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Mechanism of Hyperbolic Growth Explained

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Abstract. Fundamental law of growth is used to explain the mechanism of hyperbolic growth of human population and of the Gross Domestic Product (GDP). Hyperbolic growth is described by a simple mathematical formula and the explanation of its mechanism turns out to be also simple. Historical economic growth was prompted by the familiar net market force, which was on average directly proportional to the existing wealth expressed usually as the GDP. The larger was the GDP, the stronger was the driving force and the faster was the economic growth. It is shown that this simple force generates hyperbolic growth. No other force is required. Hyperbolic growth is not assumed but derived when using this force. Historical growth of population was prompted by the biologically driven force of procreation, which was on average approximately constant per person. This force includes the natural, familiar, biologically controlled process of births, aging and dying. Here again, hyperbolic growth is not assumed but derived when using this force. Explanation of two demographic transitions in the past 12,000 years in the growth of population and of the currently experienced transition is also proposed. Currently, economic growth and the growth of population are no longer unconstrained. Other additional forces contribute significantly to the growth process and the growth is no longer hyperbolic.
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1. Introduction

Hyperbolic growth gives a remarkably good description of population and economic data (Nielsen, 2014, 2016a, 2016b, 2016c). It describes historical growth of the Gross Domestic Product (GDP) and of population, global and regional and even in individual countries. This conclusion is based on the analysis of the extensive data published by Maddison (2010). They describe the economic growth and the growth of population during the AD era, starting from AD 1 and extending to 2008. Hyperbolic growth describes also remarkably well the growth of global population in the past 12,000 years (Nielsen, 2016a). This analysis is supported by population data coming from a wide range of sources (Biraben, 1980; Clark, 1968; Cook, 1960; Durand, 1974; Gallant, 1990; Haub, 1995; Livi-Bacci, 1997; Maddison, 2010; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994, United Nations, 1973, 1999, 2013).

Hyperbolic growth of population was first noticed by von Foerster, Mora & Amiot (1960) close to 60 years ago and it was soon confirmed and accepted by other authors (Kapitza, 2006; Kremer, 1993; Podlazov, 2002; Shklovskii, 1962,

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2002; von Hoerner, 1975). Hyperbolic growth turns out to be exceptionally stable and generally undisturbed. Many driving forces might be considered as influencing growth. For the growth of human population, and as pointed out by Kapitza (2006), all these forces can be arranged in such categories as industrial, economic, cultural, social and biological. However, he also pointed out that the simple formula describing the growth of the world population suggests that many of these forces must have been "suppressed by the process of averaging" (Kapitza 2006, p. 77).

Economic growth is also described by the simple hyperbolic formula and in general it has been also stable over a long time in the past suggesting a simple explanation of the mechanism of growth and indicating that the growth must have been also controlled by single net force. It is the aim of this publication to identify these dominating forces of growth and to explain the mechanism of the historical hyperbolic growth of population and of the GDP.

2. Mechanism of growth

2.1. Mechanism of the historical economic growth

Gross profit may depend on many factors but it obviously depends on the size of investment. "Money makes money. And the money that makes money makes more money" (Benjamin Franklin). Economic growth is directly related to the size of our investments. With the sufficiently high investment, we can build more retail stores, or larger retail outlets, we can buy more goods for sale, employ more people in our business, buy more tools and machinery, invest in a better equipment to increase production, build more houses either for sale or for rent, build more factories, improve agriculture, improve our services, pay for advertising, pay for the transportation and distribution of goods and support all other necessary activities aimed at generating profit. According to the well-known theory of Cobb & Douglas (1928), production yield can be described by the following simple equation:

$$Y = aL^{\alpha}K^{\beta}. \tag{1}$$

Where Y is the production yield, a is the so-called total factor productivity, L is labour expressed as person-hours during a given time, e.g. during one year, K is capital input (the money invested in the equipment, buildings or anything else to support production), α and β are constants, $\alpha + \beta = 1$, $0 < \alpha < 1$ and $0 < \beta < 1$.

In this equation, wealth generates wealth or money makes money not only through the investment K, which could be passed from one year to another, but also through the ongoing costs of labour.

In essence, therefore, the right-hand side of the eqn (1) represents the investment of a certain amount of money to produce profit. The left-hand side does not represent the total wealth but the increase in wealth, which could be the annual increase. This increase is proportional to the money locked as K and to the annual investment of money expressed as L. We need money to make money. We need wealth to generate wealth.

In order to explain the mechanism of economic growth we shall look at it from the point of view of a driving force, because driving force represents the mechanism of growth. For the economic growth, it is the net market force. We can have many market forces but in order to explain the mechanism of growth it is best to start with the simplest assumption and make it complicated only if necessary. This is the fundamental principle in scientific research, known as the Occam's razor or the law of parsimony: *Entia non sunt multiplicanda praeter necessitatem.*

The simplest way to describe mathematically the driving force of economic growth is to assume that it is directly proportional to the invested wealth. The larger is the circulated wealth, the greater wealth can be produced.

$$F = cW \, , \tag{2}$$

where W is the total existing wealth and c is a constant.

