Journal of
Economics and Political Economy
www.kspjournals.org
Volume 5 June 2018 Issue 2

Substitution elasticities in an energy-augmented CES production function: An empirical analysis for Turkey

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Abstract. This study estimates a production function for Turkey taking capital, labor and energy as input factors. The production function estimated is of the CES form with Hicks-neutral technology and constant returns to scale. A nonlinear least squares regression is employed on a dataset for the entire Turkish economy covering a time period of 27 years. The production function parameters provide insights into the elasticity of substitution of capital, labor and energy in Turkey. In particular, it is found that the elasticity of substitution between the capital-labor bundle and energy is $\sigma = 0.645$, slightly higher than values found in other studies for various countries. This finding shows the relative ease of substitutability of capital-labor with energy for one another in Turkey and provides new insight on a critical parameter for future energy-economy modeling studies related to Turkey and other similar countries with no elasticity estimate. It is thought that the high substitutability for the case of Turkey may be related to the flexibility of its rapidly growing economy with investment needs that can easily be adapted to market conditions.
Keywords. Substitution elasticities, CES, Energy economics.
JEL. D22, E23, Q40.

1. Introduction

The role of energy in the production process is highly important for countries like Turkey with a high budget deficit arising among others from energy imports which account for about 20% of total imports. Turkey is highly dependent on oil and gas imports as domestic fossil fuel reserves are negligibly small; domestic production has been covering less than 26% of total energy supply. With a highly import dependent situation in terms of energy trade dynamics, the assessment of the role of energy in the production process becomes essential for policy analysis and development of credible projections.

While neo-classical capital–labor aggregate production functions do not take energy as an input factor, due to the view of energy as an intermediate product, energy crises throughout history have emphasized the role of energy in economic growth. Therefore, nowadays, besides labor and capital, energy constitutes an

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important input factor in the production process. The key measure in the production process revealing information about the relationship between energy and non-energy inputs is the elasticity of substitution, which is an essential parameter for economic and policy analysis. Moreover, elasticity of substitution shows to what degree two inputs can be substitutes for one another (Brockway *et al.,* 2017). As it is identified by Koesler & Schymura (2015), any policy-oriented numerical model must pay attention to the elasticities, because they are the key parameters determining comparative static behavior. These parameters can be calculated from the production function under focus.

Moreover, while inputs and outputs are similar for some countries, technological development levels, capital value shares and energy efficiencies can cause wide variations between some others. For example, while the adjusted labor share for selected G20 countries is at approximately 0.55, as it will be further elaborated on in this study, this value is different for Turkey (ILO, & OECD, 2015). This fact together with other country-specific differences create a need for the estimation of customized substitution elasticities obtained based on country specific data.

At this junction, this paper estimates the customized substitution elasticities for Turkey using a production function in the constant elasticity of substitution (CES) form with Hicks-neutral technology and constant returns to scale using data from 1988 to 2014. The choice of the production function being of the CES form has been made because CES functions are a more generalized type of production function and do not come with restrictive assumptions like the Cobb-Douglas and Leontief functions (Besanko & Braeutigam, 2005). While the inclusion of energy into the production function can be done in different ways, this paper includes energy into the production function through widely used (KL)E nesting structure due to its research question targeting the substitutability between energy and non-energy goods. The estimations are performed using aggregated data for the entire Turkish economy. Indeed, this study contributes to the literature by presenting a production function for the entire Turkish economy with capital, labor and energy as inputs. There is only very limited literature on the estimation of a production function with capital, labor and energy as input factors based on Turkish data. While there are some studies on Turkey performing a computable general equilibrium analysis or a production modelling on Turkey, neither of them includes an estimation of the substitution elasticities for the particular case. An example can be given as the study by Kumbaroglu *et al.,* (2008) where the substitution elasticities are chosen at particular values based on expert-guess, but evidence shows, as will be presented later on, that these elasticities can vary widely among countries and hence can require country specific estimations. This study aims to fill this gap.

The paper is structured as follows. After this introduction, Section 2 presents the existing literature on this field to better situate the importance of the analysis conducted for Turkey. Section 3 then explains the methodology adopted throughout this study and Section 4 introduces the data used for the case of Turkey. The empirical results are presented in Section 5, followed by a discussion and conclusion in Section 6.

2. Literature review

The study of production functions with three inputs dates back to the 1950s with Robert Merton Solow (Solow, 1956). The first applications of inserting energy as input factor into the production function follows within the following decade. A comprehensive study on the incorporation of energy into the production function was undertaken by Berndt & Wood (1975) who were the first to undertake an empirical study on estimating the elasticities of substitution between energy and non-energy inputs. The motivation in their study was to put together a research, which would give an understanding of consequences of higher priced energy inputs.

Manne, Mendelsohn & Richels (1995) contribute to the literature through their study, where they take capital (K), labor (L) and energy as input, yet additionally also separate electric (E) from non-electric (N) energy. They apply a (KL)(EN) nesting structure, where the elasticity of substitution between the two input factor bundles is taken to be constant (Manne, Mendelsohn, & Richels, 1995). More specifically, the elasticity of substitution between the (KL) and (EN) composites is taken as 0.4 on the basis of a "back casting" experiment for the USA, and this reference value is then maintained throughout for the USA and OECD countries. Gerlagh & Van der Zwan (2003), on the other hand, takes the same nesting structure but chooses to separate energy based on its type into fossil (F) and non-fossil (N) fuels and uses a production function of the (KL)(FN) nesting structure.

