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# Public service broadcasters and commercial media outlets with media bias in the changing media environment

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Abstract. In the current media environment, some commercial media outlets are offering more biased news than ever before with some public service broadcasters (PSBs) even appearing to disseminate biased news under political pressure. Under such conditions, can the media system ensure political accountability? This study theoretically examines how increased commercial media bias and politically pressured PSB bias affect political accountability and social welfare and discusses the role of PSBs in the changing media environment. We construct a two-period retrospective voting model in which voters obtain information from biased news. First, we present the conditions under which political accountability works even when biases exist. Second, we examine a case in which a PSB is politically pressured and present the bias selected by the PSB and the level of political accountability. We find that even if commercial media bias becomes extremely high, there are still cases in which political accountability works. Third, regarding two situations in which a PSB is mandated not to broadcast biased news and is politically pressured, we demonstrate how social welfare changes with an increase in commercial media bias and explore the social significance of PSBs. We discover that in the case of the anti-incumbent commercial media, when commercial media bias becomes relatively high, the existence of PSBs can always improve social welfare, even if PSBs are politically pressured.

**Keywords.** Public service broadcaster (PSB); Commercial media; Media bias; Political accountability; Social welfare.

JEL. L82; L32; D73.

#### 1. Introduction

any people use the news media to obtain political information. While many people prefer to obtain news via television, print newspapers, and radio, the number of people who obtain news through the Internet, especially social media and news websites, is increasing steadily (Mitchell et al., 2018a). Along with the increasing online news sources, online news is likely to be more biased or partisan than news offered by broadcasters or newspapers (Barclay, 2018; Faris et al., 2017). Some social media outlets produce hyperpartisan news or false and misleading news (so-called fake news), which spreads rapidly.[1] The number of people who feel that online news is biased is increasing (Shearer & Mitchell, 2021), and numerous people are distrusting the media (Mitchell et al., 2018b).[2]

While the number and variety of news outlets have increased, the news disseminated by public service broadcasters (PSBs) still attracts viewers or listeners in many countries. Mitchell *et al.* (2018b) show that for most Western Europeans, long-standing public news organizations are their main news sources.[3] Moreover, PSBs generally have a stated mission—to report news

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independently of political forces (Tambini, 2015). However, PSBs' revenue is determined by the political process, and their directors and executive committees are usually appointed by governments. Thus, politically pressured PSBs may broadcast news that is more in line with government preferences, despite governments not directly intervening in the content that they broadcast. Some studies have presented examples of PSBs that are politically captured or pressured and reporting biased news (Durante & Knight, 2012; Kitamura & Kuroda, 2020; Latham, 2013).

If media news is fair and neutral, it would play a vital role in letting politicians implement policies of interest to voters or holding politicians accountable.[4] Conversely, if media news is biased, it may not be able to effectively play this role. Thus, this study theoretically analyzes the following questions. First, when the commercial media offers more biased news and a PSB is politically pressured, can the media system still play the informational role and ensure political accountability? Second, when a politically pressured PSB can choose the level of bias regarding the news that it offers, what bias level does it choose? Specifically, when the commercial media offers more biased news, does the PSB also offer more biased news? Third, if a PSB is mandated not to broadcast biased news, can the media hold politicians accountable, and how does social welfare change? Furthermore, when a PSB is politically pressured and changes its bias with an increase in commercial media bias, in what situation can the media hold politicians accountable, and how does social welfare change? Moreover, does the existence of PSBs play a vital role in holding politicians accountable and improving social welfare, especially when commercial media bias increases?

To simplify this analysis, our study considers a situation in which a representative commercial media outlet and a PSB coexist and a situation in which only a representative commercial media outlet exists. As previously mentioned, in the reality of the current media environment, partisan or biased news has increased, especially online, due to the appearance of more ideologically biased commercial media (especially, social media)[5] or the increased probability of the commercial media broadcasting false news from lack of research. Many people obtain news through their preferred commercial media outlet, and on average, people are watching more biased news than ever before. Thus, for simplicity, we examine one representative commercial media outlet, the bias of which can expand. We assume that the commercial media outlet has an ideological preference for the incumbent politician and reports false news at some probability depending on its preference.[6] Our model calls the false-reporting probability the commercial media bias and explores the effects of its expansion.[7]

In addition, we examine both cases in which the commercial media is antiand pro-incumbent. While both right- and left-wing (or anti- and pro-incumbent) media coexist, in the U.S., extensive hyperpartisan right-wing online news has recently emerged (Faris et al., 2017; Marwick & Lewis, 2017).[8] Since we can observe the results that hyperpartisan news increases on one side of political spectrum, we explore two cases in which either anti- or pro-incumbent commercial media bias increases and discuss the differences between the results obtained in each case.

