Does Clower's Dual-Decision Hypothesis lead to the change in saving conclusion in Keynes's General Theory?

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Abstract. Keynes' General Theory (1936) is probably the most challenging economics book ever written, with an abundance of hypotheses, concepts and theories. Twenty five years after its publication, Clower proposed an insightful explanation on Keynes, the Dual-Decision Hypothesis (DDH). Hall (1978) and Flavin (1981) seemingly reached the conclusion that, under certain conditions, consumption was independent of income. In contrast, Wu (2016) has shown that, change in saving has to be a function of income growth. In fact, applying Wu's corrected consumption for period t+1, it is possible to show DDH equations leading to Keynes' change in saving (and disequilibrium) conclusion.

Keywords. Keynes, Clower, Keynesian, Disequilibrium, Dual decision hypothesis, Consumption, Martingale.

JEL. A10, B2, B22, C20, E20, E60, F00.

1. Introduction

For a mathematical identity A = B, one needs to prove both that A = B and then B = A. Often, a backward proof is easier than a forward one. More than twenty years ago, I found out evidence of a correlation between saving rate and trade. But Hall (1978) and Flavin (1981) had already "proved" that change in consumption may be independent of income. So, for the evidence to fit with the theory (and not the other way around), I had to prove - in theory - that change in consumption was not independent of income.

Keynes' General Theory (1936) is a standard A to B proof. Since he is trying to distinguish his economics system from the more orthodox (classical) one, it is also a most difficult proof. Keynes' book is more impressive in that one needs to identify the problem(s) and offer a solution(s) that most economists haven't thought about it yet. In doing so, Keynes introduced and applied a wide range of new and established concepts and theories that were not adequately tested or established at that time either in theory or evidence, such as involuntary unemployment, disequilibrium, propensity to consume, fiscal stimulus, saving and dissaving and so on.

A quarter of century after General Theory publication, Clower proposed a Dual-Decision Hypothesis (DDH). It was an attempt to clarify and emphasize Keynes, first on disequilibrium and then on consumption theory. According to Clower, Keynes, for all the novel ideas and conclusions, lacked an adequate explanation on the essential mechanism of the consumption theory. Arguably, Clower's DDH is itself a theory within another theory. As we will see later, Clower did provide a clear road map but did not attempt to travel in it himself.

Thus, when Clower stated that "Keynes either had a dual-decision hypothesis at the back of his mind, or most of the General Theory is theoretical nonsense," we are left with a few more questions:

1. Where is the proof that Keynes had a DDH in the back of his mind?

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2. What to make of Keynes' propensity to consume?
3. Did Keynes have a clear view of the change in saving?
4. How does saving and dissaving help Keynes' disequilibrium proposition?

How does Clower's DDH lead to Keynes' saving conclusion and why does it matter today? Hopefully, understanding these questions will give greater emphasis and appreciation of Keynes' General Theory and Clower's DDH for providing the essential theoretical and practical framework to economics.

2. Keynes' consumption theory and disequilibrum

Keynes stated that propensity to consume "is a fairly stable function… mainly depends on the amount of aggregate income (both measured in terms of wage units)" (Keynes, p.90).

$$C = \chi(Y) \tag{1}$$

where the propensity to consume is the functional relationship χ between Y and C.

And by assuming that the propensity to consume is "fairly: stable and that "dC/dYis positive and less than unity", Keynes quickly concluded that, "short periods in view, as in the case of the so-called cyclical fluctuations of employment during which habits, as distinct from more permanent psychological propensities, are not given time enough to adapt themselves to changed objective circumstances… if he does adjust his expenditure to changes in his income, he will over short periods do so imperfectly. Thus a rising income will often be accompanied by increased saving, and a falling income by decreased saving, on a greater scale at first than subsequently [emphasis supplied] (Keynes, p.96-97)."What all this means is that, with a few assumptions and without a formal proof, Keynes arrived at the conclusion ("simple principle") that change in saving should be a function of change in income,

$$\Delta S = f(\Delta Y) \tag{2}$$

Keynes extend its short period conclusion to involuntary unemployment and long term changes in equilibrium: "On the other hand, a decline in income due to a decline in the level of employment, if it goes far, may even cause consumption to exceed income not only by some individuals and institutions using up the financial reserves which they have accumulated in better times, but also by the government, which will be liable, willingly or unwillingly, to run into a budgetary deficit or will provide unemployment relief; for example, out of borrowed money. Thus, when employment falls to a low level, aggregate consumption will decline by a smaller amount than that by which real income has declined, by reason both of the habitual behaviour of individuals and also of the probable policy of governments; which is the explanation why a new position of equilibrium can usually be reached within a modest range of fluctuation. Otherwise a fall in employment and income, once started, might proceed to extreme lengths" (Keynes, p. 97-98).

