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Randomness, Determinism and Undecidability in the Economic Cycle Theory

By Ignacio Escañuela ROMANA †

Abstract. The scientific literature that studies the Business cycles contains a historical debate between random and deterministic models. On the one hand, models built with explanatory variables follow a stochastic trajectory and produce, through transmission mechanisms, the studied cycles. Its rationale: the so-called Slutsky-Yule effect. In addition, models in which the system phase at time T fixes, applying the "ceteris paribus condition", the phase at time t + 1. The cycle would be the product of variables, making it possible to predict and enabling economic policies to combat recessions. The thesis of this work is as follows. The application of the theorems of Chaitin of undecidability shows that it is not possible to conclude such debate. It is impossible to determine with absolute certainty whether the observed cycles follow a deterministic or stochastic model. To reach this result, I outline the fundamental theories of the business cycle, providing a classification and examples of mathematical models. I review the definition of randomness, and I consider the demonstration of Chaitin about the impossibility of deciding whether a data set is stochastic or not. A consequence, he says, of Gödel incompleteness theorems. I conclude considering a string of economic data, aggregated or not, as random or deterministic, depends on the theory. This applies to all cyclical phenomena of any nature. Specific mathematical models have observable consequences. But probabilism and determinism are only heuristic programs that guide the knowledge progress.
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1. Introduction

The Economic cycle consists of the variations that the activity of attached economic variables, such as the national income or employment, have over time, around a long–term tendency from which diverts. This movement is given as in nominal values as, deflecting, in real quantities. The second characteristic of the cycle lays on the fact that different economic strings show a positive or negative correlation in the cycle. It means, there is an ensemble movement along the variations (Diebold & Rudebusch, 1996: 67). An outfit movement which appears from cycle to cycle. To sum up, the historic recurrence of fluctuation lets and requires a general theory of the cycle. Theory which explains and preferably predicts.

However, the cycles, although are recurring, are also irregulars in stage, space and time of repetition. So, the existence of cyclic determined movements is in doubt and it makes, at least, difficult to design models which make possible the prediction.

† Calle Ramón de Paz 30, 41820 Carrión de los Céspedes, Sevilla. Spain.
☎. 954755807, 674155783
✉. ignacioesro@gmail.com

The cycle is, as well, a problematic feature, because the theory builds models in which the economic agents, rationally acting, reach a position of balance among them. Why don´t the unstable cycles stop of repeating?

Can we establish a level theory which lets describe properly the different observed cycles historically? Kingleberger (1985) denies it. However, I consider the Economic science moves forward with the constant attempt to refine general mathematical models to describe and predict with accuracy and generality, the economic events. Mathematics and modern science go hand by hand because Mathematics let know, forecast and promote the facts, and processes which wish being launched (Frey, 1972:177).

Being the business cycle theory necessary, an agreement has not been reached about a conceptual and general mathematic model until now. Alternative research programs exist. In the scientific literature, there are, at least, three perspectives (Ekkehard & Stockhammer, 2003:1).

Keynesian theory of lack of effective aggregate demand, which explains the economic cycles as market failures due to rigidities in prices and/or salaries, or other factors, may reach a high involuntary unemployment.

Secondly, the classic theory of return to the equilibrium of full employment throughout the operation of markets. Economy is always in a Walrasian equilibrium: combination of prices and quantities which simultaneously equalize supply and demand in the different markets of economy. The economic agents decide maximize their settlement, in the temporary horizon, with possible restrictions in the production possibilities and limits of resources.

The business cycle is explained as a product with imperfections of market (asymmetric information, doubt, oligopolies, public procedures) or random exogenous shocks which cannot be provided (Tobin 1995: 32). It deals, therefore, about unpredictable exogenous factors or institutional eliminable elements. Nothing substantial to the market. The economic agents act with rational expectations and choose the best possible positions. The markets tend, by themselves, to a balance where they get empty in natural rates.

Thirdly, Austrian theory, it is rather minor in the scientific literature, based on the alteration of intertemporal prices which are far from the preferences. Changes in the economic policy and/or modifications in bank credits which lead to different monetary interest types far from natural interest types and run into fluctuations by inefficient resource allocation. A correct monetary policy and institutional improvements, especially in the financial sector, can lead us to a world described by the classic theory.

The classic theory says only the prices and real wages determinate the real quantities of production and employment, because the agents understand the impact of the prices and they discount them. The economy is constantly in balance of the system of Walrasian equations. While the Keynesian theory is still postulating real classic equilibriums (Tobin 1995: 38) based on the equality of the marginal productivity and disutility of labor, determining real salaries, volume of employment and output. But in that point, the markets don't empty and involuntary unemployment can exist. The economic cycles are persistent and can´t only be explained by the evolution of real variables. The explanatory key is the slow adjustment in prices and wages. Even though mechanisms have been suggested which will produce the same result even with a complete flexibility of prices and wages. Greenwald & Stiglitz (1993) research the general impact of externalities (institutional incomplete markets and with incomplete information) over the global economic balance.

The current classic theory is called Real Business Cycles theory (RBC) or Dynamic Stochastic General Equilibrium models (DSGE). The key contributions

are Kydland & Prescott (1982), Long & Plasser (1983) and King, Plosser & Rebelo (1988). They are introduced in a growing neoclassic model, with rational optimization by the agents, random and exogenous factors as stimulus of the cycle. Mainly, technological shocks.

The classic theory is of the equilibrium one in natural rates. The Keynesian theory is of the disequilibrium one in prices and quantities or equilibrium one without emptying. The classic theory states the dichotomy between real and nominal variables. The Keynesian one not.

The cycle theories are analyzed and put into groups according to their deterministic or stochastic character. I will discuss whether the inclusion of endogenous or exogenous variables have any relation with this character. I will classify the theories according to these two judgements.

What is considered is whether the explanatory mathematic model of the observed cycles means the equal to a short program which let us deduce the long program or data set. If from this set of variables, connected among equations, we can find out how the cycle has happened and will happen. We can predict it.