It is essential to understand that we are dealing here with average quantities. In explaining economic growth of a country or region or of the world we are not dealing with individual economic units but with the whole assembly of these units. The eqn (2) describes the *average* force of economic growth. The quantity W represents the total wealth of a country, a group of countries or of the whole world, expressed usually as the GDP and c could represent the average fraction of this wealth used to drive economic growth. The larger is the already generated wealth, the larger is the driving force of economic growth when this wealth is invested to produce more wealth. Wealth generates wealth. This principle and this process appears to be well known and universally accepted. However, this principle has been never expressed in mathematical form, which could be compared directly with data.It was never used to describe economic growth trajectories. It was never used to describe and explain the mechanism of the historical growth of the GDP.

In our earlier publication (Nielsen, 2016d), we have formulated a general law of growth:

$$F = rG \, , \tag{3}$$

where G is the growth rate and r is the resistance to growth.

The advantage of using this simple law of growth is that it links the force of growth with trajectories of a growing entity. The force of growth represents the mechanism of growth and the law of growth allows for defining this force, i.e. for defining the expected or postulated mechanism, and to compare it with data as described by growth trajectories. This simple law allows for a mathematical formulation of postulated mechanism and for translating this mechanism into growth trajectories, which can be readily tested by data. Thus, this law allows for testing various mechanisms of growth by data.

The growth rate G is defined as

$$G \equiv \frac{1}{W}\frac{dW}{dt} \, , \tag{4}$$

where t is time.

If we now insert the postulated driving force of economic growth defined by the eqn (2) into the eqn (3), we shall get the following equation describing economic growth.

$$\frac{1}{W}\frac{dW}{dt} = kW \, , \tag{5}$$

where $k \equiv c / r$.

We have now linked the driving force with economic growth trajectory. The parameter k is inversely proportional to the resistance to growth r and could be called the compliance factor or simply the compliance. In the formulation of the general law of growth (Nielsen, 2016d) we have defined $1/r$ as compliance. However, k differs only by a constant c so it plays the same role as $1/r$. The larger is the parameter k, the more efficient is the generation of wealth and the faster is the growth of W. We could easily extend this model by considering that c or r or both of them depend on time, but at this stage it is preferable to use the simplest possible assumption.

The eqn (5) does not describe the growth of an individual economic unit but the average economic growth of a country, region or globally. Economic growth of a single unit might be affected by many random forces but for a large assembly of such units, random forces might be averaging out. If they are not or if there is some other strong force not included in our simple assumption, then our predictions of growth will be contradicted by data and we shall have to modify our assumed mechanism of growth. We can check whether our assumption is correct by comparing the calculated trajectory with data.

The eqn (5) can be solved using the substitution $W = Z^{-1}$. Its solution is

$$W = \frac{1}{C - kt}. \tag{6}$$

This is hyperbolic growth. Data describing historical economic growth (Maddison, 2010) and their analysis (Nielsen, 2016b) show that our choice of the driving force was correct and that there is no need to assume the presence of any other type of forces. An example of comparing calculations with data is presented in Figure 1.

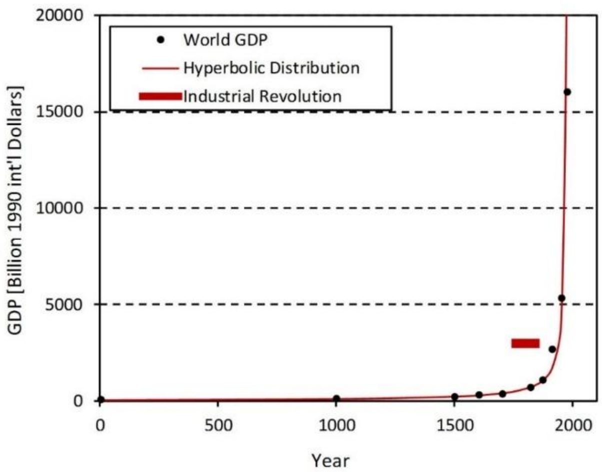

Figure 1. *World economic growth as described by the Gross Domestic Product (Maddison, 2010) compared with hyperbolic distribution. Single and simple driving force explains the mechanism of growth. This force was so strong that even the Industrial Revolution had no impact on changing the growth trajectory.*

Data and their analysis show that the historical economic growth was indeed hyperbolic and now we can understand why. *Historical economic growth was prompted by a single dominant force directly proportional to the existing volume of wealth, expressed usually as the GDP.* Hyperbolic economic growth describes the net historical growth of a large number of economic units. The larger was the existing wealth of a country or a region or globally, the larger was the driving force of economic growth. Currently, economic growth is no longer hyperbolic. It is no longer controlled by the simple force given by the eqn (2). Driving forces appear now to be now more complicated.