Regarding the PSB setting, we consider both a case in which a PSB is free from political power and is mandated not to report false news and a case in which a PSB is politically pressured and reports false news at some probability.

We call the false-reporting probability the PSB bias and explore the level of bias that a PSB chooses, especially when commercial media bias increases.

A growing number of studies have examined the political economy of the mass media,[9] and substantial literature exists on media bias.[10] Gentzkow, Shapiro, & Stone (2016) divide the literature into studies focusing on supply-and demand-driven biases. Supply-driven media bias may be derived from ideological bias and the preferences of journalists (Baron, 2006), news outlet owners (Djankov *et al.*, 2003), advertisers (Reuter & Zitzewits, 2016), or governments (Besley & Prat, 2006). Meanwhile, demand-driven bias may be explained by consumers who prefer to choose biased news (Gentzkow & Shapiro, 2006; Mullainathan & Schleifer, 2005). Our study contributes to the literature on media bias by constructing a model in which both commercial media outlets and PSBs are assumed to have a supply-driven bias. The novelty of our study is in considering the situation in which commercial media bias and PSB bias can both exist and commercial media bias increases.

Our study is related to the literature on media capture and political accountability.[11] Media capture occurs when the government actively pressures the media industry to suppress unfavorable information (Prat, 2016). Dunham (2017) demonstrates the presence of media capture in various countries. Although the media should ensure government accountability, it would not be able to play this role effectively if media capture exists. Our study considers a case in which a PSB is free from political power and a case in which an incumbent politician politically pressures a PSB. Ours is characteristic in considering changes in PSB bias, political accountability, and social welfare under a situation in which commercial media bias can vary.

The study proceeds as follows. Section 2 presents the model settings. Section 3 explores the relationship between biases and political accountability. Section 4 focuses on a case in which a PSB is politically pressured and examines the bias selected by the PSB and political accountability under an increase in commercial media bias. Section 5 examines social welfare in two situations in which a PSB is mandated not to broadcast biased news and is politically pressured. Moreover, it discusses the role of a PSB in the changing media environment. Finally, Section 6 concludes the study. Proofs of all propositions and corollaries are provided in the appendix.

#### 2. Model

#### 2.1. Basic setting

We use a two-period retrospective voting model. The actors include an incumbent politician, a challenger politician, a representative voter,[12] a PSB (*P*), and a commercial media outlet (*C*). Notably, the players of the game are only the incumbent, voter, and PSB (in Sections 4 and 5). For simplicity, we use female pronouns to refer to the incumbent and challenger and a male pronoun to refer to the voter. At the end of period 1, an election is held, and the voter chooses to either reelect the incumbent or elect the challenger.

At the beginning of period 1, Nature determines the types of incumbent and challenger independently (private information): a congruent ( $\mathcal{C}$ ) or dissonant ( $\mathcal{D}$ ) type with a probability of 1/2, the probability of which is common knowledge. The incumbent and challenger learn their own type, and in each period, the officeholder takes an action (private information). The incumbent gains benefit 1 if she is reelected and gains 0 if defeated. In both periods, the

congruent officeholder always takes the voter's preferred action a. Conversely, the dissonant officeholder has a privately preferred action b, which is not preferred by the voter, and suffers from emotional damage D (D > 0) when she does not take her preferred action b. In period 2, as the game ends after period 2, the dissonant officeholder takes her preferred action b. In period 1, the dissonant incumbent chooses action  $x \in \{a, b\}$  to maximize her expected payoff by comparing the expected benefit obtained from reelection with the emotional damage D from not taking her preferred action b.[13] We say that political accountability works (does not work) if x = a (x = b). For simplicity, we assume that when the expected payoff received from choosing x = a and x = b are the same, she chooses x = a.