Chaterjee (2000) states that an evolution in the economic cycle theory exits. First analysis would be based on postulating factors which would produce determined fluctuations. Economic system factors which would develop peaks and crisis in an endogenous and unambiguous way. While later theories gave the cycles to the compound effect of random disturbances. Although the identification of these shocks would have been, until now, ambiguous or inaccurate. This author says an evolution appears from the first to the second way to explain the cycle and an almost current agreement about a string of random shocks are the key. Slutsky showed in the 20s that random fluctuations can be at the base of the cycles. Later, Slutsky thesis or Slutsky-Yale effect are analyzed. In a similar way, Benhabib & Farmer (1999) say the Modern Macro-economy is based in DSGE models designed by the Real Economic cycles theory. In my opinion, however, both perspectives or stochastic and determinist programs coexist and debate (Mankiw, 1989) with various proposals of mathematic models.

The final thesis of this work lays on, due to the impossibility of giving an empiric content to the concept of randomness and determinism, the main discussion is unsolvable and it will always happen in the economic theory. Models will be provisionally accepted and others rejected, according to the theory and data, but not to the related ideas about random or its denial.

2. A simple guideline to analyze the empirical application of economic theories

When we state an event happens randomly, it is aimless; does this statement admit an empiric proof, or is it an unanswerable and irrefutable conclusion, from data?

The criterion used to know if a scientific concept is observable is done by Bridgman´s operationalism. For that, I consider that random is a concept with observable reference only if there is an operation or operation set to find and measure such reference properly. The operations can be physical or mental. (Bridgman, 1927: 36)

I don't demand that the concepts have empiric contents, this is not the topic concerned. But it is demanded that, to have empiric contents, an operating procedure must exist. This guideline is restrictive and is effective to decide the empiric concept of the concept random. Also, it is equal to, dealing with this work, other formulations such as Hempel´s: A statement with the concept "random"

would have the empiric meaning whether it was expressed in terms of observable characteristics of physical objects" (Hempel, 1950:10).

In short, a concept has an empiric meaning "if we can assign numerical values to particular instances of it-if we can, in effect measure it under certain circumstances" (Gillies, 1972:8). If not, "random" won´t have this reference and will build other statements, without empiric foundation.

Without an empiric reference, and out of analytical (tautologies) statements, some few concepts can have, however, a positive function in the science. Their functionality would be heuristic: basic concepts which give foundation to programs of research and alternative proposals. They must, therefore, have principles which let build contrastable and refutable alternative models up. The heuristic value of a concept is given in an environment of epistemic doubt, when there isn´t a final and complete solution to the problem given. In that situation, the construction of programs with possible alternative solutions would be essential. The mathematical models are refutable, but not concepts such as random and determination.

3. The Determinism

Following to Ernest Nagel (2006: 371-441) and transmitting what he says about physics to the social sphere, a set of rules is deterministic for a system of variables, related to a kind of characteristics, if a phase of the system determinates unambiguously its phase at any other moment. The theory lets know what will happen to the considered system, whether we know all initial conditions. Indeterminism would be the mathematical model that doesn´t permit us to know the following phase of the system accurately.

It means, the system is deterministic if a function F (model) exists, that knowing the values of variable in time T1, it gives values to these variables in time T2 and then in time T3 and so on (Bricmont, 2004: 3). Once we know the values T1, it implies predictability. Under the assumption "Ceteris Paribus": the non-included variables in the model don´t move".

This definition of determinism let get away from polemic about if determinism and coincidence have a necessary relation between them. In my opinion, the coincidence gives an unnecessary conceptual complexity.

4. Random concept and Undecidability

I am starting from the concept of Randomization Kolmogorov-Chaitin (also found as Solomonoff). Volchan (2002: 60-61) believes that Martin-Löf definition about randomization is preferable, but it is equal to that concept.

Program means instructions (rules) plus input. Elegant program is the smallest one which is able to generate a string of data. Data are produced randomly if the same string is its elegant program. It means, if the available short program, which generates the string, is as long as the string itself, it isn´t compressible. "A string is random if no program of size substantially smaller than the string itself can generate or describe it" (Volchan, 2003:56, Chaitin, 2004:22). Calude (2002) states, it means the absence of a command or model. These definitions are equivalent. "Intuitively, the slightest possibility to calculate an infinite fraction of a succession makes this is not random" (Mario-Parra & Suarez, 2006. 166). Random implies unpredictability (Calude, 2002).

Consequently, demonstrating that a string is random is to probe that it is its own elegant program. Proving is not random; it is to find a shorter elegant program.

This definition leads to what we cannot know, if a string is stochastic or not accurately. We can´t know because any distribution of data could be done randomly, despite being more or less probable. Throwing a coin, the string

"1001101011000" is as probable as "111111111111": both options have a probability of $1/2^n$. In this case $1/2^{12}$. However, we intuitively observe that a string has a bigger stochastic appearance.

Chaitin thinks this characteristic comes from incompleteness of every formal reasoning system, so a border is placed to not overcome. Is a random string or it seems only? We don´t know, it is a limitation of the mathematics related with the incompleteness theorem of the formal systems minimally powerful for Gödel (Chaitin, 1975). This is an accepted result by the scientific mathematical literature.

The demonstration can be done in different ways, and, as such, it is not subject in this work. The base is on the function which decides the complexity of a string: if it is a product of an elegant program and what is the size of that program. This function is not itself computable, so we cannot accurately decide the randomness of a string (Volchan, 2002: 2). "If we could prove that a program is elegant, then that would enable to find a smaller program that produces the same output, contradiction!" (Chaitin, 2004: 109). In conclusion, it isn´t possible to determinate if a program is elegant or not. Because if it was possible, we would go into a contradiction.

It means, there isn´t an algorithm that we can explain a string of data and answer to the question of whether a string is random or not. It is not possible to decide if a string is stochastic or not because we cannot prove it. We can only watch if a string seems more or less stochastic, so we can speak about pseudo-random strings when they have such statistical appearance. This inability to choose is the fundamental and unappealable result.

It doesn´t exist a final, quantitative and qualitative procedure to find a string of data randomly. Nor otherwise. Operationally, it is a concept without operation. Random can have theoretical content, empiric approximations but never a complete empiric model

5. Slutsky-Yale effect

Slutsky (1937) asked himself in 1924 (reproduced in 1937 in Econometry) whether random fluctuations may generate regular waves. It means, if strings with deterministic appearance (regular) could be, actually, generated by stochastic process. It is said coherent string in which the appearance of a value in a position depends on the previous or subsequent values of the variable (auto-correlation). An incoherent string doesn´t show this dependence.

