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New Result in Consumption Theory: Change in Savings and Income Growth – Nineteen Years Later

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Abstract. This new version uses the definitions and some of the results found in Sargent's Macroeconomic Theory. Hall's (1978) proof of the corollary 4,$c_{t+1} = c_t$, can be found in Flavin (1981). Writing the same consumption stated in Flavin, for period *t+1*, in a different way for the summation of the expected future incomes, it is possible to show that changes in savings is a function of income growth. This new result has implications, for instance, in Keynes' (1936) saving and dissaving.
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1. Introduction

Hall's (1978) proof of the corollary 4,$c_{t+1} = c_t$, can be found in Flavin (1981). Writing the same consumption stated in Flavin, for period *t+1*, in a different way for the summation of the expected future incomes, it is possible to show that changes in savings is a function of income growth.

In consumption theory, consumption for period *t+1* is partially determined by the sum of $y_{t+1} + (1/R) E_{t+1} y_{t+2} + (1/R)^2 E_{t+1} y_{t+3} + \ldots + (1/R)^{n-1} E_{t+1} y_{t+n} + \ldots$. It is possible to show that this part of the consumption at *t+1*can be written as the summation (∑)of functions with two different lower limits for the index of summation j, which yields two completely different set of economic results.

Furthermore, it is possible to arrive to an alternative resultwithout Flavin's assumption that expectations of future income are rational and that change in expectations for income is zero.

First, I will find outconsumption at any period *n* and then apply it to*t+1*, and then compare it with Flavin (1981) equation. I will offer a new result and conclusions.

2. What is consumption for period t+1?

A consumer maximizes[1]

$$\sum_{t=0}^{\infty} b^t [u_0 + u_1 c_t + \frac{u_2}{2} c_t^2], \qquad 0 < b < 1, \qquad u_0, u_1, u_2 > 0 \tag{1}$$

subject to

$$A_{t+1} = R\,[A_t + y_t - c_t] \tag{2}$$

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and where y_t, under a stochastic process, is $E_t y_t$

Where, c is consumption, A is non-human assets, y is labor income, R is gross rate of return (all at the beginning of period), E is expectation, t is time.

Under the “Euler equation approach,” optimal consumption for period t is given by (Sargent, p. 215),

$$c_t = (1 - R^{-2} b^{-1})A_t - \frac{u_1}{u_2}\frac{R^{-1}b^{-1}L^{-1}}{(1-R)} + \frac{(1-R^{-2}b^{-1})}{1-L^{-1}R^{-1}} E_t\, y_t, \tag{3}$$

where L is the lag operator

Repeating the Euler optimization, consumption, c_{t+1}, c_{t+2}, . . ., c_{t+n}, should be given by,

$$c_{t+1} = (1 - R^{-2} b^{-1})A_{t+1} - \frac{u_1}{u_2}\frac{R^{-1}b^{-1}L^{-1}}{(1-R)} + \frac{(1-R^{-2}b^{-1})}{1-L^{-1}R^{-1}} E_{t+1}\, y_{t+1} \tag{4}$$

$$c_{t+2} = (1 - R^{-2} b^{-1})A_{t+2} - \frac{u_1}{u_2}\frac{R^{-1}b^{-1}L^{-1}}{(1-R)} + \frac{(1-R^{-2}b^{-1})}{1-L^{-1}R^{-1}} E_{t+2}\, y_{t+2} \tag{5}$$

. . .

$$c_{t+n} = (1 - R^{-2} b^{-1})A_{t+n} - \frac{u_1}{u_2}\frac{R^{-1}b^{-1}L^{-1}}{(1-R)} + \frac{(1-R^{-2}b^{-1})}{1-L^{-1}R^{-1}} E_{t+n}\, y_{t+n} \tag{6}$$

Assuming $Rb = 1$

$$c_{t+n} = (1 - R^{-1})\,[A_{t+n} + \frac{E_{t+n}y_{t+n}}{1-L^{-1}R^{-1}}] \tag{7}$$