It should be noted that the growth described by compound interest is of a different kind. It is not a spontaneous and unconstrained growth controlled by the net driving force proportional to the size of the existing wealth. The force controlling the growth described by compound interest is constrained. It is dictated by human-imposed regulations. No bank in the world would pay interest increasing in the direct proportion to the balance of our deposits. For the money deposited in the bank, interest varies within a small range of values and consequently it is approximately constant. This type of growth is described by a constant or approximately constant force of growth, which generates exponential growth, the growth described by compound interest. Likewise, no bank in the world would give a loan with interest decreasing with the decreasing balance. These two types of transactions are controlled by man-made regulations. They are not controlled by the assumed by us, and confirmed by data, force describing the spontaneous and unconstrained historical economic growth. However, it does not mean that the current economic growth cannot be exponential. It can and it often is because, as indicated by data, the current economic growth is no longer prompted by the historically prevailing single force.

2.2. Mechanism of the historical growth of population

The most obvious and essential force, which has to be considered to explain the mechanism of the growth of population is obviously the biologically-controlled or prompted force of procreation, which is defined here as the difference between biologically controlled birth and death rates. Other forces might be included, if necessary, but this force is indispensable.

Let us assume that *on average*, the biologically controlled force of procreation is constant *per person*. Biologically controlled birth and death rates may vary over time but we assume that on average and per person the difference remains the same. This is a very simple assumption but again in science it is always advisable to use the simplest possible assumptions and make them more complicated only if necessary. Under this assumption,

$$\frac{F}{S} = c\,, \tag{7}$$

where F is the biologically controlled force of procreation, S is the size of the population and c is certain average constant. It describes how, on average, each person contributes to the growth of population.

If we use this force in the general law of growth given by the eqn (3) we shall get

$$cS = rG\,, \tag{8}$$

where G is now given by

$$G = \frac{1}{S}\frac{dS}{dt}\,, \tag{9}$$

which leads to the following differential equation describing the growth of population

$$\frac{1}{S}\frac{dS}{dt} = kS\,. \tag{10}$$

Solution to this equation is

$$S = \frac{1}{C - kt}\,. \tag{11}$$

It is also a hyperbolic distribution, which gives excellent description of data (Nielsen, 2016a, 2016c). Example is shown in Figure 2.

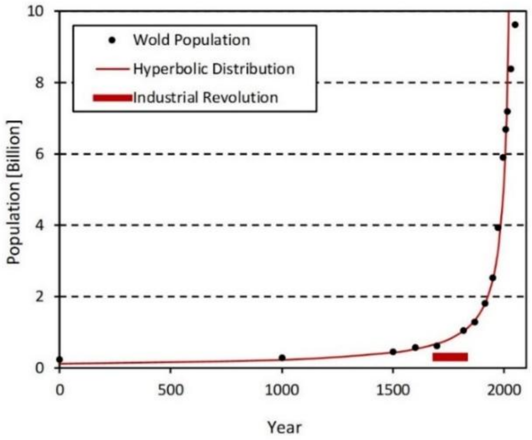

Figure 2. *Growth of the world population (Maddison, 2010) compared with hyperbolic distribution. Single and simple driving force explains the mechanism of growth. This force was so strong that even the Industrial Revolution had no impact on changing the growth trajectory.*

The mechanism of historical hyperbolic growth of population is explained as an unconstrained growth prompted solely by the biologically controlled force of procreation. This force is given by the average difference between biologically controlled birth and death rates and is assumed to be constant per person. This simple mechanism explains global and regional historical growth of population (Nielsen, 2016a, 2016c).

2.3. Mechanism of demographic transitions

If we include in our analysis a wider range of data describing the growth of the world population (Biraben, 1980; Clark, 1968; Cook, 1960; Durand, 1974; Gallant, 1990; Haub, 1995; Livi-Bacci, 1997; Maddison, 2010; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994, United Nations, 1973, 1999, 2013) we shall soon discover certain interesting details showing two demographic transitions in the past and the current ongoing transition (Nielsen, 2016a).

As we have shown earlier (Nielsen, 2016a), growth of the world population was hyperbolic between 10,000 BC and 500 BC, between AD 500 and 1200, and between AD 1400 and 1950. During these large sections of time, taking approximately 90% of the past 12,000 years, the mechanism of growth of population can be explained as being prompted by the simple, biologically controlled, force of procreation, which was on average constant per person. All other forces, even if present, had no influence on the growth of global population. They were either too weak or they were averaged out.

The time when the prevailing hyperbolic growth was significantly disturbed in the past 12,000 years was only between 500 BC and AD 500, between and AD 1200 and 1400 and now after around 1950. These are the only recorded demographic transitions in the past 12,000 years. The first transition was from a fast to a slow hyperbolic trajectory. The second transition was from a slow to a slightly faster trajectory and the current transition is to a yet unknown trajectory.

The first transition appears to coincide with the massive and widespread changes in the *style of living* associated with the intensified changes in the political landscape in various parts of the world, graphically and comprehensively explained by Teeple (2002). It is also probably not without significance that this transition coincides with the rise and fall of Roman Empire, the longest lasting political system in history, which by the first century BC ruled already over vast areas of land surrounding Mare Nostrum (the Mediterranean). After its fast expansion and after subjugating many independently-living societies under its rule, this powerful and seemingly unconquerable political structure disintegrated into many fragments of independent countries. However, during that long time, significant changes in the political landscape were also occurring outside the realm of the Roman Empire.