A country-specific study for the elasticity of substitution parameters in the production function is undertaken by Kemfert & Welsch (2000) for Germany. To estimate the substitution elasticities in the German industry, they develop two approaches, one with aggregate time series data for the entire German industry and one with disaggregated time series data for the chemical, stone and earth, non-ferrous metal, vehicles, food, and paper industries. They start with three different nesting structures (KE)L, (KL)E and (EL)K, and conclude that while for some sectors the (KL)E nest is more appropriate, for the entire German industry the (KE)L nest is the most useful nesting structure in contrast to the widely spread view (Kemfert & Welsch, 2000).

The approach regarding the estimation method has evolved over the time as well. Kmenta (1967) uses the Taylor expansion formula for the estimation of the production function. He obtains an approximation formula through taking the logarithm of the CES function and applying a first-order Taylor series expansion to the logarithmized CES function accordingly. This approach has a generalized solution method for the CES function under different circumstances. It transforms the non-linear functional form of the CES function to a linear form and makes the use of simple least squares estimation possible. Following Kmenta (1967), Van der Werf (2008) tries to discover through a thorough study the optimal nesting structure given the three input factors capital, labor and energy. Unlike Kmenta (1967), his study uses a cost function-based approach. Van der Werf finds out that based on industry level data on 12 OECD countries, the nesting structure where capital and labor are combined first, fits the data best, but at the same time, the nest where all three inputs are combined simultaneously cannot be rejected for most countries and industries.

The research mentioned so far on the estimation of production functions with more than two inputs have one thing in common: They use comprehensive price data, which is in some cases difficult to obtain. While obtaining data on sector prices can be a problem in the case of sector specific analysis, in the case of macro analyses, the aggregation of factors creates a need for a price index to be calculated, bringing with it the problem of choosing the most appropriate method out of a wide sea of indexation ways. Henningsen & Henningsen (2011) as well as Koesler & Schymura (2000) try to get around this problem by developing a non-linear least squares estimation method. Neither one of these two studies require extensive price data to be at hand. The method developed by Henningsen & Henningsen makes the estimation through the R package called micEconCES which they developed themselves. This R package contains various estimation methods including Kmenta's Taylor series expansion for an appropriate type of function and other methods such as the Levenberg-Marquardt for non-linear estimation cases (Henningsen & Henningsen, 2011). An important feature to this research is that it re-estimates the Kemfert and Welsch study using the Kmenta (1967) and various other approximations with the exact same data provided in the annex of Kemfert & Welsch (2000). The re-estimation results are in large deviations from the original results no matter which estimation and optimization method is used, and no reasonable explanation could be found for this divergence. In this context, Henningsen & Henningsen (2011) conclude that linear approaches using the Kmenta approximation are not proper approaches for CES function estimation.

Following Henningsen & Henningsen (2011), Koesler & Schymura (2015) contribute to the literature by applying the methods developed by Henningsen and Henningsen to the data retrieved from the World Input-Output Database (WIOD) with the goal of obtaining elasticities for the (((KL)E)M) nesting structure. Here, E stands for energy and M represents intermediate inputs, which can be used by researchers during their studies of various topics (Koesler & Schymura, 2015). Their data set covers 40 countries and 35 industries with detailed information on primary, secondary as well as tertiary sectors. Their analysis reveals that Cobb-Douglas and Leontief production functions should be rejected for the majority of sectors, just as Van der Werf (2008) found out, and provides a detailed set of substitution elasticities covering a wide sectoral breakdown.

All in all, even though there are some, such as Koesler & Schymura (2015) who claim that there are no substantial variations in substitution elasticities between regions, country specific research on various countries reveal that significant results for production functions based on country specific data can in some cases only be obtained through certain nesting ways. Su, *et al.,* (2012) start by estimating all three nesting forms of a capital, labor and energy composed CES function with the extension to the existing literature in the form of a relatively larger dataset. Since they focus on China, they approach their estimation with two subdivided periods, before and after China's reform, more specifically, from 1953 to 1978 and then from 1979 to 2006. Su *et al.,* (2012) use the estimation method applied by Mishra (2006), which shows that for the loss function minimization the Differential Evolution (DE) and Repulsive Particle Swarm (RPS) methods outperform the other methods. As a result, Su *et al.,* (2012) indicate that while all nesting structures are insignificant the only economically meaningful result can be obtained for the (KE)L nesting structure, where E represents energy. Shen & Whalley (2013) contribute to the literature through their working paper at the National Bureau of Economic Research (NBER) by taking the research by Su *et al.,* (2012) and extending it to the extent that they use normalized CES production functions and perform grid-search based optimization methods. Their results therefore turn out to have lower standard errors with statistically significant results for the (EL)K nesting structure.

A study on Turkey has only recently been undertaken by Andic (2016) where the estimation of a normalized CES production function for Turkey is set as goal. Andic takes just capital and labor as inputs and does not include energy, at which point it diverges from the so far mentioned references as well as the research goal of this study. She employs a system approach and determines the elasticity of substitution and the total factor productivity. Besides this recent research on Turkey, there is no literature on the estimation of a production function with capital, labor and energy as input factors based on Turkish data. As previously mentioned, this study aims to fill this gap.