Since $\frac{1}{1-RL}$ can be expanded as

$$\frac{1}{1-RL} = \frac{-(RL)^{-1}}{1-(RL)^{-1}} = -\frac{1}{R}L^{-1} - (\frac{1}{R})^2 L^{-2} - (\frac{1}{R})^3 L^{-3} - \ . \ . \ . \tag{8}$$

and by definition of lag operator

$$L^{-n}\, y_t = y_{t+n}$$

implies

$$\frac{(RL)^{-1}}{1-(RL)^{-1}} y_t = \frac{1}{R}L^{-1}y_t + (\frac{1}{R})^2 L^{-2} y_t + (\frac{1}{R})^3 L^{-3}\, y_t + \ . \ . \ . = \sum_{j=1}^{\infty} (\frac{1}{R})^j \ y_{t+j} \tag{9}$$

(Sargent, pp. 178-179).

Eq. (9) shows that if we were to calculate it repeatedly for $y_{t+2}, y_{t+3}, \ldots, y_{t+n}, \ldots$, the lower limit for the index of summation (j) would move forward by one for each new period, and for any n,

$$\frac{E_{t+n}y_{t+n}}{1-(RL)^{-1}} = \sum_{j=n}^{\infty} (\frac{1}{R})^{j-n} E_{t+n}y_{t+j} \tag{10}$$

then

$$c_{t+n} = (1 - R^{-1})\,[A_{t+n} + \sum_{j=n}^{\infty} (\tfrac{1}{R})^{j-n} E_{t+n} y_{t+j}] \quad (11)$$

and for $n = 1$,

$$c_{t+1} = (1 - R^{-1})\,[A_{t+1} + \sum_{j=1}^{\infty} (\tfrac{1}{R})^{j-1} E_{t+1} y_{t+j}] \quad (12)$$

The short way to reach the summation of eq. (12) is to take eq. (9), found in Sargent, and multiply it by R (and use the definition of lag operator),

$$\frac{L^{-1}}{1-(RL)^{-1}} y_t = \frac{y_{t+1}}{1-(RL)^{-1}} = L^{-1} y_t + (\tfrac{1}{R})\, L^{-2} y_t + (\tfrac{1}{R})^2 L^{-3} y_t + \ldots = \sum_{j=1}^{\infty} (\tfrac{1}{R})^{j-1} y_{t+j} \quad (13)$$

3. Flavin Consumption forperiod *t+1*

When permanent income is equal to consumption, Flavinstated that consumption at period $t+1$(Flavin's eq. (4)) is given by,

$$c_{t+1} = (1 - R^{-1})\,[A_{t+1} + \sum_{j=0}^{\infty} (\tfrac{1}{R})^{j} E_{t+1} y_{t+j+1}] \quad (14)$$

Two observations on Flavin's approach:

It is relevant to point out that, Flavin had to make the critical assumption, "*if the expectations of future income are rational, the expectation of next period's revision in expectation* $(E_{t+1} - E_t)\, y_{t+j+1}$*is zero*" to reach the conclusion that,

$$E_t\, c_{t+1} = c_t \quad (15)$$

Clearly, one needs to question whether Flavin's assumption is really necessary; whether it holds true from period t to period t+1 or for any period t+n and under what conditions.

Second, as mentioned earlier, eq. (12) and (14) give the same result, i.e.,

$$c_{t+1} = (1 - R^{-1})\,[A_{t+1} + y_{t+1} + (1/R)\, E_{t+1}\, y_{t+2} + (1/R)^2\, E_{t+1}\, y_{t+3} + \ldots$$

$$+ (1/R)^{n-1}\, E_{t+1}\, y_{t+n} + \ldots] \quad (16)$$

Is there a meaning for eq. (12) and Flavin's eq. (14)?