To Keynes, a rate of saving should be desired only in special situations, such as full employment equilibrium,: "Forced saving is the excess of actual saving over what would be saved if there were full employment in a position of long-period equilibrium… a forced excess of saving would be a very rare and a very unstable phenomenon" (Keynes, p. 80).

In Keynes' general theory, deviations to the propensity to consume, i.e., saving and dissaving, may occur for a variety of reasons, including "changes in fiscal policy. In so far as the inducement to the individual to save depends on the future return which he expects, it clearly depends not only on the rate of interest but on the fiscal policy of the government. Income taxes, especially when they discriminate against 'unearned' income, taxes on capital-profits, death-duties and

the like are as relevant as the rate of interest; whilst the range of possible changes in fiscal policy may be greater, in expectation at least, than for the rate of interest itself. If fiscal policy is used as a deliberate instrument for the more equal distribution of incomes, its effect in increasing the propensity to consume is, of course, all the greater" (Keynes, p. 94).

3. Clower's dual-decision hypothesis

Clower questioned whether "buying and selling are all carried out simultaneously... planned sales and purchases cannot possibly be true of realized sales and purchases, unless the system as a whole is always in a state of equilibrium." Further, "differences between realized and planned purchases and sales of individual households may properly be supposed to occur more or less at random."

Clower stated that, initially, households will maximize the preference function $U(d_1, \ldots, d_m; s_{m+1}, \ldots, s_n)$ subject to the budget constraint

$$\sum_i^m \mathbf{p}_i \, d_i \; - \; \sum_j^n \mathbf{p}_j \, s_j \; -\mathbf{r} \; = \; 0 \tag{3}$$

If realized or actual income is less than the notional or planned income, Clower stated that a "*second round of decision making is indicated: namely, maximize*"

$$U(d_1, \ldots, dm; \mathbf{s}_{m+1}, \ldots, \mathbf{s}_n)$$

subject to the modified budget constraint (based on new information available),

$$\sum_i^m \mathbf{p}_i \, d_i \; - \; \sum_j^n \mathbf{p}_j \, s_j \; -\mathbf{r} \; = \; 0 \tag{4}$$

These two maximizations are at the core of Clower's famous DDH or the road map if you prefer.

Writing DDH maximization equations in a more recent terminology, i.e., under the "Euler equation approach," optimal consumption for period t is given by

$$\sum_{t=0}^{\infty} b^t [u_0 + u_1 c_t + \tfrac{u_2}{2} c_t^2 \,], \qquad 0 < b < 1, \qquad u_0, u_1, u_2 > 0 \tag{5}$$

subject to

$$A_{t+1} = R \, [A_t + y_t - c_t] \tag{6}$$

and where y_t, under a stochastic process, is $E_t y_t$

Where, c is consumption, A is non-human assets, y is labor income, R is gross rate of return (all at the beginning of period), E is expectation, t is time.

Optimal consumption for period t is

$$c_t = (1 - R^{-1})[\, A_t + \sum_{j=0}^{\infty} \left(\tfrac{1}{R}\right)^j \; E_t \; y_{t+j} \,] \tag{7}$$

Optimal consumption for period t+1 is

$$c_{t+1} = (1 - R^{-1}) \, [A_{t+1} + y_{t+1} + (1/R) \, E_{t+1} \, y_{t+2} + (1/R)^2 \, E_{t+1} \, y_{t+3} + \ldots$$
$$+ (1/R)^{n-1} \, E_{t+1} \, y_{t+n} + \ldots] \tag{8}$$

Please note that, on purpose, the mathematical notation for period t+1, eq. (8), is written in an expanded form, as opposed to summation, sigma (Σ), notation on period t, eq. (7).That is, depending on how one writes the summation notation of functions with two different lower limits for the index of summation j, we may yield two completely different set of economic results, which we will discuss in the following two sections.

