Journal of Economics Library
www.kspjournals.org
Volume 3 September 2016 Issue 3

Mexican Hat Wavelet Mathematical Formula Applied to Business and Economics

By Patrick D. CAOILE [†]

Abstract. The Mexican hat wavelet mathematical formula is used in physics, music, and earthquake prediction to allow different equations to equal because the wavelet can approximate the exogenous and the endogenous variables. This spontaneous symmetry breaking equation can have applications to business and economics akin to the game theory Nash equation that enabled companies to provide yields to financial instruments without necessarily go through a zero sum equation scenario. This research will allow the economics and finance practioners to apply the model to their respective disciplines to be able to draw the different uses in business and economics in particular risk analysis.
Keywords. Mexican hat wavelength mathematical formula, economics and business theory building, Nash equation.
JEL. C40, C50, C57.

1. Introduction

Fan & Lu (2008) described the wavelet as small wave localized in both time and frequency space and are useful for processing data with sharp discontinuities or compressing image data. There are two main uses as an integration nucleus of the analysis to get information about the processes and as a characterization basis of the processes. The wavelet had been used to explain several phenomena in very diverse fields such as music, atmospheric analysis, physics, medicine, earthquake detection, and even the analysis of emerging markets.

2. The Historical Background

Chun-Lin (2010) cited Morlet in 1981 as well as Morlet and Grossman in 1984 then improved on the concept from its beginnings with Alfrd Haar in 1909. Goupellaud, Grossman, & Morlet (1984) proposed an improvement from the mathematician Alfrd Haar as early as 1909. Eadie, et al. (1971) mentioned the Mexican wavelet as part of the statistical methods in experimental physics. Then Meyer further improved on the wavelet concept in 1985 as well as Mallat in 1988 and separately with Daubeechies in 1988. Daubeechies (1992) published his ten lectures on wavelets that he conducted at the University of Lowell in Massachusetts. Alexey Vikhlinin together with Forman & Jones (1998) cited applications in astrophysics, cosmology, and X ray astronomy and further expounded on this in 2001. Holschneider (1995) published his work on wavelets as tools for analysis. Fischer, et al. (1996) cited spatial filters to discuss the role of

[†] Financial Management Department, De La Salle University, Ramon V. Del Rosario (RVR) College of Business, Taft Avenue, Malate, Manila, Philippines.
☎. + 09178153125
✉. patrick.caoile@dlsu.edu.ph

wavelets in pixels. Recent works include those of Brinks (2008) and Lindeberg (2015) on imaging and the Gaussian wavelet function. Rua & Nunes (2012) used the concept to evaluate emerging markets like Portugal.

The Mexican Wavelet Mathematical Formula. In mathematics and numerical analysis, the *Ricker wavelet* (1)

$$\psi(t) = \frac{1}{\sqrt{2\pi\sigma^3}}(1 - \frac{t^2}{\sigma^2})\frac{-t^2}{e2\sigma^2} \tag{1}$$

Is the negative normalized second derivative of a Gaussian function. It is a special case of the family of continuous wavelets (wavelets used in a continuous wavelet transform) known as Hermitian wavelets.

The Ricker wavelet is frequently employed to model seismic data, and as a broad-spectrum source term in computational electrodynamics. It is usually only referred to as the *Mexican hat wavelet* in the Americas, due to taking the shape of a sombrero when used as a 2D image-processing kernel. It is also known as the *Marr wavelet* for David Marr (2).

$$\psi(x,y) = \frac{1}{\pi\sigma^4}(1 - \frac{x^2+y^2}{2\sigma^2})e^{-(x^2+y^2)/2\sigma^2} \tag{2}$$

Shown as Figure 1 is the 2D Mexican hat wavelet

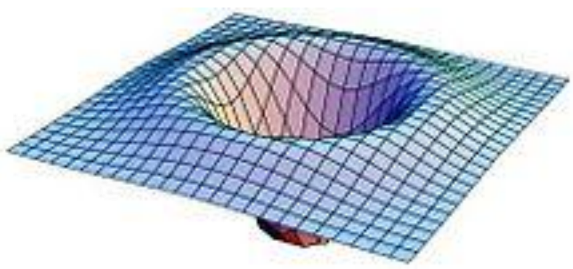

Figure 1. *2D Mexican wavelet*

The multidimensional generalization of this wavelet is called the *Laplacian of Gaussian* function. In practice, this wavelet is sometimes approximated by the *difference of Gaussians* function, because it is separable and can therefore save considerable computation time in two or more dimensions.

The scale normalized Laplacian (in L_t -norm) is frequently used as a blob detector and for automatic scale selection in computer vision applications; see Laplacian of Gaussian and scale space. The Mexican hat wavelet can also be approximated by derivatives of Cardinal B-Splines.