Barnett (2006) says Slutsky is defending two proposals. Firstly, adding random factors can generate phenomena with regular waves appearance. It means that the addition introduces in mutual independent events an oscillatory appearance, in which subsequent data have a dependence from the subsequent ones. Secondly, these fluctuations with wave aspect can seem cycles which show apparent regularity. "The summation of random causes may be the source of cyclic, or undulatory processes" (Slutsky, 1937:114).

To prove these proposals, Slutsky uses a data mobile addition and tries to demonstrate that this process turns into coherent to incoherent strings. Giving as results, movements with cyclical appearance, similar to real economic cycles studied. For that, he compares the graphic of aneconomic cycle observed with the movement of data, stablished by the selection of a series of lottery numbers and doing a mobile addition of twelve. Adding the number of random factors, we will get, finally, to cyclical series equal to symmetrical functions. Also, these wavy movements would be identical to studied economic cycles. Slutsky used other procedures with similar results. All these methods share the fact of introducing a dependent structure among data.

To conclude, Slutsky introducesan inductive proof about the possibility of variable randomly acting give a cyclical process in Economy. In general, it is a fact of wavy appearance. Inductive proof means it is an example, or several examples, of the possibility, and not a universal deductive demonstration.

Yule (1926) thought about the same idea, referring to the appearance of the cyclical behavior in a string that is, actually, the result of stochastic factors. Oscillations which don´t appear in the original data. Yule would suggest (Alvarez Vazquez, 1996:320-1) an auto-countdown model for the economic cycle or equation in finite differences of second order, with the addition of a stochastic process or white sound.

$$Y_t = \delta_1 Y_{t-1} + \delta_2 Y_{t-2} + \varepsilon_t \quad (1)$$

Where the variable Y is related with its delayed values, and δ are the coefficients. A process of white sound ε is a succession of independent values among them and distributed identically. In a lax way, a string of values with a zero auto-correlation in every retardations and normality. The random coefficients and shocks are the key, once we get output data. The white sound process produce, right now, a string of impacts of cyclical appearance.

The economic cycle would behave as a pendulum, with a white sound process ε_t hitting the pendulum randomly. It would generate a regular movement that is not true. It is calculated to solar spots, giving a period of 10,6 years. Where this autoregressive model is similar to a process of mobile additions of Slutsky type (Pollock, 1987), so:

$$Y_t = \mu_0 \varepsilon_t + \mu_1 \varepsilon_{t-1} + \ldots + \mu_q \varepsilon_{t-q} \quad (2)$$

As Pollock explains (1987), Slutsky and Yale are confirming the same type of dependent process of the random disturbances: "the affinity of the two sorts of process is further confirmed when it is recognized that an autoregressive process of finite order is equivalent to a moving-average process of infinite order and that, conversely, a finite order moving–average process is just an infinite-order autoregressive process" (p.8).

Kuznets (1929) accepted the following thesis: series serially auto-correlationed produce, through mobile additions, series with auto-correlation. Space and time of the cycles generated would depend on the distribution of random shocks and on the period of the mobile addition employed. The more elements we include in the series of a mobile addition, the higher the internal correlation in the series that we get. Frisch (1931) focused on the way to avoid spurious economic cycles. Moran (1950) agrees that applying mobile additions to a random string leads, in the limit, to perfect sinuous functions.

It has also raised a second option or "realistic" interpretation: we have a "real" effect (Barnett, 2006), it is the addition of random "shocks" what produces a studied cycled. So, business cycles are explained as the addition of random causes, without the necessity to turn to any underlying regular factor, which becomes irrelevant. Business cycles would be phenomena of random foundation. The theory of real business cycles takes Slutsky-Yale effect as foundation, considering it as tested. The macroeconomic series would be nonstationary stochastic processes, without tendency, by the effect of real random factors. (Nelson & Plosser, 1982).

This problem arises for all data series that present periodicity. "All activities in which the periodicity of time series was involved. Thus, it had direct relevance to all statistical manipulations of data" (Barnett, 2006: 6).

The result of Chaitin says every answer to the Slutsky-Yale problem can only be an approximation. Never a certainty. We have series of cyclical appearance, in which to know if they are a product of random variables falls within Chaitin Undecidability theorem. It is only an approximation.

Likewise, while Slutsky and Yale sharply set out the possibility that the studied cycles have random elements produced.

From now on, the scientific literature has discussed about two different conclusions. The first option or interpretation "statistics" (Barnett, 2006) is that we have a statistical effect to bear in mind. A result due to the application of a procedure. Moving averages or others. A logical question arises: if we take a economic series of data and we do an careful study, avoiding a procedure of addition or average, can we avoid the problem exposed by Slutsky? If we improve the procedures, would we dismiss the generation of possible spurious business cycles?

Likewise, a second option arisen or "realistic" interpretation: we have a "real" effect (Barnett, 2006): it is the addition of random "shocks" what a studied cycle produces. So, studied business cycles explain the addition of random causes, without resorting to any regular underlying effect which becomes irrelevant. Business cycles would be phenomena of random foundation. The real business cycles theory uses this Slutsky-Yale effect as a foundation, considering as approved. Macroeconomic series would be non-steady stochastic processes, without tendency, by the real random effect of factors (Nelson & Plosser, 1982).

This problem arises for every series of data which have periodicity. "All activities in which the periodicity of time series was involved. Thus, it had direct relevance to all statistical manipulations of data" (Barnett, 2006:6).

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Also, although Slutsky and Yale sharply exposed the possibility that the studied cycles could have random elements which they produce, they left out the problem whether the presumably stochastic factors were or not. Let´s use the lottery series as an example of random series, it is a questionable assumption: because the statistical values of the series don't fully coincide with the probabilistic assumption (Alvarez Vazquez, 1996, 310). Alvarez Vazquez tries to prove that the regular cycles are already in the original data of the lottery string. "The cause of the regular cycles, studied in the series of mobile additions, is in the regular movements of the original series" (Alvarez Vazquez, 1996, 301). So, "the mistake of the argument in which Slutsky hypothesis lays on would be in assuming that the string of the first prizes (its last digits) were random, when it is only irregular" (Alvarez Vazquez, 2004, 98). Since in the frequency field, the regular cycles don't equally contribute to the variety as they should do in an ideal random series. The mobile average would create or destroy nothing.

