + 1)] < M_2$	$L_2 = a\ (1-\theta_2)\ /\ [\ b\ (\theta+1)]$ $L_1 = \theta\ a\ (1-\theta_1)\ /\ [\ b\ (\theta+1)]$

Table 3.1. *Insufficient Membership of Union 2 - Stone-Geary Utility and Linear Demand*

	Conditions	Equilibrium Solution
Cournot	$(a/b)\,(1-\theta_2)\,\theta_1\,/$ $/\,[1-(1-\theta_1)\,(1-\theta_2)] > M_2$	$L = (a/b)\,(1-\theta_1) + \theta_1\,M_2$ $W = \theta_1\,(a - b\,M_2)$
	$(a/b)\,(1-\theta_2)\,\theta_1\,/$ $/\,[1-(1-\theta_1)\,(1-\theta_2)] < M_2$	$L = (a/b)\,[(1-\theta_2)\,\theta_1 + (1-\theta_1)\,\theta_2]\,/$ $/\,[1-(1-\theta_1)\,(1-\theta_2)]$ $W = a\,\theta_1\,\theta_2\,/\,[1-(1-\theta_1)\,(1-\theta_2)]$
Stackelberg	$(a/b)\,(1-\theta_2)\,\theta_1 > M_2$	$L = (a/b)\,(1-\theta_1) + \theta_1\,M_2$ $W = \theta_1\,(a - b\,M_2)$
	$M_2 > (a/b)\,(1-\theta_2\theta^1)$	$L = a\,(1-\theta_1\,\theta_2)\,/\,b$ $W = a\,\theta_1\,\theta_2$
Employment - Pushing	$(a/b)\,(1-\theta_2)\,\theta_1 < M_2$ and $M_2 < (a/b)\,(1-\theta_2\theta^1)$	$L = (a/b)\,(1-\theta_1) + \theta_1\,M_2$ $W = \theta_1\,(a - b\,M_2)$
Efficient Bargaining	$a\,(1-\theta_2)\,/\,[\,b\,(\theta+1)] > M_2$	$L = [(a/b)(1-\theta_1) + (\theta_1 + \theta_2\,/\,\theta)\,M_2]\,/$ $/\,(1+\theta_2\,/\,\theta)$ $W = (a - b\,M_2)\,(\theta_1+\theta_2)\,/\,(\theta+\theta_2)$
	$a\,(1-\theta_2)\,/\,[\,b\,(\theta+1)] < M_2$	$L = a\,[\theta\,(1-\theta_1) + (1-\theta_2)]\,/\,[b\,(\theta+1)]$ $W = a\,(\theta_1+\theta_2)\,/\,[b\,(\theta+1)]$

Table 4. *Insufficient Employment of Union 2 - Stone-Geary Utility and Linear Demand*

	Conditions	Equilibrium Solution
Cournot	$(a/b)\,(1-\theta_2) \circledcirc \theta_1 /$ $/ [1-(1-\theta_1)(1-\theta_2)] < \bar{L}$	$L_2 = \bar{L}$ $L_1 = (1-\theta_1)(a/b - \bar{L})$
	$(a/b)\,(1-\theta_2)\,\theta_1 /$ $/ [1-(1-\theta_1)(1-\theta_2)] > \bar{L}$	$L_2^* = (a/b)\,(1-\theta_2)\,\theta_1 /$ $/ [1-(1-\theta_1)(1-\theta_2)]$ $L_1^* = (a/b)\,(1-\theta_1)\,\theta_2 /$ $/ [1-(1-\theta_1)(1-\theta_2)]$
Stackelberg	$(a/b)\,(1-\theta_2)\,\theta_1 /$ $/ [1-(1-\theta_1)(1-\theta_2)] > \bar{L} > (a/b)\,(1-\theta_2)\,\theta_1$	$L_2 = \bar{L}$ $L_1 = (a/b) - [\bar{L} / (1-\theta_2)]$
	$(a/b)\,(1-\theta_2)\,\theta_1 /$ $/ [1-(1-\theta_1)(1-\theta_2)] < \bar{L}$	$L_2 = \bar{L}$ $L_1 = (1-\theta_1)(a/b - \bar{L})$
	$\bar{L} < (a/b)\,(1-\theta_2)\,\theta_1$	$L_2^S = \theta_1\, a\,(1-\theta_2)/b$ $L_1^S = a\,(1-\theta_1)/b$
Entry Deterrence	(One for each Stackelberg case)	$L_2 = 0$ $L_1 = (a/b) - \bar{L}$
Efficient Bargaining	$a\,(1-\theta_2) / [\,b\,(\theta+1)] < \bar{L}$	$L_2 = \bar{L}$ $L_1 = (1-\theta_1)(a/b - \bar{L}) / (1+\theta_2/\theta)$
	$a\,(1-\theta_2) / [\,b\,(\theta+1)] > \bar{L}$	$L_2 = a\,(1-\theta_2) / [\,b\,(\theta+1)]$ $L_1 = \theta\; a\,(1-\theta_1) / [\,b\,(\theta+1)]$