3. Economics and Business Applications

Precisely because of its symmetric qualities that enable the Mexican hat wavelet mathematical formula to be applied in various disciplines that seem unrelated such as commerce and business. Rua & Nunes (2012) used the Mexican wavelet to explain risk in emerging markets like the Philippines. Dr. Harry Markowitz introduced the concept of portfolio theory to mitigate the business risk but this meant dichotomizing risk into diversifiable and market risks. Diversifiable risks are unsystematic risks that are intrinsic and inherent in the companies that comprise the portfolio while market risks are systemic risks that involve the general conditions of business wrestling with the notion of inflation, unemployment, and business cycles. The idea is to treat investments in companies' stocks as a basket of goods with wide disparity in beta variances. An investor and the investment manager does not really chose companies with all low betas as the investment might be better managed as investing in low risk government bond or commercial paper instead of a portfolio. The systemic and market risk will go against the investment bundle. The idea is to comingle varied betas such that risk is spread and even mitigated.

Dr. William F. Sharpe introduced the Capital Asset Pricing Model (CAPM) to improve on the idea of the Portfolio Theory. The CAPM mathematical formula (3) is shown below:

$$\frac{E(R_i) - R_f}{\beta_i} = E(R_m) - R_f \tag{3}$$

The market reward-to-risk ratio is effectively the market risk premium and by rearranging the above equation and solving for $E(R_i)$ we obtain the capital asset pricing model (CAPM)((4)).

$$E(R_i) = R_f + \beta_i (E(R_m) - R_f) \tag{4}$$

Where:

- $E(R_i)$ Is the expected return on the capital asset
- (R_f) Is the risk-free rate of interest such as interest arising from government bonds
- β_i (the *beta*) is the sensitivity of the expected excess asset returns to the expected excess market returns, or also $\beta_i = \frac{Cov(R_i, R_i)}{Var(R_m)}$,
- $E(R_m)$ Is the expected return of the market
- $E(R_m) - R_f$ Is sometimes known as the *market premium* (the difference between the expected market rate of return and the risk-free rate of return).
- $E(R_i) - R_f$ Is also known as the *risk premium*

Restated, in terms of risk premium, we find that:

$$E(R_i) - R_f = \beta_i (E(R_m) - R_f) \tag{5}$$

which states that the *individual risk premium* equals the *market premium* times *β*.

The portfolio theory model and its improvement in the CAPM contain intrinsic and inherent constraints that prevent the discussions and inclusions of market risks. The model just encourages the isolation of markets risks. Consequently by adding the Ricker wavelet to either side of the equation will now account for the inherent risk of the market. The equation will now be robust and will account for market risks (6), by adding the wavelet (equation 1) being the mathematical equation that will allow the risk premiums to balance but the intrinsic risk factor as well.

$$\psi(t) + E(R_i) - R_f = \beta_i (E(R_m) - R_f) + \Psi(t) \tag{6}$$

References

Brinks, R. (2008). On the convergence of derivatives B spline to derivatives of Gaussian function. *Computational Applied Mathematics*, 27(1), 79-92. doi. 10.1590/S1807-03022008000100005

Chun-Lin, L. (2010). A Tutorial of the Wavelet Transformation. [Retrieved from].

Daubeechies, I. (1992). *Ten Lectures on Wavelets*. Philadelphia SIAM

Eadie, W.T., Drijard, D., James, F.E., Ross, M., & Saudolet, B. (1971). *Statistical Methods in Experimental Physics*, Amsterdam: North Holland.

Fan, H.Y., & Lu, H. (2008). General Formula for FindingMexican Hat Wavelets by Virtue of Dirac's Representative Theory and Coherent State. Department of Physics, Shanghai Jiao Tong University. [Retrieved from].

Fischer, R., Perkins, S., Walker, A., & Wolfart, E. (1996). *Spatial Filters Gaussian Smoothing*. John Wiley & Sons Ltd.

Goupellaud, P., Grossman, A., & Molet, J. (1984). Cycle-Octave and Related Transforms in Seismic Signal Analysis, *Geoexploration Band*, 23, 85-102.

Holschneider, M. (1995). *Wavelets: An Analysis Tool*. Oxford. Oxford University Press.

Lindeberg, (2015) Image matching using generalized scale space. *Journal of Mathematical Imaging and Vision*. 52(1), 3-36. doi. 10.1007/s10851-014-0541-0

Rua, A., & Nunes, L. (2012). A Wavelet Based Assessment of Market Risk: The Emerging Market Case, Banco Portugal Economic Research Department. [Retrieved from].

Vikhlinin, A., Forman, W., & Jones, C. (1998). Evolution of Cluster X-Ray Luminosities and Radii: Results From 160 Square Degrees ROST Survey, ApJ Letters, 498, 162.

Vikhlinin, A., Forman, W., Markenitch, W., & Jones, C. (2001) *Zooming in on the COMA Cluster with CHANDRA: Compressed Warm Gas in the Brightest Cluster Galaxies.* Harvard-Smithsonian Center for Astrophysics, 60 Garden St. Cambridge MA. Submitted to ApJ letters, February 27. 2001.