Therefore, the main previous problem to the interpretation is referred to the assumptions arose from Slutsky and Yale. Business cycles can be considered deterministic or random, but the factors which seem random can be questioned, as predicted Chaitin Undecidability theorem.

6. Classification of the cycle, determinism and endogenousity theories

There isn´t any basic theory accepted by a scientific literature consensus and the discussion among alternative research programs is so complex.

The two main research programs are Keynesian and classic. I have explained their conceptual basis. The classic theory uses the form of RBC theory, about the conceptual basis of the rational expectations of Lucas and models of general dynamic stochastic DSGE equilibrium are generated. The conceptual basis is in the general theory of the economic growth which comes from Solow, and, when the agents can choose all variables, specially savings, from Ramsey-Cass-Koopmans. The fluctuations of the cycle are necessarily the best answers for the economic agents to real exogenous shocks. On the Keynesian hand, from the 80s, it has looked for microeconomic fundaments for the temporary stiffness of the prices and salaries, and the effects of intuition about the risk in the corporate behaviour. Coordination mistakes, the imperfect competence, the stiffness in the assignment mechanism and nominal shocks are studied.

A part of this Keynesian literature has adopted the DSGE form. In this mathematical model, for example Gali & Rabanal (2004), studies demanding random shocks instead of technological random shocks. Cho & Cooley (1991) introduced the nominal stiffness in the employment contracts. Blanchard & Kiyotaki (1987) studied monopolistic competition with nominal stiffness, by contracts whose effect lasts in the time. Hairault & Portier (1993) took into account dynamic monopolistic competition with exogenous and random technology and money supply. And a long list. Models which link Keynesian principles in classic models.

Another part of the literature, which has developed DSGE models, is distinguished by adopting the principle of indetermination in the evolution to the equilibrium or stationary state, or in the same stationary state. A "sunspot" shock, in this literature, is the impact that the expectations of the agents have when, not being based on the real evolution of the macroeconomic variables, have a final impact in the allocation. It means, it is a random variable that doesn´t affect to the economic foundations (endowment, preferences, technology) and isn't established by them, and give that some different balances of consumption and production are possible.

The expectations are auto-predictive: they are done on the mere fact that the agents believe. This capacity to have a so important impact comes from the fact that the models are incomplete and incorporate factors which expand the impact. The two more common elements are an incomplete working market with externalities, where there isn´t a balance between supply and demand, and scale economies in production function. Shell (1977) used the simple model of generations that comes before Lucas´ one (1972), using Samuelson`s one (1958) to introduce "sunspots" shocks which affect to the level of prices. Benhabib & Farmer (2994) proved that indetermination in a simple model, type DSGE, with scale economy can arise. Multiple equilibriums are possible and the final equilibrium is random. In closed models, with complete markets, the "sunspots" shocks would not have impact because the agents avoid any risk (Benhabib & Farmer, 1999). Otherwise, if the fundamental variables give a solution, the expectations could not significantly modify the final equilibrium, nor the way to it.

Called by some authors as "Endogenous Business Cycle theory", EBC theory, is endogenous because includes variables which turn the initial boosts into cyclical movements, whose values are decided within the equational system. But it still introducing exogenous triggering factors (Whitta-Jacobsen, 2004, Farmer, 2012), and the expectations behave in random way and move out of the system.

It is true that the technology is a key variable and the expectations are extrinsic. But there aren't relevant conceptual differences between the impact of the technological factor in the RBC and the impact of the expectations of the agents in the models EBC, because in both cases are exogenous and random. The "sunspots"

factors are added to technological shocks. Random movements which close the system." Deterministic" and "indeterministic" doesn´t have to do with "determinate" and "indeterminate" in the EBC literature. "We say that the model is determinate if there is a unique equilibrium and indeterminate if there are multiple equilibria" (McGougha *et al.*, 2012: 8). It means, it is determinate if the model is a solution, and indeterminate if it has several possible options. A RBC model is determinate, in this sense, and, however, is indeterministic in the global defined sense in this work.

The endogenous variables that would transmit and expand the initial shocks, producing the cycle, would be, among others, externalities in the production (Bernabib & Farmer, 1994, Farmer & Guo, 1994), complementarities in the intertemporal election (Azariadis, 1981), unfinished labour market, without labor supply (Azariadis, 1981; Farmer & Woodford, 1984), risk loans without collateral (Azariadis *et al.*, 2015). An example is Benhabib & Nishimura (1998), with a model in which the expectations about the marginal value of the investments cause the double effect in increasing this investment, but, at the same time, reducing, through externalities of the labour market and leisure growth, with a final indeterminate effect.

For Farmer (2012) and Farmer & Platonov (2016) the EBC models that involve indetermination in the way to steady determinate state are conceptually based on the classic theory. A function of beliefs closes the models and explains the stiffness of prices, in a context of rational expectations. The models which have, also, an indetermination in the steady state would be based on Keynesians foundations, producing the possible involuntary long-term unemployment. However, the decisive factor lies on indeterministic models in which the expectations develop randomly and are decisive.

Thinking about these theories, I consider the model of the cycle can be classified according to two principles. If endogenous mechanisms about explanation of the cycle are introduced or not. And if the explanation is deterministic or probabilistic. Determinism and probabilism have already been defined. One or more auto-regressive equations, which have one or more white noise processes, are used to be introduced in the stochastic models.

Endogenous variables are those whose value is determinate in the mathematical model: the variables determinate among themselves. Exogenous variables use non-fixed values within the system of equations. Exogenous variables can be random or deterministic, if their development hasn´t got observable or postulated model, or if they have it. The endogenous variables must be deterministic, because, from a value, el model determines the following.