In period 1, media outlets offer news about the incumbent's action to the voter as follows.[14] Although both the PSB and the commercial media outlet cannot observe the politicians' type, when the incumbent takes action  $\alpha$ (action b), each media outlet independently observes signal a (signal b) regarding the incumbent's action with a probability of  $\pi \in (\frac{1}{2}, 1)$  and signal b(signal a) with a probability of  $1-\pi$ , the probability of which is common knowledge. Here,  $\pi$  represents the media outlets' ability to obtain the correct information through their own research.[15] Now, we assume that the PSB is neutral or pro-incumbent.[16] When the PSB observes signal a, which is the voter's preferred action, it offers message a regarding the incumbent's action with a probability of 1. Conversely, when it observes signal b, which is the dissonant incumbent's preferred action and the voter's unpreferred action, it offers message a with a probability of  $\eta_P \in [0,1]$  and message b with a probability of  $1 - \eta_P$ ;  $\eta_P$  is PSB bias in this study. In Sections 2 and 3, we treat bias  $\eta_P$  as given, and in Sections 4 and 5, we consider the selection of PSB bias  $\eta_P$ , which occurs at the beginning of the game.

The commercial media outlet is anti-incumbent, neutral, or pro-incumbent.[17] In the case of an anti-incumbent commercial media outlet, when it observes signal a, it offers message b with a probability of  $\eta_C$  ( $\in$  [0,1]) and message a with a probability of  $1 - \eta_C$ , and when it observes signal b, it offers message b with a probability of 1. In the case of a pro-incumbent commercial media outlet, when it observes signal a, it offers message a with a probability of 1, and when it observes signal b, it offers message a with a probability of  $\eta_C$  and message a with a probability of a0 and message a1. We treat bias a1 and a2 as given.

In period 1, the voter is interested in learning the type of incumbent to increase the period 2 expected payoff. Thus, for guessing the type of incumbent, he chooses the PSB or the commercial media to watch the news about the incumbent's action and obtains message a or b from the chosen media outlet. The voter recognizes whether the commercial media is pro- or anti-incumbent and also knows biases  $\eta_P$  and  $\eta_C$ . The voter has a relative preference for the commercial media over the PSB, which is denoted by  $\sigma$  (private information). At the beginning of the game,  $\sigma$  is realized from an independent uniform distribution over the range  $[-\frac{1}{2f},\frac{1}{2f}]$ , where f>0. The voter dislikes biased news[18] and selects the PSB if  $-\eta_P \ge -\eta_C + \sigma$  and selects the commercial media if  $-\eta_P < -\eta_C + \sigma$ . At the beginning of the game, the expected probability of the voter viewing the PSB becomes  $\frac{1}{2} + (\eta_C - \eta_P)f$  and that of the voter viewing the commercial media becomes  $\frac{1}{2} - (\eta_C - \eta_P)f$ .

We implicitly assume that  $\frac{1}{2} + (\eta_C - \eta_P)f > 0$  ( $\frac{1}{2} + (\eta_C - \eta_P)f < 1$ ) because the possibility would exist that the voter selects the PSB (the commercial media) under any pair of  $(\eta_C, \eta_P)$ . The necessary and sufficient condition for the assumptions is  $f < \frac{1}{2}$  as  $\eta_C - \eta_P \in [-1, 1]$ .

In period 1, an election is held after the voter receives a message about the incumbent's action from the chosen media outlet. If the incumbent gives him a greater (smaller) expected benefit than the challenger, the voter votes for the incumbent (the challenger). For simplicity, we assume that if the expected benefit given by the incumbent and that by the challenger are the same, the voter who observed message a (message b) votes for the incumbent (the challenger). At the end of each period, the voter obtains benefit 1 (benefit 0) when the politician takes action a (action b). This setting means that the voter votes for the incumbent if the incumbent's expected probability of taking action a in period 2 is higher than the challenger's. In other words, the voter votes for the incumbent if the expected probability of the incumbent being congruent is higher than that of the challenger being congruent (i.e., 1/2).