For now, replacing Keynes' eq. (1) with DDH equations we have,

$$c_t = (1 - R^{-1})[\ A_t + \sum_{j=0}^{\infty} (\tfrac{1}{R})^j \ E_t \ y_{t+j}\ ] \tag{7}$$

$$c_{t+1} = (1 - R^{-1})\ [A_{t+1} + y_{t+1} + (1/R)\ E_{t+1}\ y_{t+2} + (1/R)^2\ E_{t+1}\ y_{t+3} + \ldots$$
$$+ (1/R)^{n-1}\ E_{t+1}\ y_{t+n} + \ldots] \tag{8}$$

$$\Delta S = f(\Delta Y) \tag{2}$$

Clearly, for eq. (7) and (8) to reach the result in eq. (2), we need to take the difference in consumption. Let's first examine Hall/Flavin change in consumption approach.

4. Hall/Flavin's consumption at period T+1

The difference in consumption from one period to another was made famous by Hall's (1978) proof of the corollary 4, $c_{t+1} = c_t$, which can be found in Flavin (1981).

When permanent income is equal to consumption, Flavin stated that consumption at period t+1 (Flavin's eq. (4)) is given by,

$$c_{t+1} = (1 - R^{-1})\ [A_{t+1} + \sum_{j=0}^{\infty} (\tfrac{1}{R})^j \ E_{t+1}y_{t+j+1}] \tag{9}$$

Taking the difference between consumption at t+1, eq. (9), and consumption at period t, eq. (7), and assuming that, "if the expectations of future income are rational, the expectation of next period's revision in expectation $(E_{t+1} - E_t)\ y_{t+j+1}$ is zero," Flavin reached the conclusion that,

$$E_t\ c_{t+1} = c_t \tag{10}$$

This result has been widely supported by rational expectations economists. For almost four decades, on grounds that, consumption may be independent of income, Keynes' consumption theory and by extension Clower's DDH were left, in effect, discredited.

Still, one should reasonably consider that, even though the number of incomes goes to infinity, as the consumer ages, there is a loss of income going forward one period. In Flavin's equations, for period t and for period t+1, the total number of incomes always remains constant, i.e., both indices of the summations, for period t and for period t+1, varying from 0 to infinity. The untenable (and implicit) assumption that a consumer won't lose any labor income while he/she ages is the reason one must always check the range of the summation. Otherwise, one may end up assuming - incorrectly - that the number of future incomes for a young worker and a middle aged one to be the same.

It is relatively straightforward to show that the difference of two summations with the same number of incomes may equal to zero. Let's take the case where the summation of incomes converges from period 0 to n (where n tends to infinite) and that each period has the same income Y. Either consumption for period t and t+1 would have exactly the same number of incomes, i.e., nY. Thus, it is no wonder the difference in consumption is zero!

In practice, Hall's consumption result cannot offer a reasonable explanation for earlier and newer empirical evidence, specifically,

1. Since the 1950s and even before then, economists, including Modigliani & Brumberg (1954), have shown evidence of a possible relationship between saving and income growth.
2. How to explain the decline of U.S. (and other countries) personal saving rate? In the past 40 years, U.S. savings has trended lower while other countries' saving, such as in Japan, increased and then declined. Other Asian countries follow similar patterns to Japan. What causes sudden shifts in saving and dissaving?
3. More recently, how to explain Obama's fiscal stimulus and its effect on the Great Recession? Undoubtedly, the Great Recession would be deeper and wider without the actions of the American Recovery and Reinvestment Act of 2009 (ARRA).

5. Wu's consumption at T+1 and change in saving

Wu (2016) has shown that the generalized maximized consumption equation should be,

$$c_{t+n} = (1 - R^{-1})\ [A_{t+n} + \sum_{j=n}^{\infty}\ (\tfrac{1}{R})^{j-n}\ E_{t+n}y_{t+j}] \tag{11}$$

and for n = 1,

$$c_{t+1} = (1 - R^{-1})\ [A_{t+1} + \sum_{j=1}^{\infty}\ (\tfrac{1}{R})^{j-1}\ E_{t+1}y_{t+j}] \tag{12}$$

Clearly, Eq. (12) takes into consideration individual's age while eq. (9) does not.