Moreover, it is necessary to discriminate between a trigger factor of the cycle and mechanisms of spread and persistence. A trigger factor is the movement which causes the first variation in macroeconomic variables. In the RBC theory, it is a exogenous technological shock. In the Keynesian theory, it can be a change in the expectation of marginal productivity of capital. Spread mechanism is the evolution of variables that cause the business cycle in the sense of persistent, continued and correlated movement of the macroeconomic variables. In the RBC theory, it is the inter-temporal replacement of leisure for work. In the Keynesian theory, it is the temporal stiffness of prices and/or salaries, and the perception of risk of the companies.

In this classification, every research model or program can be placed as follows. The RBC theory and the before models to DSGE are exogenous and probabilistic in trigger factors. Technological shocks, if they behave in stochastic way. And endogenous one, in the spread mechanisms: decisions of replacement of leisure for work along the time.

The classic theory, prior to RBC, problems of information, business competence and economic policies were admitted as spread variables. They have been added again.

The Keynesian and neo-Keynesian theories don´t start from a representative agent or maximizing agents inter-temporally, but from macroeconomic variables in a holistic approach. Except, the school of thoughts that uses DSGE models. But these approaches, to belong to this research program, must share the conceptual foundation: exogenous and deterministic theories in the trigger factors. The models of accelerator-multiplier are an exception because they are endogenous. There is a drop in the aggregate demand, which generally isn't described but deterministic. A stochastic mathematical model about this initial reduction is not raised And endogenous in the spread mechanisms. A process of failures of market, with limitations in the factor, monetary and/or good markets. Slow and pricey adjustments in prices and salaries, productive externalities, nominal shocks, imperfect competition (Standler, 1994).

Finally, EBC models or of "sunspots" factors would display random exogenous trigger factors. Auto-predictive expectations, but also technology, that acting on the basis of incomplete markets and economies of scale, determine the result of such markets.

7. Some mathematical developments in the theory of the cycle.

The starting Keynesian models caused endogenous and deterministic cycles, in regular and temporal waves. They were based on the ideas of the multiplier and accelerator.

About Samuelson´s model (1958), Hicks designs the theory with explosive macroeconomic movements, but it causes continuous cycles by the existence of floor and ceiling.

Idea of the multiplier:

$$C_t = c \cdot Y_{t-1} \qquad C_t = c \cdot Y_{t-1} \tag{3}$$

Investment is the induced addition (accelerator) and the autonomous one (independent from the system):

$$I_t = I'_t + I''_t \qquad I_t = I'_t + I''_t \tag{4}$$

$$I'_t = v \cdot (Y_{t-1} - Y_{t-2}) \tag{5}$$

$$I''_t = A_0 \cdot (1+g)^t \tag{6}$$

Being National income, Y_t, the addition of consume and investment. Model that generates a difference equation of second order. If v = 1,we have regular oscillations. If v > 1, we have divergent oscillations, in an explosive model that is what Hicks believes more probable (Samuelson disagreed). If v < 1, they are convergent oscillations.

Divergent oscillations will produce regular and deterministic cycles by the existence of floor and ceiling, which make the movements to go bouncing (in a non-lineal model). The floor is the accelerator retention which makes the induced investment turn negative and equal to the depreciation. And the ceiling is the potential product or full employment, determined by the quantity of productive factors and the existing technology.

The regularity of the fluctuations is the product of a deterministic model, but it doesn´t seem supported in data. Although it could be argued that ceteris paribus cycles occur, so other factors may be changing the frequency. We have the challenge of introducing such variables.

Later Keynesian models were divided about the trigger factor. Some models still include an endogenous and deterministic factor. Others don't allow anticipate the first fact which triggers that market failure. And in this sense, the models are incomplete. But randomization is not postulated and later mechanism is deterministic. Finally, there are models which introduce exogenous and random demand models, but it is debatable that they fall within this approach. In all cases, the idea which links them lays on what the balance of the market can be unreachable, in an irreversible way, without ad hoc interventions of public policies. The cycle would be a natural fact of economy.

The fact the Keynesian models are divided because of introducing exogenous and endogenous factors comes from Keynes´ thought. Keynes (1936) explained the cause of the economic cycles was the movement in the marginal efficiency of the capital. But it didn´t deal with an automatic mechanism, nor necessarily cyclical because the key wasn´t only the marginal value to the last invested currency unit, but also the perception we have of this performance. What in later literature is sometimes called "animal spirits". Keynes claimed in his General Theory, 1936, the possible impact of expectations: "it is not so easy to revive the marginal efficiency of capital, determined, as it is, by the uncontrollable and disobedient psychology of the business world" (Keynes, 2008: 288). The return of confidence would be a difficult factor to manage (Minsky, 1986).

Market failures would provoke non-efficient situations in Pareto terms. There is not market emptiness. Goods without being sold, unemployed work, idle capital. The economic agents would like to be in another transfer point, but they can´t. Recessions would produce a lower social welfare: there are economic agents that could improve without being detrimental to others. As opposed to this classic school of thought which defends that Economy is always in an ideal Pareto situation. The cycle would have as last explanation, monetary factors (nominal variables) which work in necessary way, from a first trigger factor. There isn´t classic dichotomy.

As an example, the simple model exposed by Romer (2006) about stiffness prices, flexible wages, competitive labour market.

$$\pi = \pi^* \tag{7}$$

$$L = L^s\left(\frac{W}{p}\right) \tag{8}$$

$$Y = F(L),\ F'(\quad) > 0, F''(\quad) < 0 \tag{9}$$

Maximum Y exists. π is the price level, L is the number of workers and L^s is the labor supply, which is growing referring to the real wage. $L^s>0$.

The production function depends on the only productive factor which is work, and it has positive labor productivity, but decreasing. The labor demand is determined by the points which meet that the labor marginal productivity is similar to the real wage.

The stochastic factor doesn´t exist in this model. The evolution is again deterministic. But the trigger cause must be necessarily exogenous. An initial moving in demand develops a cut in the income Y. The model doesn´t generate steady cycles, so a continued mechanism which provokes imbalances is left.

Some alleged neo-Keynesian theories mix, by contrary, stiffness in prices and wages, oligopolistic factors with random shocks of technological and monetary character (e.g., Hairault & Portier, 1993).