#### 2.2. Timing

The timing of the game is summarized as follows. In period 1:

- 1. In Sections 4 and 5, the PSB chooses its bias  $\eta_P$ .
- 2. Nature determines the types of incumbent and challenger and the type of voter,  $\sigma$ .
- 3. The incumbent takes action *a* or *b*; the PSB and the commercial media outlet independently receive signal *a* or *b* about the incumbent's action.
- 4. The voter chooses the media outlet to watch.
- 5. The PSB and the commercial media outlet independently offer message *a* or *b* about the incumbent's action.
- 6. An election is held, and the voter votes for or against the incumbent.
- 7. Payoffs in period 1 are paid.In period 2:
- 1. The officeholder takes action *a* or *b*.
- 2. Payoffs in period 2 are paid.

## 3. Political accountability in a perfect Bayesian equilibrium (PBE)

The equilibrium concept used is a perfect Bayesian equilibrium (PBE) in pure strategies. Subsection 3.1 provides the choices of the voter and incumbent in the PBE. Subsection 3.2 models the case in which only the commercial media exists, and we compare the dissonant incumbent's equilibrium choice of x when only the commercial media exists and when the PSB coexists with the commercial media.

#### 3.1. Political accountability in the PBE

We define  $x^*$  as the dissonant incumbent's equilibrium choice of x. Then, the following proposition is obtained.

**Proposition 1.** We consider a case in which a PSB coexists with the commercial media. With both anti- and pro-incumbent commercial media, given biases  $\eta_C$  and  $\eta_P$ , we obtain the following in any equilibrium.

- 1. At the election, a voter reelects the incumbent if he receives message a from the media that he selected and elects the challenger if he receives message b.
- 2. The dissonant incumbent chooses  $x^* = a$  if

$$D \le (2\pi - 1) \left[ \left\{ \frac{1}{2} + (\eta_C - \eta_P) f \right\} (1 - \eta_P) + \left\{ \frac{1}{2} - (\eta_C - \eta_P) f \right\} (1 - \eta_C) \right]$$
 (1)

and chooses  $x^* = b$  if

$$D > (2\pi - 1) \left[ \left\{ \frac{1}{2} + (\eta_C - \eta_P) f \right\} (1 - \eta_P) + \left\{ \frac{1}{2} - (\eta_C - \eta_P) f \right\} (1 - \eta_C) \right]. \tag{2}$$

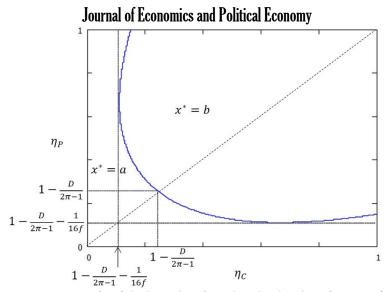
Result 1 is obtained as follows. The voter updates his knowledge regarding the expected probability of the incumbent being the congruent type depending on the media's message a or b; if the media that he chooses offers message a (message b), its probability exceeds (falls below) or equals the probability of the challenger being the congruent type, and he reelects the incumbent (elects the challenger).

Result 2 reveals that when the incumbent is dissonant, whether political accountability works (i.e.,  $x^* = a$ ) depends on the value of the emotional damage D and that of the expression  $(2\pi - 1)[\{\frac{1}{2} + (\eta_C - \eta_P)f\}(1 - \eta_P) + \{\frac{1}{2} - (\eta_C - \eta_P)f\}(1 - \eta_C)]$ . Hereafter, we call the following equation the boundary line (3):

$$D = (2\pi - 1) \left[ \left\{ \frac{1}{2} + (\eta_C - \eta_P) f \right\} (1 - \eta_P) + \left\{ \frac{1}{2} - (\eta_C - \eta_P) f \right\} (1 - \eta_C) \right].$$
 (3)

Note that the dissonant incumbent's equilibrium choice of x is independent of whether the commercial media is anti- or pro-incumbent. This is because the difference in the reelection probability when x = a and x = b does not vary between anti- and pro-incumbent commercial media.