Between 10,000 and 500 BC, growth of population is described by a fast-increasing hyperbolic trajectory, as defined by the parameter k. After the BC/AD transition, the growth was directed to a significantly slower trajectory characterised now by the parameter k, which was about 6.4 times smaller. (The resistance to growth was now significantly larger.) Thus, the proposed explanation of the BC/AD transition is that it was caused by strong exogenous forces of political nature, forces causing the wide-spread and profound changes in the style of living. During that time, the resistance to growth was changing and eventually settled along a significantly larger value.

Demographic transition between AD 1200 and 1400 is much easier to explain. During that time, there was a temporary delay in the growth of human population. When closely inspected, it can be found that this delay coincides with *themost unusual convergence of demographic catastrophes*. It appears to have been caused by a *combined impact of five* large demographic catastrophes (Nielsen, 2016a): Mongolian Conquest (1260-1295) with the total estimated death toll of 40 million; Great European Famine (1315-1318), 7.5 million; the 15-year Famine in China (1333-1348), 9 million; Black Death (1343-1352), 25 million; and the Fall of Yuan Dynasty (1351-1369), 7.5 million.

During this transition, hyperbolic growth changed to a slightly faster trajectory, characterised by k only about 30% higher. This is the only available evidence that the growth of human population might have been affected by demographic catastrophes. However, their combined impact was small. The transition to a faster trajectory quickly compensated for the loss of time in the growth of population. This quick process of recovery could be explained by the regenerating impacts of Malthusian positive checks (Malthus, 1798; Nielsen, 2016f).

Currently, after a minor boosting around 1950, the growth of human population is slowing down. The possible explanation of the current diversion to a slower trajectory appears to be of *endogenous nature associated with human choices and motivations, voluntary or enforced*. While in many countries there is an increasing tendency to opt for smaller families, in China, small families have been enforced by legislation. This additional force appears to be the force of preventive checks (Malthus, 1798). They may have been active in the past but they were too weak to shape the growth trajectories.

So, the three demographic transitions in the past 12,000 years, including the ongoing transition, can be probably explained by three different forces: political forces active during the first transition, which lasted for about 1000 years; forces of demographic catastrophes, which were active for about 200 years; and the endogenous forces of personal choices, either voluntary or enforced by law during the current transition.

It would be difficult to describe mathematically all these complex forces. However, as already mentioned, the first two transitions were between hyperbolic trajectories characterised by different *k* factors. During these transitions, the *k* factor was changing. During the first transition, *k* factor dramatically decreased, which means that the resistance to growth dramatically increased. It increased by a massive factor of about 6.4. During the second transition, *k* factor slightly increased. The resistance to growth decreased by about 30%. Thedescription of the past and present demographic transitions can be reduced to the description of changes in the compliance factor or in the corresponding resistance to growth. Resistance to growth was changing and we can study how it was changing. Such a study will not give a complete mathematical explanation of the mechanism of demographic transitions but will reduce this explanation to a single parameter: to changes in the compliance factor *k* or in the corresponding resistance to growth.

We can study these changes using a slightly modified eqn (8). If we assume that the resistance to growth was dependent on time (or equivalently that *k* depended on time), then we shall have the following equation describing growth trajectories during demographic transitions:

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = k(t)S(t). \qquad (12)$$

Now, for better clarity, we are showing explicitly the dependence on time. Solution of this equation is

$$S(t) = -\left[\int k(t)dt\right]^{-1}. \qquad (13)$$

If we assume that $k(t)$ is represented by an *n*-order polynomial,

$$k(t) = \sum_{i=0}^{n} a_i t^i\ , \qquad (14)$$

then

$$S(t) = \left[\sum_{j=0}^{n+1} b_j t^j\right]^{-1}, \qquad (15)$$

where $b_j = -a_{j-1} / j$ for $j > 0$ and b_0 is the constant of integration.

Even though we cannot describe mathematically the mechanism of growth during the demographic transitions, we can understand them a little better by studying changes in the growth factor $k(t)$, whose reciprocal values represent resistance to growth. Results are shown in Figure 3. The corresponding parameters are listed in Table 1. These calculations do not explain *why* the resistance to growth was changing (they do not explain the mechanism of the demographic transitions) but at least they are describing *how* the resistance to growth (or the compliance factor) was changing.

In the lower section of Figure 3, we show the growth trajectory during the AD era. It is made of two hyperbolic trajectories, between AD 500 and 1200 and between AD 1400 and 1950. The remaining segments of time represent demographic transitions described by the reciprocal values of polynomials, as given by the eqn (15). This section shows also one of the projected trajectories.

In the middle section, we show the overall fit to the data, which is represented by hyperbolic trajectories between 10,000 BC and 500 BC, between AD 500 and 1200 and between AD 1400 and 1950. The remaining segments of time represent demographic transitions described by the reciprocal values of polynomials [see eqn (15)].