Lucas (1972) develops a theory of the cycle in a conceptual classic context. The cause of the economic cycle would be the imperfect information related to the prices. The agents would believe, erroneously, that general movements in the level of prices are movements in relative prices, taking decisions to increase the production, and labor demand and generating cycles. Investments based on an incorrect prevision of prices lead, first to boom and later to bust. It deals with an exogenous and random theory. The difference between the price of the sector and the real price follows a stochastic evolution:

$$p_{jt} = p_t + \varepsilon_{jt} \quad (10)$$

Where j refers to a good, p_t is the general level of prices. The price of the good (sector) stochastically varies according to the general level of prices, since ε is a white sound. The supply of each company depends on the expectation that the evolution of prices of its products has, against the general level of prices. The bigger dispersion of prices of the sectors according to the general prices, the bigger differences will be given and more cycles. This is the institutional component which expands or reduces the exogenous shocks. Although it is difficult to think this mechanism is enough to generate studied fluctuations in real variables (Mankiw, 1989).

In the classic or Walrasian theories, the agents behave rationally, in such way that a position of equilibrium is always placed (e.g. Kydland & Prescott, 1982). Changes in nominal variables are immediately adapted: general movements in the price level don´t modify the supply and demand equilibrium (Lucas, 1977). It is an efficient Pareto situation, the economic cycles would be efficient and optimum positions of the economic system (Long & Plosser, 1983). Unemployment is never involuntary.

We get to the so called classic dichotomy (Mankiw, 1989:80). The monetary supply is exogenous, the monetary demand is determined by the output level and price level, but it doesn't operate in the Walrasian equilibrium. Real and nominal variables don't influence between them because money doesn´t impact on resource allocation, monetary variations leave to the real variables unchanged.

It is unavoidable that the Walrasian school of thought, focused on the general inter-temporal equilibrium: where the possibility disequilibrium in markets is excluded, explains the economic cycles from exogenous factors which impact in a random way. Randomness is the necessary motor of the studied cycles. Among these factors, the main one is in technological changes (technology is empirically quantified as productivity of all factors totality, within a Solow quantification of growth). Productivity would move randomly giving the studied economic cycles (Stadler, 1994:1752). Recession would be a period where productivity capacities of society have fallen. Other secondary variables exist such as movements in consumers´ tastes.

Exogenous and stochastic movements modify relative prices and force rational agents to adjust labor supply and consumption level. This is the mechanism of transmission which gives depth and cycle persistence. What it leads to changes in produced quantities. New decisions about the job offered (decisions about how much leisure must be sacrificed along the labor life) are the key. The mechanisms of transmission turn the punctual and stochastic shocks into persistent cycles.

Unemployment grows in recessions because people decide to sacrifice less leisure, because the labor productivity and real wage have gone down (relative cost of leisure). It is decided to change current work for future. Labor supply is reduced. National output falls. However, this mechanism should be weak. An increase in real wage give bigger value to work (price effect), but also the income increases (income effect). A slowdown gives less value to work, but also it generates a downfall in income, giving a positive boost to the labor supply which counters, at least partially, to replacement effect. So, even if the real wage behaved in a pro-cyclical way, its impact would be reduced.

Other mechanisms of transmission: goods in stock that the company keeps, decisions of investment which affect to the capital stock, delays in process of investments. The global weakness in mechanisms of transmissions explain the introduction of nominal shocks and stiffness of Keynesian inspiration.

Mathematically, these models vary about structure and common assumptions: neoclassic model of capital accumulation to which stochastic models of productivity are added. About rational expectations and market emptiness, families maximize a function of usefulness; companies maximize a function of benefits. Shock expand themselves and persist by the intertemporal substitution of work and leisure, and by the impact about investment and the capital shock.

The model of Long & Plosser (1983) is one of the paradigmatic contributions of RBC theory, it consists of:

- A function of Cobb-Douglas production, with neoclassic conditions.
- A technology which evolves with deterministic temporal tendency and an additional random perturbation which behaves as an auto-regressive of first order (AR(1)) process, and white noise ε_t variable is also added.
- A representative consumer who maximizes a usefulness that depends on consumption and leisure (so, on work), limited by the budget constraint which relates consumption with work.
- A closed economy is supposed.
- The rate of accumulation in capital, which depends on the national output, and the savings rate.
- Facilitating assumptions which make the savings rate and labor supply fixed.

In this model, the process AR(1) in technology generates a solution for the national product "per capita" of AR (2), or auto-regressive process of second order:

$$y_t^c = (\alpha + \delta) y_{t-1}^c - \alpha\delta y_{t-2}^c + (1 - \alpha)\varepsilon_t \quad (11)$$

Where α is the participation of capital in the national income (exponent in the function of Cobb-Douglas production), δ is the auto-regressive coefficient of the technological perturbations. The variables are expressed in cyclical values, data without tendency. The second parameter is negative, what it generates cycles.

In RBC models, the technological progress is exogenous and, also, must be. According to Euler theorem, since capital and work are rival factors which gain, in competence, the same as their marginal product. So:

$$Y = K\left(\frac{\partial F}{\partial K}\right) + L\left(\frac{\partial F}{\partial L}\right) \quad (12)$$

There are no resources to pay the technological progress, so it must be necessarily endogenous (Sala-i-Martin, 1994:42-43). The economic growth is at long-term, and the cycles are explained by non-explained changes nor explainable by the technology.

Romer (2006) presents a standard RBC model characterized by the following equations, with a function of neutral Cobb-Douglas production of Harrod.

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha} \quad (13)$$
$$Y_t = C_t + I_t + G_t \quad (14)$$

The evolution of capital comes determined by:

$$K_{t+1} = K_t + I_t - \delta K_t \quad (15)$$

Technology follows a growing temporal tendency and it is subjected to random shocks:

$$A = e^{A_0 + gt + A^*} \quad (16)$$
$$A_t^* = \rho A_{t-1} + \varepsilon_t \quad (17)$$

Where g is the coefficient of temporal growing in technology, y $-1\langle\rho\langle 1$. ε_t is a process of White noise. A_t* is the neoclassic component in technology, again AR(1). It can be reduced to a weighted sum of different random process in a potentially infinite succession.

$$A_t^* = \alpha A_{t-1}^* + \varepsilon_t = \varepsilon_t + \alpha\varepsilon_{t-1} + \alpha^2\varepsilon_{t-2} + \cdots + \alpha^{t-1}\varepsilon_0 = \sum a^i \varepsilon_{t-i} \quad (18)$$

Later, introducing in the production function, we have:

$$\frac{\delta lnY}{\delta \varepsilon_0} = (1-\alpha)\rho^{t-1} \quad (19)$$

All technological shocks have long-term effect.