(Why is that if A = C and B = C then A ≠ B, where A is Flavin's eq. (9), B is Wu's eq. (12) and C is expanded eq. (8)? It is well known that a formula can yield different structural formulas but not all structural formulas will yield the same result.)

The change in consumption is,

$$c_{t+1} - c_t = (1 - R^{-1})[\ A_{t+1} - A_t + \sum_{j=1}^{\infty}\ (\tfrac{1}{R})^{j-1}\ E_{t+1}y_{t+j} - \sum_{j=0}^{\infty}\ (\tfrac{1}{R})^{j}\ E_t\ y_{t+j}\ ] \tag{13}$$

Assuming

$$\sum_{j=1}^{\infty}\ (\tfrac{1}{R})^{j-1}\ E_{t+1}y_{t+j} - \sum_{j=1}^{\infty}\ (\tfrac{1}{R})^{j}\ E_t\ y_{t+j} = 0 \tag{14}$$

and applying the definition of total income or "measured" income

$$y_{mt} = (1 - \tfrac{1}{R})\ A_t + y_t \tag{15}$$

then change in consumption can be written as

$$\Delta S = (\ y_{t+1} - \tfrac{y_t}{R}) \tag{16}$$

6. From Clower's DHH to Keynes' change in saving equations

From Clower's DDH equations,

$$c_t = (1 - R^{-1})[\ A_t + \sum_{j=0}^{\infty}\ (\tfrac{1}{R})^{j}\ E_t\ y_{t+j}\ ] \tag{7}$$

$$c_{t+1} = (1 - R^{-1}) \left[A_{t+1} + \sum_{j=1}^{\infty} \left(\frac{1}{R}\right)^{j-1} E_{t+1} y_{t+j} \right] \tag{12}$$

Change in consumption is,

$$c_{t+1} - c_t = (1 - R^{-1}) \left[A_{t+1} - A_t + \sum_{j=1}^{\infty} \left(\frac{1}{R}\right)^{j-1} E_{t+1} y_{t+j} - \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^{j} E_t\ y_{t+j} \right] \tag{13}$$

If we assume that,

$$\sum_{j=1}^{\infty} \left(\frac{1}{R}\right)^{j-1} E_{t+1} y_{t+j} - \sum_{j=1}^{\infty} \left(\frac{1}{R}\right)^{j} E_t\ y_{t+j} = 0 \tag{14}$$