In the top section, we show time dependence of the compliance factor $k(t)$, which can be calculated using the fitted $S(t)$. As we can see from the eqn (13)

$$k(t) = -\frac{dZ(t)}{dt}, \tag{16}$$

where $Z(t) \equiv S^{-1}(t)$.

In Figure 3, we show the compliance factor $k(t)$ only down to 2000 BC. However, this factor was constant between 10,000 BC and 500 BC but then started to decrease. The compliance was decreasing, the resistance to growth was increasing and the growth of population was slowing down. Around 80 BC, the compliance factor decreased to zero, the resistance to growth increased to infinity and the growth of population reached its maximum. The compliance factor continued to decrease and the size of population was decreasing. When the compliance factor reached its minimum, around AD 200, there was a turning point in the growth of population. The compliance factor was still negative but now it was increasing. Slowly, the deceleration in the growth of population was decreasing. Around AD 450 the compliance factor reached its second value of zero. The size of the population reached a minimum value and started to increase. By around AD 500, this demographic transition was over and the growth of population settled again along an unconstrained hyperbolic trajectory, but now is was a significantly slower trajectory characterised by a significantly smaller compliance factor or equivalently by the significantly larger resistance to growth.

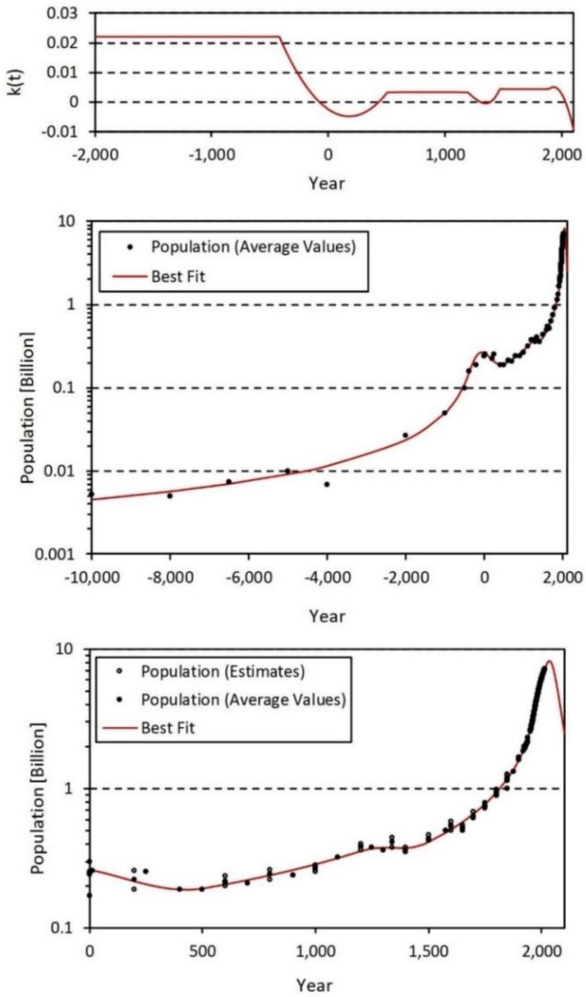

Figure 3. *Growth of the world population in the past 12,000 including mathematical description of the past two demographic transitions between hyperbolic trajectories and the ongoing transition to a yet unknown trajectory.*

Table 1. *Parameters describing the growth trajectory of the world population in the past 12,000 years.*

Unconstrained, hyperbolic growth (~89% of the total combined time) $k = const$	Demographic transitions (~11% of the total combined time) $k(t) = \sum_{i=0}^{n} a_i t^i$
10,000 BC – 500 BC	500 BC – AD 500
$a = -2.282$; $k = 2.210 \times 10^{-2}$	$a_0 = -2.347 \times 10^{-3}$, $a_1 = -2.659 \times 10^{-5}$, $a_2 = 7.479 \times 10^{-8}$
AD 500 – 1200	AD 1200 – 1400
$a = 6.940$; $k = 3.448 \times 10^{-3}$	$a_0 = -1.022$, $a_1 = 2.618 \times 10^{-3}$, $a_2 = -2.198 \times 10^{-6}$, $a_3 = 6.068 \times 10^{-10}$
AD 1400 – 1950	1950 – present
$a = 9.123$; $k = 4.478 \times 10^{-3}$	$a_0 = -1.820$, $a_1 = 1.891 \times 10^{-3}$, $a_2 = 4.899 \times 10^{-7}$

The onset of the second demographic transition occurred around AD 1200. Again, the compliance factor started to decrease and the growth of population started to slow down and even briefly decline. However, the growth quickly recovered and by around AD 1400 this short-lasting transition was over. Growth of population resumed its spontaneous and preferred hyperbolic trajectory, which was even a little faster than before, as indicated by the slightly larger *k* factor.

Around 1950, the compliance factor was boosted but only for a short time. The growth of population started to be a little faster than before, but very soon this temporary boosting was halted and the growth of population started to slow down as expressed in the continually decreasing compliance factor *k*.