3. Methodology

Production functions in general are categorized according to three criteria: technology, elasticity of substitution and returns to scale (Besanko & Braeutigam, 2005). Technology can be incorporated into a production function in three different way: The Hicks-neutral technology, Harrod-neutral technology and Solow-neutral technology, which can also be referred to as factor augmenting, labor augmenting and capital augmenting respectively. Functions can have either constant or variable elasticity of substitution. And returns to scale can be decreasing, constant or increasing. The CES function can be regarded as a generalization for a production and does not make certain assumptions regarding the nature of the function, such as Cobb-Douglass and Leontief functions, which turn out to be not very appropriate production functions for many sectors as the study by Koesler and Schymura demonstrates (Koesler & Schymura, 2015).

The production function estimated in this study is of the CES form with Hicks-neutral technology and constant returns to scale and is denoted as follows,

$$Y = A\left[\alpha(K^{KPVS}L^{1-KPVS})^{\frac{\sigma-1}{\sigma}} + (1-\alpha)E^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

The dependent variable Y denotes output, whereas the independent variables K, L and E represent respectively capital, labor and energy. The parameters A, KPVS and σ stand for total factor productivity, capital value share and elasticity of (technical) substitution between the capital-labor bundle and energy inputs respectively.

KPVS is calculated following the paper by Atiyas & Bakis (2013). While Atiyas and Bakis concentrate on the labor share (LS) and apply their notation accordingly, our focus is on the capital value share. Hence, we apply the notation $LS = 1 - KPVS$ to their approach. The numerical value for KPVS is obtained using the below formula,

$$1 - KPVS = \frac{W}{(Y-T)}\frac{1}{(1-z)}$$

W denotes the compensation of employees, Y denotes GDP, T stands for net indirect taxes and z is an adjustment factor representing the share of self-employment in the labor. The adjustment created by multiplying with $\frac{1}{1-z}$ brings the assumption that the wage earned by self-employed people is equal to the wage earned by employees.

The method to solve the problem of estimating the production function at focus is the nonlinear least squares (NLS) regression. The non-linear solution to the estimation problem of the parameters, will come through the minimization of the following sum of squares, $S(A, \alpha, \sigma)$.

$$S(A, \alpha, \sigma) = \frac{1}{2}\sum_{t=1988}^{2014} e_t^2$$

$$S(A, \alpha, \sigma) = \frac{1}{2}\sum_{t=1988}^{2014}\left[Y_t - A\left[\alpha(K_t^{KPVS}L_t^{1-KPVS})^{\frac{\sigma-1}{\sigma}} + (1-\alpha)E_t^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}\right]$$

Minimizing $S(A, \alpha, \sigma)$ means choosing the parameters A, α and σ in such a way that the sum of squares of the error terms, e_t, i.e. difference between the Y_t and the value for $A * \left[\alpha(K_t^{KPVS}L_t^{1-KPVS})^{\frac{\sigma-1}{\sigma}} + (1-\alpha)E_t^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$ calculated with the iterated values for the parameters plugged in, will be minimized. For the case of this study, this maximization will be through the derivatives of $S(A, \alpha, \sigma)$ with respect to the parameters to be estimated, which are as mentioned before A, α and σ. The equations obtained from these derivatives do not have explicit solutions, leading us to a nonlinear least squares regression. At this point, it is important to point out that the solution to the above minimization can only be found, if at all, given that the number of observations, *t*, is greater than the number of parameters, *n*, to be estimated. In the case of this study, we have $t = 27 > n = 3$ and can conclude that this condition is satisfied.

For situations where the solution to the first order derivatives cannot be calculated analytically, numerical methods must be applied. These numerical methods consist of iterative algorithms which require starting values for the parameters to be estimated. The iterative process takes the starting values and tries to reach an optimum through certain rules for repeatedly making the same calculations with the next available values for the parameters. These rules are defined as optimization methods (Kuan, 2004).

Numerous optimization algorithms exist in the theory. Henningsen & Henningsen (2011) use several optimization algorithms for their nonlinear least squares estimation, besides also applying the Kmenta approximation. They make use of the Levenberg-Marquart algorithm, which is the most commonly used optimization algorithm and is also set as default algorithm in numerous statistical softwares (Henningsen & Henningsen, 2011). Additionally, they also use the Conjugate Gradients method (Nocedal & Wright, 2006), Newton method (Schnabel, Koontz, & Weiss, 1985), Broyden-Fletcher-Goldfarb-Shanno algorithm (Broyden, 1970, Fletcher, 1970, Goldfarb, 1970, Shanno, 1970), Nelder-Mead algorithm (Nelder & Mead 1965), Simulated Annealing algorithm (Belisle, 1992), Differential Evolution algorithm (Mullen, *et al.,* 2011) and numerous other algorithms, which additionally impose a parameter constraint (Henningsen & Henningsen, 2011). Koesler & Schymura (2015) on the other hand go with the make their estimations based on the commonly used Levenberg-Marquart algorithm.

In this study, the NLS method was used, which applies the Gauss-Newton optimization method with the Marquart step method. The Gauss-Newton method is based on a linear Taylor series approximation to the nonlinear regression function, which is in our case the production function under focus. The iterative estimator is calculated through the transformation of the optimization to a series of linear least squares regressions (Greene, 2011). If we rewrite our production function as below,

$$Y = A\left[\alpha\left(K^{KPVS}L^{1-KPVS}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)E^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$
$$y = h(x,\beta) + e$$

then the Gauss-Newton method will make a linear estimation to $h(x,\beta)$ at a particular value for the parameter vector β^0. As it is described by Greene (2011), the estimation will look as mentioned below.