Table 4.1. *Insufficient Employment of Union 2 - Stone-Geary Utility and Linear Demand*

	Conditions	Equilibrium Solution
Cournot	$(a/b)(1-\theta_2)\theta_1 / [1-(1-\theta_1)(1-\theta_2)] < \bar{L}$	$L = (a/b)(1-\theta_1) + \circledcirc\theta_1 \bar{L}$ $W = \theta_1 (a - b\bar{L})$
	$(a/b)(1-\theta_2)\theta_1 / [1-(1-\theta_1)(1-\theta_2)] > \bar{L}$	$L = (a/b)[(1-\theta_2)\theta_1 + (1-\theta_1)\theta_2] / [1-(1-\theta_1)(1-\theta_2)]$ $W = a\theta_1\theta_2 / [1-(1-\theta_1)(1-\theta_2)]$
Stackelberg	$(a/b)(1-\theta_2)\theta_1 / [1-(1-\theta_1)(1-\theta_2)] > \bar{L} > (a/b)(1-\theta_2)\theta_1$	$L = (a/b) - [\theta_2/(1-\theta_2)]\bar{L}$ $W = b[\theta_2/(1-\theta_2)]\bar{L}$
	$(a/b)(1-\theta_2)\theta_1 / [1-(1-\theta_1)(1-\theta_2)] < \bar{L}$	$L = (a/b)(1-\theta_1) + \theta_1\bar{L}$ $W = \theta_1(a - b\bar{L})$
	$\bar{L} < (a/b)(1-\theta_2)\theta_1$	$L = a(1-\theta_1\theta_2)/b$ $W = a\theta_1\theta_2$
Entry Deterrence	(One for each Stackelberg case)	$L = (a/b) - \bar{L}$ $W = b\bar{L}$
Efficient Bargaining	$a(1-\theta_2)/[b(\theta+1)] < \bar{L}$	$L = [(a/b)(1-\theta_1) + (\theta_1 + \theta_2/\theta)\bar{L}] / (1 + \theta_2/\theta)$ $W = (a - b\bar{L})(\theta_1+\theta_2)/(\theta+\theta_2)$
	$a(1-\theta_2)/[b(\theta+1)] > \bar{L}$	$L = a[\theta(1-\theta_1) + (1-\theta_2)] / [b(\theta+1)]$ $W = a(\theta_1+\theta_2)/[b(\theta+1)]$