The mechanism of propagation, work and leisure decisions, is understood from utility function of a representative agent which relates consumption c and work l, with the restriction of wage w and real interest rate r.

$$U = lnc + bln(1-l) \quad (20)$$

With the restriction:

$$c_1 + \frac{1}{1+r}c_2 = w_1 l_1 + \frac{1}{1+r}w_2 l_2 \quad (21)$$

Getting, in a scenery of some temporal periods, to the expression:

$$\frac{1-l_1}{1-l_2} = \frac{1}{e^{-\rho}(1-r)}\frac{w_2}{w_1} \quad (22)$$

Where ρ is the discount rate. The labor supply in every period responds to the corresponding wage, but also to the interest rate. But in the model, interest rates and relative salaries evolve in opposite directions, offsetting their effects on intertemporal substitutions between leisure and work: on the labor supply.

Finally, Walrasian school of thought hasn´t explained nor The Great Depression of 1929, nor the great recession of 2008 (Farmer, 2012). Also, some regularities in data must be explained, they don´t seem coherent with the theoretical model. Firstly, a recession is related with less consumption and more leisure. What is

difficult to understand being both goods normal (Mankiw, 1989:82). The only one explanation of this phenomenon, for the model, would be in which the real wage goes down. But there are less workers in a recession, its labor marginal productivity goes up and therefore, the real wage should also go up. That would be anti-cyclical. One of the facts to explain the theory of business endogenous cycles or with "sunspots" shocks (Schmitt-Grohe, 2000).

Secondly, neither pro-cyclical movements of prices. It is a polemic about Philips curve at short and long term. So, this school of thought tries empirically to prove that prices don´t behave in this way.

Thirdly, real cycle theory says currency fluctuations are based on the variations of money demand for transactions (King & Plosser, 1984). But it is questionable the inexistence of movements, in some cases, independent from the central bank or other factors.

To which two fundamental problems are added. On the one hand, big technological shocks aren´t detected empirically to explain big recessions. On the other hand, can the productivity or technological possibility of the economic system be allied as if it hasn´t got any relationship with it? It is true that full productivity of factors ranges in time (Prescott, 1986) and behaves in a pro-cyclical way dropping in recessions. But: Is it the factor that provokes the crisis, or is it the result because the crisis gives low return sub-employments to workers and an important part of the leisure capital? It deals with a theoretical problem, since it is impossible to make empirically a difference between cause and effect.

After all, one of the neoclassic growing models of criticism is technology is placed as a main factor of growing, but it is unexplained and unexplainable. Kondratieff and Stolper thought the technical fluctuations are not random and have the economic development as origin. The technique will evolve at the wavy long-term rhythm. Thus, they state a serious mistake would be to think the direction and intensity of findings and inventions is "accidental" (p.112). Considering technical changes are given randomly lacks of basis, being rather a product of economic necessities (p.112).

It seems, therefore, difficult to keep that technology is exogenous and not endogenous. In such way, the cycles would generate endogenously, at least, a part of technological oscillations. Endogenous technological growth models should be introduced, for example Romer´s model (1986) among many others.

If the endogeneity of technological fluctuations is subject of discussion, public spending is much more. Its endogenous and anticyclical character, according to automatic stabilizers and to the evolution of tax capacity, is clear.

At last, the investigation program, that comes from DSGE models and adds "sunspots", is there. The indetermination means the model is not closed and a number of endless possible equilibriums can be given. This indetermination had been traditionally considered as a weakness of the model, which should be avoided. However, this element is used in this program as a key to explain business cycles (and other empiric facts such as the transmissions of monetary boosts and economic growth) as Benhabib & Farmer (1999) defend. Azariadis (1981) presented a Lucas model (1972) of simple economy with succeeding generations, under extrinsic uncertainty.

Beliefs about the future will determinate the final equilibrium in these incomplete models that don't give any solution but a set of possible equilibriums. Beliefs that act randomly, prediction is impossible. Benhabib & Farmer (1999) present a simplified model which must be valid for different exposed models, along the way of the balanced growth. We have in this model:

$$y_t = Ay_{t-1} + BE_t y_{t+1} + Cx_t + u_t \quad (23)$$

$$x_t = Dx_{t-1} + v_t \quad (24)$$

Where "y" is the vector of endogenous variables, "x" is the vector of economic policy variables, "u and v" are stochastic shocks. A, B, C and D are parameters. Equations must meet the conditions to get the emptying of markets. It is observed that this model introduces two white noise processes, accompanying an expectation about future. If the rational expectation hypothesis is accepted in a uncertainty situation: the agents exactly know the model but they don´t know the exact value of parameters. Using an adaptive learning in the agents, by which they suppose the values in t+1 will be the same than in t, and introducing values of x_1 expressed as the result of random present and heavy perturbations:

$$x_t = v_t + Dv_{t-1} + d^2v_{t-2} + \cdots + d^tv_0 \quad (25)$$

$$y_t - \frac{A}{1-b}y_{t-1} = \frac{C}{1-B}(v_t + Dv_{t-1} + d^2v_{t-2} + \cdots + d^tv_0) + \frac{1}{1-B}u_t \quad (26)$$

$$y_t = \frac{C}{1-B-A}(v_t + Dv_{t-1} + d^2v_{t-2} + \cdots + d^tv_0) + \frac{C}{1-B-A}u_t \quad (27)$$

Where the values y_t and y_{t-1} are equal and therefore a temporal equilibrium happens.

Consequently, rational agents, adaptive learning, temporal equilibrium, the final solution would be an addition of random perturbations in time. We are in the same intellectual schedule of RBC models.

Duffy & Xiao (2005) study if a RBC (DSGE) model of equilibrium with rational expectations, in a reduced way, and with a dynamic of adaptive learning, can generate an undetermined and stable equilibrium. Indetermination lets non-fundamental variables "sunspots" boost the model and generate cycles (Farmer & Guo, 1994).