3. Characteristic properties of hyperbolic growth

Hyperbolic growth might be more common than we think. In order to understand this type of growth it is useful to compare it with other processes and particularly with the more familiar exponential growth.

For the exponential growth, the size added per fixed unit of time is directly proportional to the total size of the growing entity, e.g. to the size of the population or the GDP (if they are assumed to increase exponentially). If the total size doubles, then the added size per unit of time also doubles. For the hyperbolic growth, to size added per fixed unit of time depends quadratically on the total size of the growing entity. If the size of the growing entity doubles, the added size per fixed unit of time quadruples. If the size triples, the added size per fixed unit of time increases nine-folds.

For the exponential growth, the doubling time is constant. For the hyperbolic growth, it decreases linearly with time. As the size of the growing entity increases, the doubling time decreases. Each consecutive doubling time is twice as small as the immediately preceding doubling time. So, for instance, if at a certain stage of growth, the doubling time is 24 years, then after 24 years it will be reduced to 14 years, after 14 years to 7 years, and so on. That is why, hyperbolic growth, or any other type of growth, but exponential, should never be characterised by the doubling time. Constant doubling time applies exclusively to the exponential growth.

For the exponential growth, the total driving force is constant. No matter how large is the growing entity, the force remains unchanged. Driving force per single unit decreases exponentially. If exponential growth were to describe economic growth then the driving force per unit of invested wealth, e.g. the driving force per invested dollar, would decrease exponentially with the size of the investment. The potential to generate economic growth per dollar would be decreasing exponentially with the size of the GDP. If exponential growth were to describe the growth of population, then the biologically driven force of procreation (the difference between the biologically generated or controlled birth and death rates) per person would be decreasing exponentially.

For the hyperbolic growth, the total driving force increases hyperbolically, i.e. in the direct proportion with the size of the growing entity, which means that the driving force per unit or per element of the whole assembly of hyperbolically growing entity, e.g. per person or per dollar, is constant. Each unit, on average and for a large assembly of growing units, contributes equally to support growth. For the hyperbolic growth, the potential of each invested unit of wealth, e.g. the potential of each dollar to create more wealth is constant. It does not depend on the size of invested wealth; it does not depend on the size of the GDP. For the hyperbolic growth of human population, the force of procreation per person remains constant; it does not decrease with the size of population.

For the hyperbolic growth, each element, each added component, makes on average, *a fixed contribution* to the overall driving force. Individual contributions may vary, but on average *the contribution of each component is constant over time*. The larger is the size of the growing entity the larger is the *combined force* pushing the growth forward. It is the growth that propels itself in a very specific way. In the unconstrained hyperbolic growth, the growth is propelled by the approximately equal contribution of *all* individual members of the growing entity. It is an interesting and distinct process where *growth generates growth* in a very specific way, i.e. where the driving force of growth per person or per unit of the growing entity is *constant*. In contrast, for the exponential growth, the *combined* driving force is constant but the driving force per unit of the growing entity *decreases* exponentially.

Now, we can see that there might be more examples of hyperbolic growth. Take, for instance, technology or knowledge. Knowledge generates knowledge by stimulating new ideas. Technology generates technology by stimulating new solutions to technological problems. This is the well-known process, which *even a single person can experience*. The more we learn, the easier it is to learn more. The more problems we solve, the easier it is to solve new problems. Ideas create new ideas, solutions create new solutions, and knowledge creates new knowledge. It is, therefore, not surprising that knowledge and technological innovations appear to have been increasing hyperbolically (Kurzweil, 2006; Vinge, 1993). There is a close correlation between the growth of population and technology (Kremer, 1993). The two processes are similar but they are prompted by different kind of forces.

Technology is certainly not prompted by the force of procreation (the biologically prompted sex drive and the biologically prompted process of aging and dying). The growth of population is obviously controlled by these processes. It could be also controlled by some additional forces but the historical growth of population shows that these other forces were either too weak or that they were averaging out.

Technology is prompted by concepts, solutions and by research activities. Growth of population is definitely not prompted by technological concepts, solutions and by research activities but by the force of procreation. Economic growth is similar to the growth of population but it is obvious that economic growth is not prompted by the biologically controlled force of procreation.

Another example of hyperbolic growth could be the growth of biodiversity. We could expect that biodiversity should generate greater biodiversity through competition, adaptation and biological solutions based on life-supporting mutations. We can also expect that the force driving the growth of biodiversity is proportional to the existing biodiversity. If it is directly proportional, then the growth of biodiversity is hyperbolic. Even if we consider minor or major extinctions of species one might expect that over a sufficiently long time the prevailing trend might be hyperbolic. If we think in terms of driving forces, we could probably identify other examples of hyperbolic growth. We can also understand easier the distinctions between various types of growth.