$$h(x,\beta) \approx h(x,\beta^0) + \sum_{t=1988}^{2014} \frac{dh(x,\beta^0)}{d\beta_t^0}(\beta_t - \beta_t^0)$$
$$h(x,\beta) \approx \left[h(x,\beta^0) - \sum_{t=1988}^{2014} \beta_t^0 \frac{dh(x,\beta^0)}{d\beta_t^0}\right] + \sum_{t=1988}^{2014} \beta_t \frac{dh(x,\beta^0)}{d\beta_t^0}$$

Setting the notation to be so that $x_t^0 = \frac{dh(x,\beta^0)}{d\beta_t^0}$ we will have for a given value of β^0, x_t^0 to be a function of data only. Then the above estimation equation can be rewritten as follows.

$$h(x,\beta) \approx \left[h(x,\beta^0) - \sum_{t=1988}^{2014} x_t^0\beta_t^0\right] + \sum_{t=1988}^{2014} x_t^0\beta_t$$
$$h(x,\beta) \approx h(x,\beta^0) - x^{0'}\beta^0 + x^{0'}\beta$$

This implies,

$$y \approx h^0 - x^{0'}\beta^0 + x^{0'}\beta + e$$

By rearranging this equation, we can obtain a linear equation.

$$y^0 = y - h^0 - x^{0'}\beta^0 = x^{0'}\beta + e^0$$

Where

$$e^0 = e + \left[h(x,\beta) - \left\{h^0 - \sum_{t=1988}^{2014} x_t^0 \beta_t^0 + \sum_{t=1988}^{2014} x_t^0 \beta_t\right\}\right]$$

Since in the equation of y^0 all errors are included and accounted for, this equation can be written as an equality instead of an estimation. This value is then estimated through linear least squares. The Marquardt algorithm has been utilized under EViews, which serves as a modifier to the Gauss-Newton method, by adding a correction matrix to the Hessian of the production function. Thereby, the obtained parameter estimations are brought closer towards the gradient vector improving the result.

4. Data

This section introduces the data used in this paper together with the calculation of some intermediary parameters. The first part presents the approach applied and data used for the capital value share calculation. The second part concentrates solely on the data used for the main nonlinear regression estimation for the production function. All data used were either directly in terms of real values or were converted to their real equivalents with reference base year 2011. Both parts include detailed definitions of the data used together with its sources.

4.1. Data for KPVS calculation

The data, which is used for the calculation of the KPVS, is obtained from the Turkish Statistical Institute (TSI), the Turkish Ministry of Finance (MOF) database and the OECD Stats. As recommended by Atiyas & Baris (2013), real GDP values obtained from the income approach are used. The value for the adjustment factor, z, is obtained from OECD Stats. The data obtained from TSI and MOF was in nominal terms. Therefore, the output, compensation of employees and net indirect taxes were turned into their real values with base year 2011 through the necessary adjustments with the consumer price index for Turkey retrieved from OECD Stats. Key measures and sources of the data used in the KPVS calculation are summarized in Table 1.

Table 1. *Data Definitions and Sources for KPVS Calculation*

Abbreviation	Variable	Definition	Data Source
Y	Gross domestic product	Real gross domestic product at constant prices (base year 2011)	TSI, OECD
W	Compensation of employees	Real total compensation of employees at constant prices (base year 2011)	TSI, OECD
T	Net indirect taxes	Real taxes - subsidies on production and imports (base year 2011)	TSI, MOF, OECD
Z	Share of self-employment	Employment of employers, workers who work for themselves, members of producers' co-operatives, and unpaid family workers	OECD

This calculation was undertaken for data on the years 2009 to 2015 and yearly values for KPVS for this period were obtained as a result. Since the base year of this study is 2012 the value of KPVS for that particular year, which equals $1 - KPVS = 0.50$, is used throughout this study.

4.2. Data and descriptive statistics for production function

The estimation of the production function requires data on the variables output (Y), capital stock (K), labor (L) and energy (E). For output, real GDP data taken from the Penn World Tables is used. While for the KPVS calculation GDP calculated through income method is taken, for the production function GPD calculated through expenditure method is used. This is partially due to the fact that investment series, which are used for the capital stock calculation, are obtained from the GDP calculated through expenditure method. Data on employment for the labor (L) input factor was taken from TSI and covers all working women and men above the age of fifteen. Data on energy has been retrieved from OECD Stats as primary energy supply in tonnes of oil equivalent. This data is prepared and published by the IEA on a yearly basis. The Turkish Ministry of Energy and Natural Resources also regularly publishes this data, but for the sole purpose of consistency of data sources, the data from OECD Stats is used.

While there are numerous studies which incorporate energy data in terms of energy units, there are also just as many studies which use energy data in terms of energy cost. The EMF (1977) outlines clearly in the report "Energy and the economy" how energy can be taken as energy cost and present various aggregation and indexation methods for prices. Among these methods some studies, including studies with limited access to energy price data, use the method of taking energy supply in terms of tonnes of oil equivalent and multiplying with the real crude oil import prices. One such study is the paper by Edwin Van der Werf (2008). With reference to Edwin Van der Werf's research, the same approach was adopted, and energy data has been taken as energy cost incurred to the Turkish economy (Van der Werf, 2008). Therefore, primary energy supply in tonnes of oil equivalent is multiplied with Turkey's real crude oil import prices in US$ per barrel of oil, obtained from OECD Stats, with base year 2011 just as capital stock and real GDP data used in the production function estimation. The barrel prices are converted to tonnes prices using the OPEC conversion table taken from the annual statistical bulletin (OPEC, 2017). At this point it is worth mentioning that a weighted approach for the calculation of energy cost according to energy source was evaluated and acknowledged as well. A weighted cost could be calculated based on the source breakdown of energy supply. Yet this would require data on prices for Turkey for each one of these sources which were not obtainable for the earlier periods analyzed in this paper. Moreover, detailed price data for each particular energy source for Turkey is only available for the more recent years and some particular years for the earlier periods. Therefore, the adaptation of this method would have restricted the number of observation years for the data.