Under rational and adaptive expectations, the agents identify the model but not the exact value of the coefficients. The equilibrium will be stable if the estimating of the coefficients keeps a close and sustained setting of the real coefficients. It means, if the differential equation is locally stable.

$$\frac{da}{dT} = T(a) - a \quad (28)$$

Where "a" are the estimated coefficients and "T(a)" are the real coefficients.

Duffy & Xiao (2005) analyze the solutions of the three more relevant models: Farmer & Guo (1994), Schmitt-Grohe & Uribe (2000) and Wen (1998). They study the conditions under which RBC models with "sunspots" shocks and rational expectations get undetermined and stable equilibriums.

The reduced model exposed by Duffy & Xiao (2005) is:

$$k_{t+1} = a_1k_t + a_2c_t \quad (29)$$

$$c_t = a_3E_tk_{t+1} + a_4E_tc_{t+1} \quad (30)$$

Where "k" is the capital shock and "c" is the consumption one. The coefficients are "a". The impact of expectations on the consumption is determinant and therefore, on the investment and evolution of the capital shock. The expectations are formed in an autoregressive process AR(1).

$$y_t = b_1 + b_2y_{t-1} + b_3s_t + \varepsilon_t \quad (31)$$

The vector of endogenous, capital and consumption variables is "y". the vector of extrinsic expectations is "s", "ε" is white sound. The typical autoregressive progress, with accumulation of stochastic boosts, of the RBC (DSGE)models is observed. It is a model similar to the used one, to analyze the same problem, by McGough et al. (2013). Those who modify in the first equation $k_{t+1}$by $E_t\, k_{\,t+1} = a_l\, k_{\,t} + a_2\, c_t$, introduce "e" in the second equation to the capital shock in time "t".

$$c_t + a_5 k_t = a_3 E_t k_{t+1} + a_4 E_t c_{t+1} \tag{32}$$

Duffy & Xiao (2005) reach the following conclusion. Under adaptive learning, in RBC models with "sunspots" shocks, restrictions in parameters avoid undetermined and stable equilibriums simultaneously. Agents don´t get the expectation and parameters to bring closer to their real values. The coefficient a_4 must be negative (nothing credible: the consumer wants to maintain a steady path of consumption) to get stability, but it must be positive to get indetermination. Economy is either stable and determined or unstable and undetermined. Or the idea of rational expectations would not hold. McGough *et al.* (2013) maintain the condition that a_4 must be negative. And Benhabib & Farmer (1994) condition: the demand curve has positive slope and higher in absolute value than the labor supply curve. They are empiric and theoretically is hardly plausible. In conclusion, however, it is limited to some existing models so far.

8. Conclusions

The classic theory has produced the real RBC business cycle theory and the stochastic models of general dynamic DSGE equilibrium. These models state the business cycles are the result of random and exogenous variables. These factors are used because their conceptual fundament is the theory of Walrasian equilibrium, throughout neoclassic models of economic growth. Rational agents will always take decisions that empty markets, in positions of Pareto boost. Hence they use a realistic interpretation of Slutsky-Yale effect. On the one hand, the economic growth is based on random exogenous factor unexplained and unexplainable. On the other hand, a real random movement behind the fluctuating image of the empiric cycle is equally presented. The theory of the cycle is unable to predict and with great difficulties to explain big recessions and some macroeconomic correlations.

In this scheme, the "sunspots" shocks involve the addition of stochastic extrinsic variables, but don´t modify the conceptual base, nor the mathematical models. With the problem of making compatible the indetermination and the stability.

Keynesian theory explains the business cycle as positions of disequilibrium, in which markets don´t empty, nor get a situation of Pareto boost. Keynesian classic models establish endogenous and deterministic focuses, producing regular cycles. Its main problem lays on they aren´t apparently compatible with the studied business cycles, with an irregular deepness, time and recurrence. Thanks to inapplicability of the "ceteris paribus" term is possible to explain, but then the target is to incorporate unknown factors gradually. As Russell (1914: 230) points when the constant relationship among groups of facts fails at a certain moment, "it is usually possible to discover a new, more constant relation by enlarging the group".

The later Keynesian theory has kept the conceptual heart which states the markets cannot empty, and the involuntary unemployment is possible at long-term; but it was made removable models. On the one hand, microeconomic fundaments

have been looked for the temporal stiffness of prices and the uncertainty of the noticed risk. Leaving the beginning of cycles to an initial change in the effect demand, exogenous, and a determined change would be supposed.

The adopted operationalist approach demands the existence of a measure procedure, to have an empiric content, which establishes the observable reference with accuracy. Approach which requires from the random concept, and therefore, from the determinism as univocal relationship with the system values over time. But Chaitin theorems show that the random concept doesn´t give that measure procedure: we cannot know certainly if a set of data accomplish or not, so it is necessary to conclude that it is strictly a theoretical concept. Equally, the determinism is so. The statements that use random process, or deny them to assert deterministic strings lack of empiric reference: they cannot, and will never do, be verified nor falsified with accuracy.

As statements and equations used in concepts or variables related with the random concept, positive or negative, don´t have empiric reference, nor own definition value or tautology (because information is added); only heuristic value can be owned. They would be research proposals. We can suppose the studied data are produced by a deterministic model. And we can also suppose, by contrary, that they are produced by a set of random variables. The theory determines these assumptions. It is empirically impossible to decide between both possibilities. So, they are not approved statements, but research proposals.

Random and determinism would be, therefore, two contrary programs which help to build models, evaluable theoretically and contrasting quantitatively. More or less plausible. Even with clear reprehensible results, in some cases, from the coherence with the general theory, and from sets of data. But the selection and possible rejection of models do not imply the final and definitive answer to the problem if the cyclical phenomena behave in one way or another. It is an untouchable limit of the human knowledge.

The historic discussion between models of the stochastic business cycles and deterministic models do not get to a conclusion. As in any other scientific field which uses these concepts as research projects.

Finally, if we ask science to predict, or at least we consider it useful if it does, with the final goal to avoid mistakes in economic policies, then, deterministic theories give a step forward to the usefulness of science. The models of the cycle and the growth of the classic theory cannot predict. Keynesian models with endogenous factors and deterministic evolution make up a more ambitious proposal.

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