The blue solid line in Figure 1 illustrates boundary line (3),[19] which divides the selection of x. The boundary line is a parabola that is convex to its origin, with a vertex of  $(1-\frac{D}{2\pi-1},1-\frac{D}{2\pi-1})$  and an axis of  $\eta_P=\eta_C$ . In addition, boundary line (3) is tangent to  $\eta_C=1-\frac{D}{2\pi-1}-\frac{1}{16f}$  and  $\eta_P=1-\frac{D}{2\pi-1}-\frac{1}{16f}$ . For simplicity, we assume that  $1-\frac{D}{2\pi-1}\in(0,1)$  because the incumbent will choose x=a if  $(\eta_C,\eta_P)=(0,0)$  and will choose x=b if  $(\eta_C,\eta_P)=(1,1)$ . Then, Figure 1 shows that, especially when  $1-\frac{D}{2\pi-1}-\frac{1}{16f}>0$ , if at least either  $\eta_C$  or  $\eta_P$  is small enough, the dissonant incumbent chooses x=a. In other words, when PSB bias  $\eta_P$  is small enough, political accountability works even if commercial media bias  $\eta_C$  increases in the changing media environment.



**Figure 1.** Example of the boundary line that divides the selection of x

The exogenous variables D and  $\pi$  determine the coordinates of the vertex of boundary line (3), and the exogenous variable f determines the line's shape. The smaller the value of D and the larger the value of  $\pi$ , the more the vertex's coordinates move toward the upper right, thereby widening the range in which political accountability works. Moreover, the larger the value of f, the narrower the shape of the parabola, also widening the range in which political accountability works. The intuitions underlying these facts are as follows. First, since a smaller D indicates that the emotional damage when the dissonant incumbent cannot take her preferred action b is smaller, the range in which she chooses x = a becomes broader. Second, since a larger  $\pi$ indicates that the action chosen by the dissonant incumbent is learned more precisely by the media outlets, the range in which the dissonant incumbent chooses x = a becomes broader. Third, a larger value of f indicates that the distribution of the relative preference for the commercial media over the PSB is so small that the voter is likely to select a less biased media and gain more precise information regarding the incumbent's action, broadening the range in which the dissonant incumbent chooses x = a.

3.2. Comparison with a case in which only the commercial media exists We now model a case in which only the commercial media exists. In this case, every voter watches the news offered by the commercial media. Then, we obtain the following proposition.

**Proposition 2.** We consider a case in which only the commercial media exists. With both anti- and pro-incumbent commercial media, given bias  $\eta_C$ , we obtain the following in any equilibrium.

1. The voter's choice is the same as that in Proposition 1.

2. The dissonant incumbent chooses  $x^* = a$  if  $\eta_C \le 1 - \frac{D}{2\pi - 1}$  and  $x^* = b$  if  $\eta_C > 1 - \frac{D}{2\pi - 1}$ .

This proposition indicates that when the incumbent is dissonant, if commercial media bias  $\eta_C$  is small enough to satisfy  $\eta_C \leq 1 - \frac{D}{2\pi - 1}$ , political accountability works (i.e.,  $x^* = a$ ); otherwise, political accountability does not

work (i.e.,  $x^* = b$ ). Now, we compare Propositions 1 and 2. When a PSB coexists with the commercial media, if the PSB bias is small enough to satisfy  $D \leq (2\pi-1)[\{\frac{1}{2}+(\eta_C-\eta_P)f\}(1-\eta_P)+\{\frac{1}{2}-(\eta_C-\eta_P)f\}(1-\eta_C)]$ , political accountability works even if commercial media bias  $\eta_C$  is large enough to satisfy  $\eta_C > 1 - \frac{D}{2\pi-1}$ , where political accountability does not work if only the commercial media exists. In other words, when commercial media bias increases in the changing media environment, the coexistence of a PSB is vital in holding politicians accountable if the PSB bias is small enough. In addition, especially when a PSB is mandated to choose zero bias, political accountability works within the larger range of commercial media bias.

Although our study focuses on the situation in which commercial media bias increases, we can also examine a situation in which only a PSB exists. In this case, a simple calculation reveals that the dissonant incumbent chooses  $x^* = a$  if  $\eta_P \le 1 - \frac{D}{2\pi - 1}$  and  $x^* = b$  if  $\eta_P > 1 - \frac{D}{2\pi - 1}$ . Then, when the PSB bias becomes large enough to satisfy  $\eta_P > 1 - \frac{D}{2\pi - 1}$ , the coexistence of the commercial media is needed to hold politicians accountable if the commercial media bias is small enough. Remarkably, if we can make the PSB bias small enough, we can hold politicians accountable without the commercial media.