For processes described by hyperbolic trajectories, each system will be prompted *by its own mechanism reflected in a specific driving force*, but each system will be prompted by the same *type* of force. In each case, the force per unit of the growing entity will be constant. Hyperbolic similarities and close correlations between hyperbolic systems should never be interpreted as necessarily reflecting precisely the same mechanism of growth represented by precisely the same driving force. In general, each hyperbolic process will be expected to be propelled by a distinctly different force reflecting a distinctly different mechanism, but all these forces will be of the same *type*: their intensity will increase in the direct proportion to the size of the growing entity; their intensity per person, per biological object, per unit of measurement (such a dollar, for instance) will be always constant during the entire time of the unconstrained growth.

Hyperbolic growth is characterised by singularity where the growth escapes to infinity at a fixed time. Such a growth might be deemed impossible. However, historical economic growth and historical growth of populations were hyperbolic so obviously, they were possible. Growth trajectories can change and there is nothing unusual about that. A new force may be added to the existing force or the previously active force might be replaced by a new force. In the growth of global population there were only two instances in the past 12,000 years when a new force of growth was added temporarily to the force of procreation. First time, this additional force appears to be of political nature changing radically and on a large scale the style of living. Second time, it was in the form of demographic catastrophes, the only known case when demographic catastrophes were reflected in the trajectory describing the growth of population. Currently, there is also a diversion to a new trajectory. The force of procreation continues to be active but the new and significant force added to the force of procreation appears to be the force of preventive checks (Malthus, 1798).

It is also absolutely not necessary to imagine that in order to avoid the problem of singularity we have to find some mathematically-described force, which over a certain time would mimic hyperbolic growth but at around a certain time would gradually become non-hyperbolic, and that this unusual and yet unknown mathematical distributions would also reproduce the growth of human population. It is absolutely not necessary "to eliminate the unrealistic 'demographic explosion' from the model" (Karev & Kareva, 2014, p. 76), because it is not at all unusual for a trajectory to remain undisturbed over a certain time but then to be diverted to a new trajectory. The mechanisms of growth can change or can be modified by adding new type of force to the already existing force. We do not have to imagine that we should have a single force, which over a long time would describe hyperbolic growth and then would also describe a diversion to a new, non-hyperbolic growth. Karev attempted to find such a force but failed (Karev, 2005). He tried two such forces but they did not explain the mechanism of growth because they were incomprehensibly complicated (Nielsen, 2016g). They were also unsuccessful in describing the growth of population. A single and easy to understand force of procreation results in a far better description of data.

Current growth of population and economic growth is no longer described so consistently by a single type of force. For instance, economic growth in Greece was logistic over a certain time but then it changed to a pseudo-hyperbolic growth with singularity in 2017 (Nielsen, 2016h). This fast growth could not have been supported in any way and it collapsed. The current global economic growth is exponential (Nielsen, 2016i). Such a growth is insecure because it does not lead to a maximum or to a safe and sustainable level of the GDP. It continues to grow until it can be no longer supported.

The current global growth of population is less clearly defined and its projections are less certain. Analysis of the growth rate shows that growth of population may reach a certain maximum but it may also continue to increase for as long as it can be supported by the availability of natural resources (Nielsen, 2006).

4. Summary and conclusions

Historical economic growth and historical growth of population were hyperbolic (Nielsen, 2016a, 2016b, 2016c). We have explained their mechanism by postulating simple forces of growth. Hyperbolic growth is mathematically simple and its mechanism of growth is also simple.

For the economic growth, the mechanism of the historical hyperbolic growth is explained by *the net market force*, which on average was *directly proportional to the invested wealth* usually expressed as the Gross Domestic Product. For the growth of population, the mechanism of the historical hyperbolic growth is explained by *the biologically prompted force of procreation* defined as the difference between the biologically prompted birth rate and the biologically controlled process of aging and dying. It is assumed that this force was on average *constant per person.*

We do not explain the net market force and neither do we explain the biological force of procreation. We do not dissect these processes, isolate their components, study minute interactions between the mand then put them together to derive the net driving force. We only describe these forces in the simplest possible way using simple mathematical expressions based on simple and readily acceptable assumptions. We then use these simplified forces to explain the mechanisms of growth.

This type of approach is common in scientific investigations. For instance, we do not understand the force of gravity. We do not really know what it is. However, we can representthis force using a simple mathematical expression (Newton, 1687) and then use it to explain the mechanism of the movement of celestial by, we can land a man on the Moon and bring him back to Earth, explore our solar system, land our probes on Mars, detect the presence of the invisible matter and in general explain the dynamics of the Universe.

We do not understand nuclear forces but we can describe them mathematically and use this description to study, for instance, the mechanism of nuclear reactions and nuclear structure (Nielsen, 2016). Nobody understands quantum mechanics (Feynman, 1967) but this does not stop us from describing mathematically various quantum phenomena, explain them and even use the acquired knowledge to apply it, for instance, in quantum computing or cryptography. We do not understand the weak force and yet we can explain the process of radioactive decay and use radioactive isotopes in many applications, primarily in medicine but also industry and agriculture.