Notes

[1] Citing Rosen (1970) as the first author to recognize strategic interdependency among unions.
[2] Also Davidson (1988), Dixon (1988), Dowrick (1989), Jun (1989) and Dobson (1994), for example, where the effect of the existence of oligopoly in the product market is investigated.
[3] After Cournot (1838).
[4] After Nash (1950).
[5] Von Stackelberg (1934).
[6] In MacDonald & Solow's (1981) lines.
[7] Article 8º, §.2, D.L. 215-B/75, April 30th. Changes in union regulations are subject to similar requirements (10% of associates or 2000 workers) - Article 43º, §.1. "Unions" and Federations require one third of target unions – of the region or cathegory, respectively –, obbeying some majority of affiliated workers criteria, according to Article 8º, §.3. See Bettencourt & Baptista (1999).
[8] See, for example, Spence (1977), Dixit (1979 and 1980), and Schmalensee (1981) for the analysis of entry barriers in the product market, which work similarly to these restrictions.
[9] Even if in a different manner: Dixit's fixed costs introduce discontinuities in the reaction functions while in our case the type of employment constraints we consider produce kinks; discontinuities only arise with minimum employment rules.
[10] See Adnett (1996), p.27.
[11] See Prazeres (2001).
[12] CGTP-IN and UGT, the two major union confederations – that coordenate activity of ("primary") unions, filliated in "unions" or federations. See Cerdeira (1997), p.57.
[13] Founded in 1978. See Cerdeira (1997), p.16, footnote 12.
[14] See Cerdeira (1997), p. 83.
[15] The reader is referred to Martins & Coimbra (1997) for additional comments on the solutions for two unions. This section summarizes the main results for the general case, needed for the following exposition.
[16] Nevertheless, most of the results below would also apply if this function represented the marginal revenue product of labor and if firms did not behave competitively in the product market.
[17] Existence is guaranteed by concavity of each union's utility function with respect to L_i, and uniqueness is satisfied if $dR^1/dL_2 \,/\, (1+dR^1/\, dL_2) + dR^2/dL_1 \,/\, (1+dR^2/\, dL_1) < 0$, which will hold if $-1 \leq dR^i \,/\, dL_j \leq 0$, i=1,2, j=2,1. This ensures that optimal L_i falls as L rises. See Friedman (1983), p. 30-33.
[18] If we departed from a Cournot equilibrium...
[19] A reasonable assumption would make θ equal to $M_1 \,/\, M_2$, the number of members of union 1 divided by the number of members of union 2. See Martins & Coimbra (1997 and 1997a) for additional interpretation.
[20] Which could be derived from the problem

$$\max_{L_1,\, L_2,\, W} \quad U^1(L_1,W)$$

$$\text{s.t.:} \quad U^2(L_2,W) \geq \bar{U}^2$$

$$W = W(L_1+L_2)$$

[21] Or to the minimum acceptable wage for union 2 to stay in the labor market according to the shape of its utility function, i.e., $W = W[R^2(0)^{-1}]$, being $R^2(0)^{-1}$ the level of L_1 for which

union 2 reacts with $L_2 = R^2(L_1) = 0$. For simplicity, we assume it to be 0, i.e., $W[R^2(0)^{-1}] = 0$. A more general formulation would replace L(0) by $[R^2(0)^{-1}]$ in the formulas below.

Appendix

We want to show that in a Cournot duopoly:

$$\text{If } R^2(R^1(L_2^*)) > M_2, \quad \text{then,} \quad R^2(R^1(M_2)) > M_2 \tag{A1}$$

Given that reaction functions are negatively sloped, we know that

$$R^2(R^1(M_2)) < R^2(R^1(L_2)) \quad \text{if} \quad M_2 < L_2 \tag{A2}$$

With a stable equilibrium, it cannot occur, however, that at $L_2^* = R^2(R^1(L_2^*))$:

$$R^2(R^1(M_2)) < M_2 < R^2(R^1(L_2^*)) = L_2^* \tag{A3}$$

and it will be the case that:

$$M_2 < R^2(R^1(M_2)) < R^2(R^1(L_2^*)) = L_2^* \tag{A4}$$

The reason is that, with stability, a small increase in the argument of $R^2(R^1(.))$ will originate an increase in the function which is smaller than one. For an infinitesimal variation of M_2 around L_2^*:

$$R^2(R^1(L_2^*)) - R^2(R^1(M_2)) \approx (dR^2/dL_1)\,(dR^1/dL_2)\,(L_2^* - M_2) \tag{A5}$$

If

$$(dR^2/dL_1)\,(dR^1/dL_2) < 1 \tag{A6}$$

then:

$$(dR^2/dL_1)\,(dR^1/dL_2)\,(L_2^* - M_2) \approx R^2(R^1(L_2^*)) - R^2(R^1(M_2)) < L_2^* - M_2 \tag{A7}$$

Therefore

$$R^2(R^2(L_2^*)) - R^2(R^1(M_2)) + M_2 < L_2^* \tag{A8}$$

Because $R^2(R^1(L_2^*)) = L_2^*$, this implies that:

$$M_2 < R^2(R^1(M_2)) \tag{A9}$$

Therefore, when $M_2 < L_2^* = R^2(R^1(L_2^*))$, we will have the corner equilibrium described by (20).

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