#### 4. Selection of PSB bias and political accountability

Section 4 focuses on a case in which a PSB is exposed to political pressure and examines how an increase in commercial media bias influences the selection of PSB bias and the dissonant incumbent's strategy. At the beginning of the game, the PSB under political pressure selects bias  $\eta_P$  to maximize the incumbent's expected payoff.[20]

### 4.1. PSB bias and political accountability with anti-incumbent commercial media

This subsection examines a situation in which a politically pressured PSB coexists with the anti-incumbent commercial media.

Given commercial media bias  $\eta_C$ , we denote the optimal PSB bias that maximizes the incumbent's expected payoff at the beginning of the game as  $\eta_P^*$ . Furthermore, given commercial media bias  $\eta_C$ , we denote the PSB bias that maximizes the incumbent's expected payoff at the beginning of the game on condition that x = a (x = b) and the voter chooses the optimal choice shown in Proposition 1 as  $\eta_P^*|_{x=a}$  ( $\eta_P^*|_{x=b}$ ). Then, we obtain the following proposition.

**Proposition 3.** When we examine the change in the pair of optimal PSB bias  $\eta_P^*$  and dissonant incumbent's equilibrium choice  $x^*$  when anti-incumbent commercial media bias  $\eta_C$  increases from 0 to 1, only the following four patterns are possible: Case 1 is  $(\eta_P^*, x^*) = (\eta_P^*|_{x=a}, a) \rightarrow (\eta_P^*|_{x=b}, b) \rightarrow (\eta_P^*|_{x=a}, a)$ ; Case 2 is  $(\eta_P^*, x^*) = (\eta_P^*|_{x=a}, a) \rightarrow (\eta_P^*|_{x=b}, b)$ ; Case 3 is  $(\eta_P^*, x^*) = (\eta_P^*|_{x=a}, a)$ ; and Case 4 is  $(\eta_P^*, x^*) = (\eta_P^*|_{x=b}, b)$ . Note that

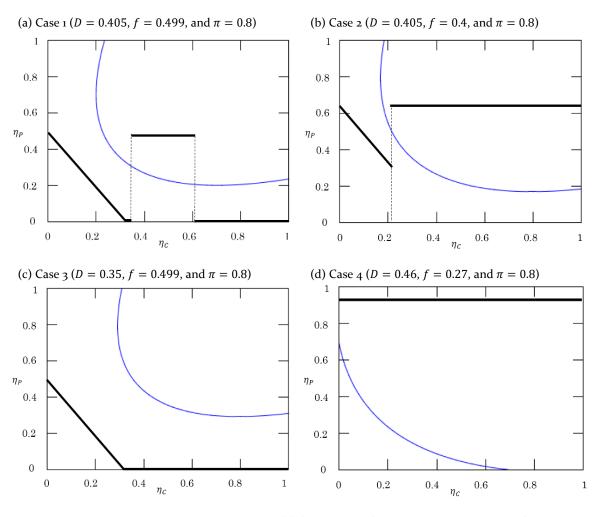
$$\eta_P^*|_{x=a} = \begin{cases}
0 & if \quad -\frac{2\pi - 1}{2 - 2\pi} \eta_C + \frac{1}{4f} \le 0 \\
-\frac{2\pi - 1}{2 - 2\pi} \eta_C + \frac{1}{4f} & if \quad 0 \le -\frac{2\pi - 1}{2 - 2\pi} \eta_C + \frac{1}{4f} \le 1 \\
1 & if \quad -\frac{2\pi - 1}{2 - 2\pi} \eta_C + \frac{1}{4f} \ge 1
\end{cases}$$
(4)

and

$$\eta_P^*|_{x=b} = \begin{cases} \frac{1}{4f} & if \frac{1}{4f} \le 1\\ 1 & if \frac{1}{4f} \ge 1. \end{cases}$$
 (5)

Note also that it always holds that  $\eta_P^*|_{x=a} \leq \eta_P^*|_{x=b}$ .