Applying the definition of total income or "measured" income,

$$y_{mt} = \left(1 - \frac{1}{R}\right) A_t + y_t \tag{15}$$

Thus, from eq. (16), we reach Keynes' change in saving conclusion,

$$\Delta S = f(\Delta Y) \tag{2}$$

Also, Keynes' propensity to consume is related to DDH. Dividing eq. (16) by ΔY and adding back eq. (14), we get the marginal propensity to save (MPS).
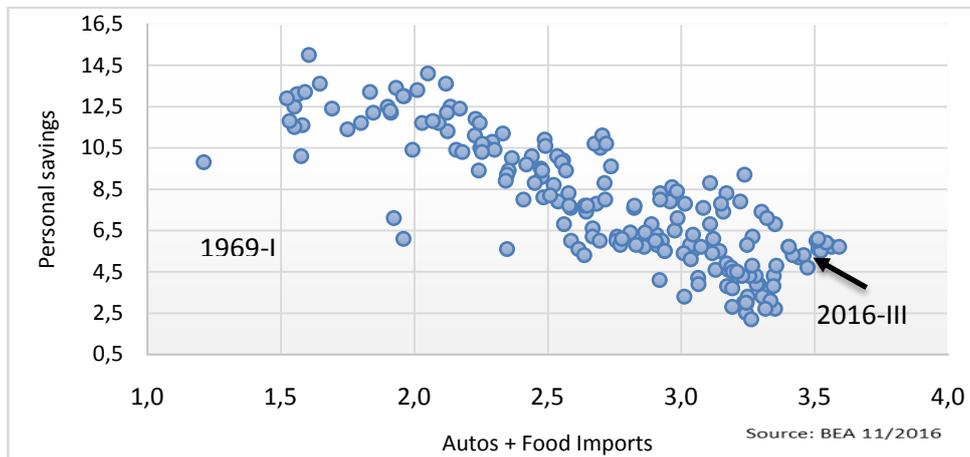
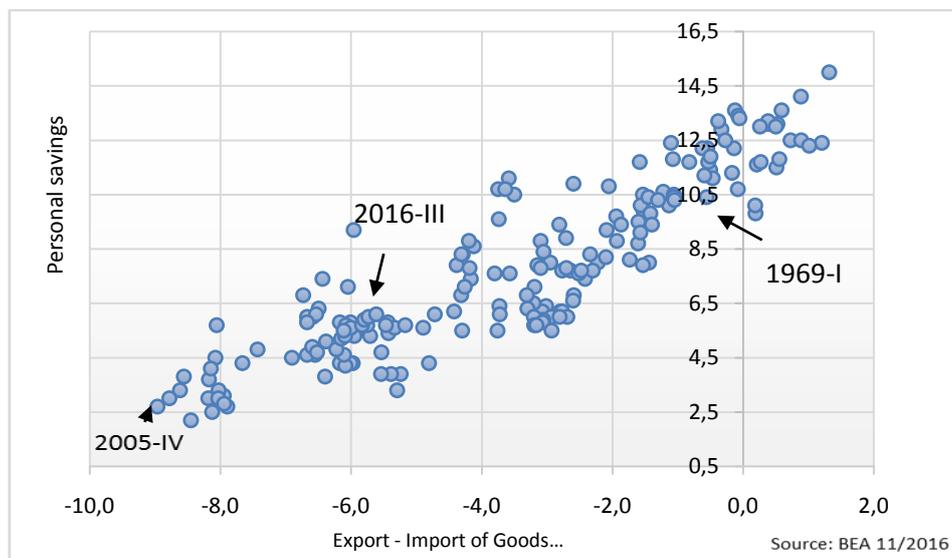## 7. The decline in US manufacturing employment and income and saving
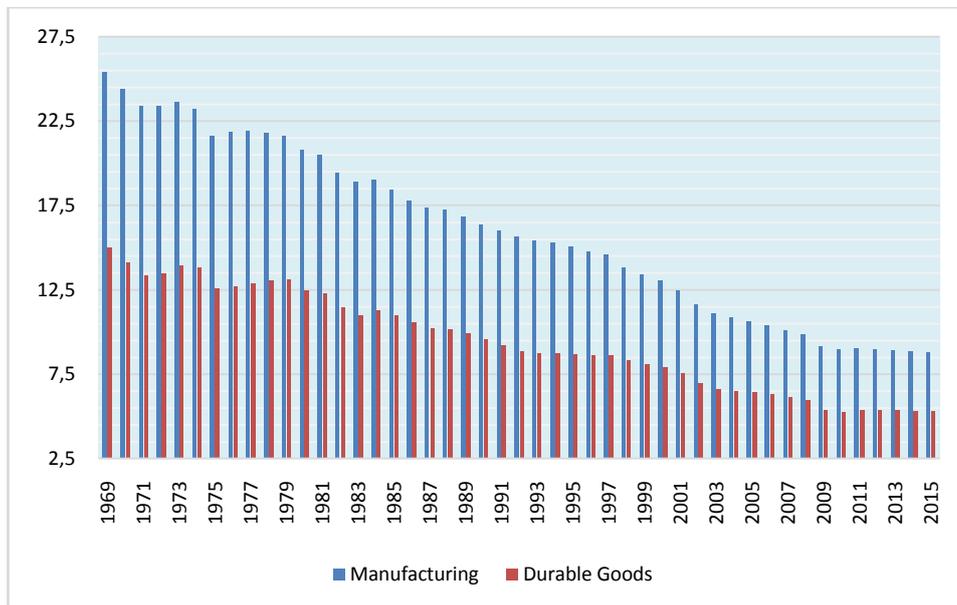
Graph 1. *Manufacturing and Durable Goods Employees (yearly, as percentage of Total Full and Part Time Employees – Source: Commerce Dept. - BEA).*
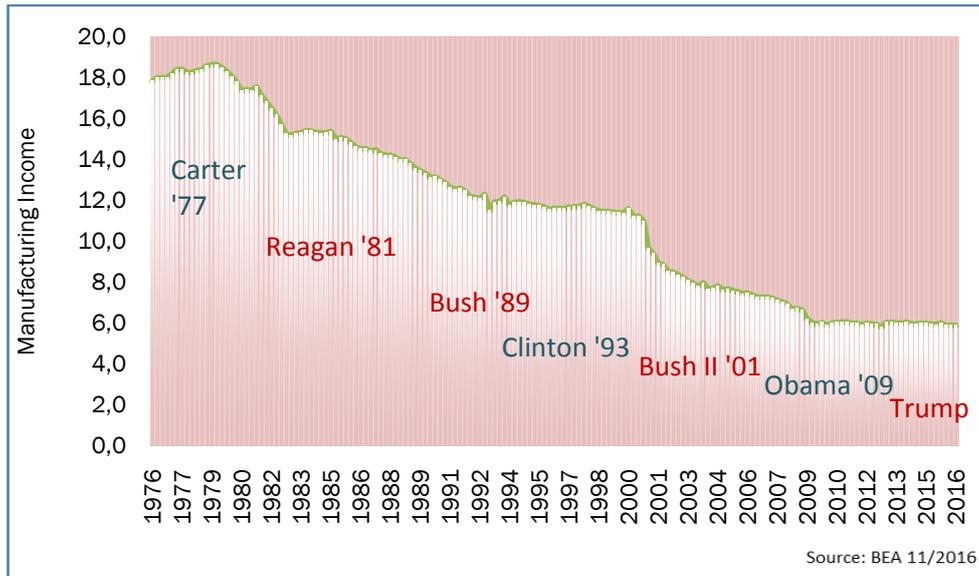
Graph 2. *Manufacturing Income (as percentage of Disposable Personal Income (DPI)).*