Since it is well known that a formula can yield different structural formulas but not all structural formulas will yield the same result, one should reasonably consider that, even though the number of incomes goes to infinity, as the consumer ages, there is a loss of income going forward one period. In contrast, in Flavin's equation, the number of incomes remains constant. This can be seen by the lower limit of the index of the summations, which, for period t+1, vary from 1 to infinity, while in Flavin the lower limit of the index remains from zero to infinity. It is relatively straightforward to show how the difference of two summations with the same number of incomes may equal to zero. The untenable (and implicit) assumption that a consumer won't lose any labor income while he/she ages is the reason why one must always check the range of the summation.

4. New Result

Applying eq. (12), the change in consumption is,

$$c_{t+1} - c_t = (1 - R^{-1})[\, A_{t+1} - A_t + \sum_{j=1}^{\infty} \left(\frac{1}{R}\right)^{j-1} E_{t+1} y_{t+j} - \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^{j} E_t\, y_{t+j} \,] \quad (17)$$

and since,

$$\sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^{j} E_t\, y_{t+j} = y_t + \sum_{j=1}^{\infty} \left(\frac{1}{R}\right)^{j} E_t\, y_{t+j} \quad (18)$$

If we assume that,

$$\sum_{j=1}^{\infty} \left(\frac{1}{R}\right)^{j-1} E_{t+1} y_{t+j} - \sum_{j=1}^{\infty} \left(\frac{1}{R}\right)^{j} E_t\, y_{t+j} = 0 \quad (19)$$

Applying the definition of total income or "measured" income (Sargent, p. 371),

$$y_{mt} = \left(1 - \frac{1}{R}\right) A_t + y_t \quad (20)$$

change in consumption can be written as,

$$c_{t+1} - c_t = y_{mt+1} - y_{t+1} - y_{mt} + y_t - \frac{(R-1)}{R} y_t \quad (21)$$

thus,

$$\text{change in savings} = (y_{mt+1} - c_{t+1}) - (y_{mt} - c_t) = \left(y_{t+1} - \frac{y_t}{R} \right) \quad (22)$$

One of the advantages of this new result, change in savings is a function of income growth, is that, even if the difference in the sum of expected future incomes assumption were not to hold true, change in savings would still be dependent on income growth. In fact, one can relax both assumptions that change in expectations of income and the sum of expected future incomes to be zero and still be able to reach this new result. Arguably, this result is not obvious, one cannot derive income growth from Flavin's approach. In addition, this approach and Flavin's will lead to opposite conclusions.

5. Conclusion

In Adam Smith's Wealth of Nations (1776), Smith thought that "*it is not the actual greatness of national wealth, but its continual increase, which occasions a rise in the wages of labour.*" It is a misconception that higher savings will lead to a "better" economy because change in savings is a function of income growth, not the other way around. Furthermore, since growth is dynamic, targeting a saving rate level is irrelevant.

In Keynes' General Theory (1936), "*a decline in income due to a decline in the level of employment, if it goes far, may even cause consumption to exceed income . ...[p. 98]*" This result shows that Keynes' theories can be mathematically derived from Modigliani's Life Cycle Hypothesis and Friedman's Permanent Income Hypothesis, where income growth is dynamic leading to the disequilibrium model in Keynes' saving and dissaving. Similarly, Keynes' fiscal stimulus policy follows the logic of employment, income growth and change in savings. In other words, trade imbalance, currency, technology, labor cost, tax, and other factors affecting income growth may help explain why countries have periods of accelerated and then slower change in savings, e.g., Japan and China.

Clower's Dual Decision Hypothesis (1965) may help explain why consumption is continually re-evaluated as income changes, leading to a 2 step decision making. That is, the difference between expected and actual income may cause errors in optimal consumption, which may require corrections on consumption.

Modigliani & Brumberg (1952) hypothesized that "*in the long run the proportion of aggregate income saved depends not on the level of income as such but, rather, on the rate of growth of income* ..." This new result has expanded Modigliani hypothesis by showing the relationship between savings and growth may not always hold true, i.e., depending on savings at the intercept of zero growth, consumers may have positive growth and negative savings and vice versa. That helps explain why the U.S. saving rate is near negative even though growth is positive.

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