Figure 2 illustrates the examples of  $\eta_P^*$  in each case. Boundary line (3) and PSB bias  $\eta_P^*$  are described by blue and black bold solid lines, respectively. Which case occurs depends on the position of boundary lines (3) to (5). We have already confirmed the relationship between the position of boundary line (3) and the exogenous variables in Subsection 3.1; lines (4) and (5) are positioned upside when f is small and the slope of the line  $\eta_P = -\frac{2\pi-1}{2-2\pi}\eta_C + \frac{1}{4f}$  (equation (4)) is steep when  $\pi$  is large.



**Figure 2.** Examples of boundary line (3) (the blue line) and optimal PSB bias  $\eta_P^*$  (the black bold line) when the commercial media is anti-incumbent

Proposition 3 suggests that in Case 3 (Case 4), political accountability always works (does not work) irrespective of the commercial media bias  $\eta_C$ . In Case 3, PSB bias  $\eta_P^*|_{x=a}$  becomes smaller with an increase in commercial

media bias  $\eta_C$  if  $\eta_C \in [\frac{2-2\pi}{2\pi-1}\frac{1-4f}{4f},\frac{2-2\pi}{2\pi-1}\frac{1}{4f}]$ . The reason is as follows. First, with an increase in commercial media bias, the probability that the voter chooses the PSB increases while the probability that the anti-incumbent commercial media offers message b when it receives signal a also increases. Then, when both the congruent and dissonant incumbent chooses action a, since the probability that the PSB and the commercial media receives signal a is  $\pi$ , which is greater than  $\frac{1}{2}$ , reducing the PSB bias to decrease the probability that the voter chooses the commercial media leads to the maximization of the incumbent's expected payoff.

In Case 4,  $\eta_P^* = \eta_P^*|_{x=b}$  is constant and independent of the value of the commercial media bias  $\eta_C$ . The reason is as follows. First, when the dissonant incumbent chooses x=b, with an increase in commercial media bias, increasing PSB bias  $\eta_P$  maximizes the incumbent's expected payoff. This result is because when the probability that the voter chooses the PSB increases with an increase in commercial media bias  $\eta_C$ , increasing PSB bias  $\eta_P$  to increase the probability that the voter receives message a when the signal that PSB receives is b increases the incumbent's expected payoff significantly. Notably, in this case, at the beginning of the game, the incumbent becomes a congruent type who chooses a with a probability of  $\frac{1}{2}$  and becomes a dissonant type who chooses a with a probability of a and becomes a dissonant type who chooses a with a probability of a and becomes a dissonant type who chooses a with a probability of a and becomes a dissonant type who chooses a with a probability of a and becomes a dissonant type who chooses a with a probability of a and becomes a dissonant type who chooses a with a probability of a and becomes a dissonant type who chooses a with a probability of a and becomes a dissonant type who chooses a with a probability of a and becomes a dissonant type who chooses a with a probability of a and becomes a dissonant type who chooses a with a probability of a and becomes a dissonant type who chooses a with a probability of a and becomes a dissonant type who chooses a with a probability of a and becomes a dissonant type who chooses a dissonant type who chooses a dissonant type who chooses a with a probability of a and becomes a dissonant type who chooses a dissonant type who chooses

In Cases 1 and 2, the dissonant incumbent chooses x = a (i.e., political accountability works) when commercial media bias  $\eta_C$  is small enough but chooses x = b (i.e., political accountability does not work) when commercial media bias  $\eta_C$  exceeds a certain level. The reason is as follows. With small commercial media bias  $\eta_C$ , a high probability exists that the commercial media will offer message a when the incumbent chooses action a. Thus, the case in which the dissonant incumbent chooses x = a with small commercial media bias  $\eta_C$  appears at certain values of exogenous variables. Meanwhile, when commercial media bias  $\eta_C$  becomes relatively large, the probability that the commercial media will offer message a when the dissonant incumbent chooses x = a reduces. Furthermore, when the dissonant incumbent chooses x = b, the PSB chooses bias  $\frac{1}{4f}$  irrespective of the value of the commercial media bias  $\eta_C$ , where the probability that the voter obtains message  $\alpha$  when the PSB receives signal b is high (i.e.,  $\frac{1}{4f}$ ) and the dissonant incumbent can avoid emotional damage D. Thus, when the commercial media bias increases, the dissonant incumbent changes the choice of x from a to b under certain values of exogenous variables. Notably, at the level of commercial media bias  $\eta_C$  that the incumbent changes the action from a to b, PSB bias  $\eta_P^*$  increases to a higher level  $\frac{1}{4f}$  or 1. This happens because a larger PSB bias can reduce the probability of the PSB offering message b when the dissonant incumbent chooses x = b.