We do not understand why matter reveals itself as mass or energy. We do not understand the intricate details of this peculiar phenomenon but we can describe it by a simple and well-known equation (Einstein, 1905a). We can then use this simple equation to calculate how much energy will be released if a certain amount of mass manifests itself as energy. We can use this knowledge, combined with our fundamental knowledge of nuclear processes, to explain the mechanism of fusion and fission reactions. We can then go a step further and construct (unfortunately) a nuclear bomb and (maybe similarly unfortunately) to construct a controversial nuclear reactor to produce energy. However, we can also explore how this huge amount of energy locked in the mass could be used in a controlled fusion reaction and maybe at last to construct a clean and practically inexhaustible source of energy. We can also use this simple mass-energy relation to explain the mechanism of the production of energy in our Sun and in the distant stars. We do not know everything but what we already know can be useful.

We do not understand why electromagnetic radiation reveals itself as waves or particles, the property, which turns out to apply not only to electromagnetic radiation but also to all matter, but we can describe this relationship by simple mathematical expressions (Einstein, 1905b; de Broglie, 1924) and explain not only why the rainbow looks so nice but also the strange phenomenon of photoelectricity (Einstein, 1905b). Einstein is well known for his theory of relatively and for his mass-energy equation but he received his Nobel Prize for explaining photoelectricity, which demonstrates that light can manifest itself as being made of tiny particle.

We may not know all the details how nature works but we can still explain many phenomena we observe and even represent our explanations by useful and often simple mathematical expressions. We might not be able to explain everything. We might not answer every single question but we can still explain many phenomena in a satisfactory manner and answer many questions. A deeper understanding might come much later but only if we make sure that our current knowledge is not based on illusions and impressions but on the methodically checked interpretations of observed phenomena.

The fundamental principle in scientific research is to look for the simplest explanations of observed phenomena. These few examples from physics show that even complicated processes can be often represented by simple mathematical descriptions and that the interpretation of their mechanism can be significantly simplified.

Distributions describing historical growth of population and the historical economic growth look complicated, so complicated that they are routinely interpreted as being made of two distinctly different components, slow and fast, stagnant and explosive, each component governed by distinctly different and complicated mechanisms. The illusion is so persuasive that even most prominent researchers are easily misguided, particularly if the data are not properly analysed or if they are presented in a grossly distorted way (Ashraf, 2009; Galor, 2005a, 2005b, 2007, 2008a, 2008b, 2008c, 2010, 2011, 2012a, 2012b, 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008).

The first indication that these distributions are not complicated is demonstrated when they are mathematically analysed. The analysis is trivially simple (Nielsen, 2014) and it shows that these distributions are hyperbolic (Nielsen, 2016a, 2016b, 2016c). Hyperbolic distributions look complicated but they are described by an exceptionally simple mathematical formula: a reciprocal of a linear function containing just two adjustable parameters.

This remarkable simplicity of hyperbolic distributions representing the historical growth of population and the historical economic growth suggests a simple mechanism of growth. We have now demonstrated that the mechanism of these two processes is indeed remarkably simple. They were prompted by the well-known and simple forces.

Data describing the growth of global population allow for a study of growth over an exceptionally long time. They show that for the most part of the past 12,000 years, growth of global population was hyperbolic: between 10,000 BC and around 500 BC, between around AD 500 and 1200 and between around AD 1400 and 1950. The remaining time of the past 12,000 years was taken by demographic transitions: between around 500 BC and AD 500, between around AD 1200 and 1400, and from around 1950.

We have proposed the explanation of the mechanism of these transitions. The first transition is explained by the dramatic and wide-spread changes in the style of living associated with significant changes in the political landscape. The second transition is explained as being caused by the combined impact of five major demographic catastrophes. This is the only example when demographic catastrophes appear to have had impact on shaping the population growth trajectory. However, this impact was insignificant. The slight delay in the growth of population was soon compensated because the growth of population was diverted to a slightly faster trajectory. We can explain the mechanism of this quick recovery by the regenerating effects of the Malthusian positive checks (Malthus, 1798; Nielsen, 2016f). The mechanism of the ongoing transition is explained by the Malthusian preventive checks.

A partial mathematical explanation of these transitions is by assuming that the growth of human population was still prompted by the biologically controlled force of procreation but that the resistance to growth (or equivalently the compliance factor) was changing. This simple assumption does not allow us to predict growth trajectories during demographic transitions but only to determine how the resistance to growth (or compliance factor) was changing during each transition.

Currently, neither the growth of population nor the economic growth can be described by the historically simple driving force. Generally, we have to use different descriptions for each specific case. For instance, current global economic growth can be described by a relatively simple but non-hyperbolic trajectory, which is now converging into the exponential growth (Nielsen, 2016i). Economic growth in Greece was logistic but then it was converted to a fast-increasing pseudo-hyperbolic growth, which inevitably resulted in the economic collapse because it came too close to the point of singularity (Nielsen, 2016h). Global growth of population can be described using different trajectories, each trajectory giving different prediction of growth (Nielsen, 2006).

The general law of growth (Nielsen, 2016d) helps to understand mechanisms of growth because it links growth trajectories with driving forces, which are usually easier to visualise and to understand. We have used this general law of growth and the simplest driving forces to explain the mechanism of the historical growth of population and the historical economic growth.

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