The Turkish government does not publish data on capital stock. Therefore, this data series had to be obtained from other sources. There are already existing studies on capital stock in Turkey by Bulutay *et al.,* (1974) and more recently by Saygili *et al.,* (2005). These studies cover respectively the periods 1923 - 1948 and 1972 - 2005. These data series were not used, as they do not cover the most recent period and their extrapolations through mathematical methods would not be robust given data availability problems. While there are several methods for the calculation of capital stock, one of these methods is the perpetual inventory methods. Starting from 1988 onwards, the capital stock is calculated through the perpetual inventory method, following the equations below. According to this method capital depreciates over one period at the depreciation rate of δ.

$$K_{t+1} = (1-\delta)K_t$$

This creates the need for the definition of the initial capital stock value for the time period considered, i.e. for the year 1988. To identify K_0 the following method is applied where it is assumed that the economy is close to a steady state (Atiyas & Bakis, 2013).

$$\frac{K_{t+1}}{K_t} - 1 = g_t = -\delta + \frac{I_t}{K_t}$$

If we assume that we are at a steady state at time $t = 0$, i.e. in the year 1988, then we can find K_0 from $K_0 = \frac{I_0}{\bar{g}+\delta}$ where $\bar{g}$ can be taken as the average GDP growth rate for ten years starting from t_0 onwards. Based on educated opinions and some other models such as the MARKAL model, the depreciation rate has been taken as 5% (Manne & Wene, 1992). Yet, the assumption of the economy being at a steady state in 1988 is a controversial topic, where no certain objective decision can be made for the case of Turkey as a developing economy. Therefore, throughout this study the data for capital stock is taken from the Penn World Tables, which is calculated through the economic definition of capital accumulation through the following formula, where I_t denotes the amount of investment in that particular year.

$$K_{t+1} = (1-\delta)K_t + I_t$$

Given this formula, capital stock series can be constructed without the assumption of a certain economic growth rate. For example, for a depreciation rate assumption of $\delta = 5\%$, a lifetime of capital means 20 years. Hence for calculating K_{t+20} there will only be need for the investment data of the past 20 years and no need for K_t. The capital stock data published by the Penn World Table database applies exactly this method and therefore can be appropriately used for this study on Turkey. To provide an overview of the data and its sources used in this study, a summary is provided in the below Table 2.

Table 2. *Data Definitions and Sources for the Production Function*

Abbreviation	Variable	Definition	Data Source
Y	Gross domestic product	Real GDP at constant national prices (in million 2011 US$)	Penn World Table
K	Capital stock	Capital stock at constant national prices (in million 2011 US$)	Penn World Table
L	Labor	Employed women and men above the age of fifteen	TSI
E	Energy	Primary energy supply (toe) Crude oil import prices (US$ with base year 2011 per barrel of oil)	OECD

A fundamental step in economic analysis is the analysis of the data itself. In this regard, a summary on the descriptive statistics is presented. Table 3 presents the key factors of descriptive statistics for the variables used in the production function.

Table 3. *Descriptive Statistics for Variables used in Production Function*

	GDP (US$; real 2011)	Capital Stock (US$; real 2011)	Labor (number of employees)	Energy (US$; real 2011)
Mean	905,780,784,722	2,101,556,937,500	21,129,407	33,876,671,837
Median	828,538,250,000	1,946,603,500,000	21,194,000	16,035,357,373
Maximum	1,442,669,875,000	3,707,828,500,000	25,932,000	94,831,506,872
Minimum	527,702,937,500	967,656,937,500	17,754,000	8,701,621,343
Std. Dev.	278,244,583,207	803,749,412,438	2,054,661	29,236,221,969
Skewness	0.433674	0.428693	0.686242	0.990473
Kurtosis	1.940591	2.058360	3.110704	2.429338

The relationship between the independent and dependent variables becomes more evident once the data series are plotted. In Figure 1 the relationships are visualized. The two monetary independent variables show an increasing trend during the time period considered. Note that while energy cost values are presented in the secondary, capital stock and GDP is presented with reference to the primary axis. Among the independent variables while capital stock shows a steady increasing trend, energy costs displays higher fluctuations. The labor series, which is not presented in this graph since it is relatively low compared to the values of the other variables, displays a steady smooth growth over the time period considered. When energy cost is compared to energy unit data we see that energy itself does not show this kind of fluctuations and instead shows a steadily increasing trend. This points towards the importance in the utilization of energy cost given the objective of an economic analysis. Energy itself would fail to capture and present these dynamics which have their reflections on the dependent variable output.