Graph 3. *Personal Savings vs Automobile and Food Imports (as percentage of DPI)*

Graph 4. *Personal Savings vs. Net Exports of Goods (as percentage of DPI).*

These graphs (Wu, 2017) illustrate how U.S. involuntary unemployment has been 'around' for almost four decades and still the households have failed to properly maximize their consumption. And, for a given level of unemployment, saving rate can turn negative.

Importantly, these results can be applied to most trade surplus economies, such as Germany, Japan, Korea, Taiwan and China, where income growth (from trade) can quickly accelerator slow down and alter savings.

8. Conclusion remarks

This article has shown that Clower's DDH does lead to Keynes' change in saving, which is a function of change in income and change in income can also be expressed in function of income growth. Some of the most relevant conclusions are:

a. If change in saving is a function of income growth then the system is mostly in 'general' disequilibrium. Further, Keynes' approach is for both short and long run, emphasizing the role of (permanent)involuntary unemployment;

b. saving and dissaving are effectively the result of errors from decision making. Keynes thought that saving is a 'determinate 'and the result of the system's determinants; Clower argued that changes in planned and realized income is the reason behind reevaluations. Thus, there can't be 'forced' saving;

c. because the relationship is about the change of both saving and income, it is possible to have positive income growth and negative saving, and vice versa;

d. saving and dissaving are affected by a multitude of determinant income factors, including fiscal stimulus and trade/economic/tax/health care policies;

e. Euler optimization shows the importance of present and future forecast of income but to reach Keynes' change in saving result, unlike in Flavin's approach, it is not necessary to assume rational expectations;

f. marginal propensity to save is also based on changes in future income; Clower showed little interest in Keynes' propensity to consume;

g. mathematically, Keynes either had a DDH at the back of his mind(and the Hall/Flavin's consumption result is defective), or most of the General Theory is theoretical nonsense.

In sum, it is clear Keynes and Clower were indeed extraordinarily clever dogs, to dive after and fish the wild duck from the weed and tangle and all the rubbish at the bottom of a deep water (Keynes, p. 183), and that, even after eighty plus years of General Theory publication, "The difficulty lies, not in the new ideas, but in escaping from the old ones, which ramify, for those brought up as most of us have been, into every corner of our minds" (Keynes, p. viii).

Given Clower's proven insight, I feel compelled to repeat his warnings that, "I am convinced that much of what now passes for useful theory is not only worthless economics (and mathematics), but also a positive hindrance to fruitful theoretical and empirical research . . . As physicists should and would have rejected Einstein's theory of relativity, had it not included Newtonian mechanics as a special case, so we would do well to think twice before accepting as 'useful' or 'general' doctrines which are incapable of accommodating Keynesian economics."

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