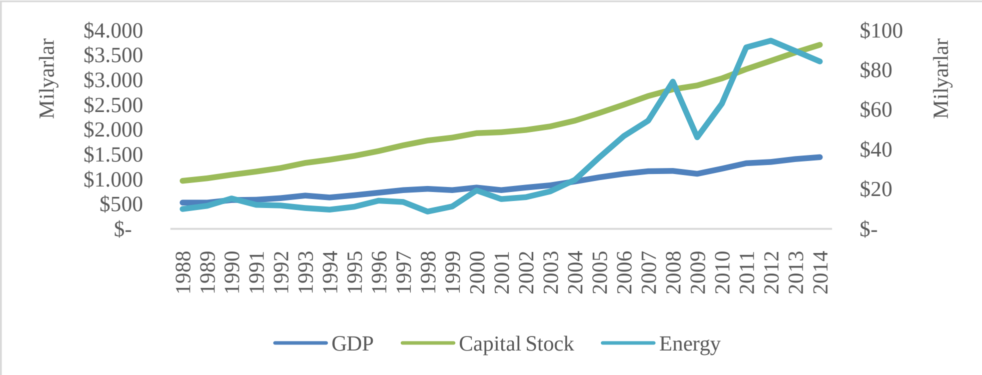

Figure 1. *Monetary data used in the production function*

With this regard, in order to completely visualize the dynamics in the variables during the time period considered the scaled data, with the scaling method being the division by the minimum value observed throughout the period considered, has been plotted. When this is carried out, it becomes evident that the changes in the dynamics of the energy cost are too extreme to be neglected.

The visualization of the data series reveals clearly that there are trends in the data. In order to test this hypothesis unit root tests have been conducted on each data series and their correlograms have been analyzed. These analyses have been performed on the raw data, the logarithmized data and the scaled data. As a result, these tests revealed that for each type of data the data series has a unit root. The t-statistic values from the Augmented Dickey-Fuller tests are summarized below in Table 4. For the raw data, the logarithmized data and the scaled data all variables had a unit root in their level data, no matter which confidence interval, 1%, 5% or 10%, was looked at. These unit roots did not persist if the test were performed on the first level differences of these variables with the sole exception to capital stock. For the capital stock series only the second level difference did not have a unit root.

Table 4. *Augmented Dickey-Fuller Test Statistic Values*

	Raw Data		Logarithmized Data		Scaled Data	
Variable	Level	Difference	Level	Difference	Level	Difference
GDP	0.941713	-4.901975	-0.237377	-5.875371	0.944847	-4.891251
Capital Stock	1.691296	-1.817959	-0.991070	-2.667983	1.696743	-1.833093
Labor	0.082672	-4.026030	-0.343214	-4.324500	0.086345	-4.018494
Energy	-0.188680	-5.243892	-0.351973	-4.791576	-0.188775	-5.244297

The stationarity of the data series could have been obtained through taking the respective number of differences or detrending the data. But both of these options were tried and had their drawbacks. Taking the first level or second level differences as well as detrending the data creates a data series with negative values. Leaving the meaning of the estimation results aside, solely from a technical perspective this was not possible since in this case negative values were tried to be raised to non-integer powers, which is not possible in the real mathematical environment. From the interpretation side, even if taking the differences or detrending the data would have given a logical value for the parameter estimates, from the economical perspective the obtained estimates would not have represented the definitions which were tried to be obtained. This idea is supported by the existing literature on elasticity of substitution estimations. To present some evidence, it can be mentioned that neither of the estimations conducted by Su *et al.,* (2012), Kemfert & Welsch (2000) or Van der Werf (2008) mention any detrending or differentiating performed on the data, even though they present extensively the data they have used. Therefore, the same approach was followed, and the raw data was used throughout the estimations.

The estimations are performed on two datasets: scaled and not-scaled data. While several normalization methods for data exist, the method adopted in this study, after a long period of search for the optimal scaling method, is the division by the minimum method. In this method each data series, capital, labor, and energy, is divided respectively by its minimum value for time period considered. This method is used in order to avoid divergences caused by taking the power of values less than 1. Several other scaling methods are tried out as well, such as using the logarithmical values, yet neither method has given any significant estimation results or not as significant results as the division by minimum method. Even though the estimations have been performed for the data as it is, and for the data scaled according to the division by its minimum values, which have revealed same estimation outputs, for the sake of brevity and due to higher significance, only scaled data estimation results will be presented in Section 5.

5. Empirical findings

The estimations in this study are made with the use of the statistical software EViews, using its built in nonlinear least square estimation tool. Due to the nature of nonlinear least squares estimation, certain input values for the parameters are required. The starting value for the parameter σ is chosen according to existing literature on a similar estimation and hence has not been changed, while the starting values for A is chosen according to the scaling method. Results are presented for a certain starting value of α but grid search has been performed on this parameter, which are not presented in order to avoid redundancies.

Scaling is applied to the raw data in order to achieve a normalization of the data. For this purpose, each data entry of each of the four variables - capital, labor, energy and output - is divided by the minimum value of that particular series over the time period considered (1988-2014). Not surprisingly, the minimum values are the values for the starting year 1988. The estimated coefficients are A, α and σ. The starting values for the parameters were set as A_0=1, α_0=0.4 and σ_0=0.3. The σ_0 was chosen to be 0.3 based on the study by Kumbaroglu *et al.,* (2008), while the other values are set as they are based on expert guess. Even though a grid search for numerous other starting parameter values is performed, the results lead to the same output. It is important to point out that even though no restrictions are manually imposed on the parameters, the estimated values are within the meaningful intervals supporting the significance of the estimation results. The estimation results are presented in Table 5.

Table 5. *Estimation Output with Scaled Data*

	Coefficient	Std. Error	Prob.
A	0.992745	0.013665	0.0000
α	0.842243	0.049231	0.0000
σ	0.645513	0.225444	0.0086

The results show that convergence was achieved after 18 iterations and each one of the estimated values for the parameters is statistically significant. No matter whether 10%, 5% or even 1% confidence interval selection, the coefficient estimates are significant as it is observable from their probabilities. The predictive power of the model is strong with a 98% adjusted R^2 value. Moreover, we can predict 98% of changes in the dependent variable output, i.e. GDP, with our production function as we have defined it. The Durbin-Watson statistics indicates that there might be serial correlation in the residuals. Comparing the Akaike Schwarz criteria of the scaled and not-scaled models, the results indicate that the scaled output results are more favorable and should be preferred to the not-scaled case. Indifferent of the choice of a 10%, 5% or 1% confidence interval, the parameter estimates remain certainly within the confidence intervals. The confidence intervals are presented in Table 6.

Table 6. *Confidence Intervals for Estimation Output with Scaled Data*

		90%CI		95%CI		99%CI	
Variable	Coefficient	Low	High	Low	High	Low	High
A	0.992745	0.969365	1.016124	0.964541	1.020945	0.954524	1.030965
α	0.842243	0.758015	0.926472	0.740635	0.943852	0.704547	0.979940
σ	0.645513	0.259805	1.031221	0.180220	1.110806	0.014960	1.276066

The actual and fitted values graph, presented below in Figure 2, shows that the fitted values are in line with the actual values throughout the entire time period considered and no significant deviation from the actual values is observable for any particular year. Particularly, over the more recent years of the considered period the fitted results are closer to the actual results than in previous periods. This may be due to the fact that the study took for several parameters and data real value 2011 as the base year.

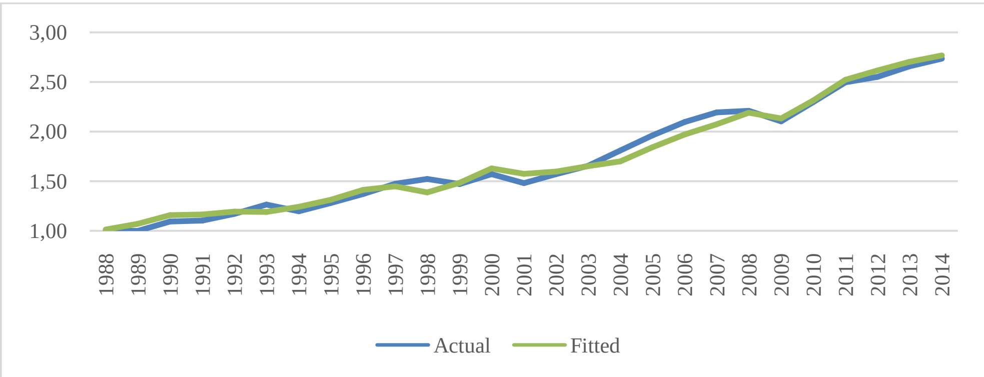

Figure 2. *Actual and fitted values graph for estimation output with scaled data*

Figure 3 presents the graph of the residuals. The residuals oscillate around the zero line. The mean and median of the residuals is in close proximity to zero, providing supporting evidence for the robustness of this model. Even though normalization tests show that a slightly skewedness to the right is observable, when plotted the distribution still appears normally distributed. The Jarque-Bera statistic supports this argument, when we compare its value to the chi-square critical value for our degrees of freedom. For each of significance levels 10%, 5% and 1%, the test statistic indicates that the null hypothesis cannot be rejected, and hence our residuals are normally distributed.

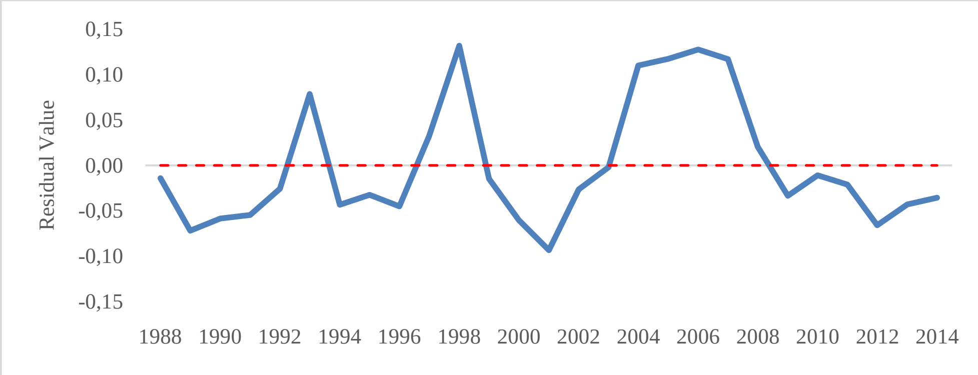

Figure 3. *Graph of residuals of estimation output with scaled data*

To conclude, the findings obtained from the estimations suggest that we can indeed formulate a production function for Turkey which is of the CES form and has capital, labor and energy as inputs entering the function in (KL)E nesting structure. The parameter estimates give for the production function the values $A = 0.992$, $\alpha = 0.842$ and $\sigma = 0.645$. Therefore, when plugging in the estimated parameter values, the function can be rewritten as follows.

$$Y = 0.992 * [0.842 * (K^{0.5} * L^{0.5})^{-0.550} + (0.158) * E^{-0.550}]^{-1.818}$$

Among the parameter estimates, the economic interpretation for $\sigma = 0.645$ can be made in the way that for the Turkish economy, the capital-labor bundle and energy inputs can be technically substituted for each other a rate of 0.645.

6. Discussion and conclusion

The result of this study provides supporting evidence for the necessity of country specific estimations. The initial motivation for undertaking this study consisted of the aim to derive parameter values for a nested CES function with capital, labor and energy as input factors for Turkey. Existing literature already provides some estimations for elasticity of substitutions. While *Bosetti et al.,* (2006) had found the substitution elasticity between the capital-labor bundle and energy to be $\sigma = 0.5$, Gerlagh and Van der Zwaan (2003) and Manne *et al.,* (1995) find this value to be $\sigma = 0.4$. While Paltsev *et al.,* (2005) finds the same outcome as Bosetti *et al.,* (2006), there are numerous research with close but slightly different results. This study reveals that with an estimation performed on the entire Turkish economy, the elasticity of substitution between the capital-labor bundle and energy is $\sigma = 0.645$. The interpretation of this value should not be made in the direction that the estimated value is completely different from what existing literature has obtained so far. The estimated elasticity of substitution value is the estimate for a particular point in time. But according to its confidence intervals with respect to the 1%, 5% and 10% confidence level, the values obtained from previous studies are in close proximity to the estimated value in this paper. Even though the result is not drastically different from the existing estimates, given the importance of this value, especially when its application will be on a macro level, digit level differences become important.

No doubt, there are many modifications, which can be performed on any research trying to estimate these parameters. Estimations can be done for data on a particular country as well as on aggregated data for a number of countries. Similarly, while the elasticity can be estimated for a particular industry or a group of several industries, it can also be estimated for an entire economy, as it has been done in this study. Even if two different studies have the same nesting and functional structure, they can diverge from each other based on the data used. While labor data comes as numbers and capital stock mostly in monetary terms, energy data can enter the function estimation in many forms. While some studies use energy consumption, other research uses primary energy supply. Similarly, energy can be taken in terms of joule or other power units, or energy can be taken in the form of energy cost. Both types of data for energy in the production function are equally common, and this paper follows amongst other the energy data approach applied by Van der Werf (2008) and takes energy cost. All these modifications can cause variations in the estimated parameter values and therefore create a need for aim-specific estimations.

In this study, even if not presented in detail, several data series have been tested. Data on capital stock and labor are not changed throughout the different estimations, yet energy data has undergone some changes. Estimations with energy in terms of peta joule and tonnes of oil equivalent have under neither scaling method led to significant conclusions for the substitution elasticity. Energy, in terms of energy cost, however led directly to significant results for data on Turkey satisfying all convergence criteria of nonlinear least squares estimation. This brings with itself the implication that even though energy consumption does increase over the time in relation with output, it does not have a nonlinear relation as it is indicated in the production function. On the other hand, total energy cost, together with capital and labor, does indeed have a nonlinear link to the economic output as it is also supported with economic theory. This draws attention to the fact that while energy consumption increases, the increase in energy consumption due to increase in output, is more closely linked to the increase in energy prices, which are omitted when solely energy in power units is taken as an input. But it is also worth pointing out that there are studies which take energy data as energy itself in terms of joule and conclude with significant results for other countries and industries, such as in Koesler & Schymura (2015).

This study imposed the (KL)E nesting structure to the production function, but there are also studies, such as the paper by Sue *et al.,* (2012), which investigate numerous different nesting forms and try to observe the most significant structure for an economy as well as for some particular sectors. It is worth mentioning that, while this study has estimated the production function for the entire Turkish economy, the nesting structure might be different from the estimated function for some particular industries, with a more industry specific input factor structure. Particularly some industries, such as the cement industry for example, are more energy intensive than others and do require more energy and capital inputs than compared to labor input. For sector specific analyses, it is therefore of use to estimate beforehand sector specific production functions and its parameters before making conclusions.

The production function structure is not derived based on trials on different versions, but it is imposed according to the research aim of this study. The derived parameter estimates are obtained through estimations performed on data for the entire Turkish economy. The main goal was to find the elasticity of substitution for the capital-labor bundle and energy. The paper presents results for substitution elasticities which are above the values applied and discovered in prior research (Bosetti *et al.,* 2006, Manne *et al.,* 1995 and Paltsev *et al.,* 2005), which can be interpreted as the elasticity of capital-labor and energy is higher in Turkey than the average value. Hence, this means that if the price of the capital-labor bundle increases, it can be relatively easily substituted for with energy. Another way of interpreting these results is that the high share of capital and labor in the production can be the result of energy prices in Turkey being relatively high due to the high share of imported energy used to meet the domestic energy demand. Hence, a reduction in energy prices can lead to a less capital-labor intensive economy for the case of Turkey.

A further point of investigation can be the application of this approach to the various sectors of the Turkish economy. Moreover, a production function estimation can be performed for the driving industries of the Turkish economy where the results can shed light on policy making fostering these industries. In this case, industry specific price can be obtained for the input factors, which could bring along besides the elasticity of technical substitution also the elasticity of substitution based on changes in prices.

Another potential topic for further research could be estimation of different types of production functions for Turkey. For instance, instead of assuming directly a Cobb Douglas form, capital and labor can be taken to be of a CES form. At the same time, based on the research question under focus, the energy input factor can be disaggregated based on electric and non-electric energy. This breakdown of energy could be detailed even further by accounting for different types of energy sources. Literature such as Oláh *et al.,* (2017) claims that changes inthe energy prices affect the level of support for biofuels with the goal of reducing dependence on crude oil imports. These and similar dynamics could be investigated further with their impact of substitution elasticities between energy and non-energy goods as well as among different types of energy sources. These possible topics and numerous other ones remain potential questions for further research.

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