In Case 1, when commercial media bias  $\eta_C$  becomes relatively large, the dissonant incumbent chooses x=a once again with a smaller or no PSB bias. The reason is as follows. With large commercial media bias  $\eta_C$ , when the dissonant incumbent chooses x=a, the PSB chooses a smaller or no PSB bias (as discussed in Case 3), where the probability that the voter chooses a

commercial media whose bias is so high becomes very low. Thus, the case in which the dissonant incumbent chooses x = a once again appears at certain values of exogenous variables. We are interested in whether political accountability works even if commercial media bias is high. Therefore, this case is interesting in that when commercial media bias is extremely high, if a PSB exists (even if it is politically pressured), political accountability works once again with a smaller or no PSB bias.

In Subsection 3.2, we confirmed that political accountability does not work with a large enough commercial media bias when only the commercial media exists. In contrast, when the anti-incumbent commercial media coexists with a politically captured PSB that chooses its bias, if Case 1 or 3 applies to the real world, the coexistence of a PSB is vital in holding politicians accountable when the commercial media increases.

### 4.2. PSB bias and political accountability with pro-incumbent commercial media

This subsection examines the situation in which a politically pressured PSB coexists with the pro-incumbent commercial media. In this case, we obtain the following proposition.

**Proposition 4.** When we examine the change in the pair of optimal PSB bias  $\eta_P^*$  and the dissonant incumbent's equilibrium choice  $x^*$  when pro-incumbent commercial media bias  $\eta_C$  increases from 0 to 1, the following two patterns exist: Case 5 is  $(\eta_P^*, x^*) = (\eta_P^*|_{x=a}, a) \rightarrow (\eta_P^*|_{x=b}, b)$  and Case 6 is  $(\eta_P^*, x^*) = (\eta_P^*|_{x=b}, b)$ , where

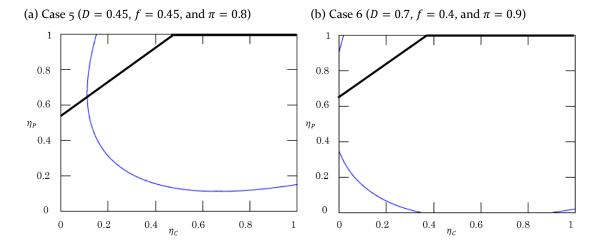
$$\eta_P^* = \eta_P^*|_{x=a} = \eta_P^*|_{x=b} = \begin{cases} \eta_C + \frac{1}{4f} & \text{if } \eta_C + \frac{1}{4f} \le 1\\ 1 & \text{if } \eta_C + \frac{1}{4f} \ge 1. \end{cases}$$

Figure 3 presents the examples of  $\eta_P^*$  (the black bold line) in each case. Note that when boundary line (3) moves to the upper right and/or the shape of the parabola narrows, the range in which the dissonant incumbent chooses x = a becomes wider.

Proposition 4 shows that when the commercial media is pro-incumbent, only two patterns exist, where political accountability does not work with the higher commercial media bias. Further,  $\eta_P^*|_{x=a}$  and  $\eta_P^*|_{x=b}$  are the same, since both the PSB and the commercial media are pro-incumbent and broadcast news about the incumbent similarly. In addition, when  $\eta_C + \frac{1}{4f} < 1$ , PSB bias  $\eta_P^*$  increases higher with an increase in commercial media bias  $\eta_C$ , since when a PSB chooses more biased news with an increase in commercial media bias, the voter is more likely to observe signal a through either the PSB or pro-incumbent commercial media. This result contrasts the result at which the commercial media is anti-incumbent. Specifically, when the commercial media is pro-incumbent, the existence of a PSB cannot play a role to hold politicians accountable.

As discussed in Subsection 3.2, we can also examine a situation in which only a PSB exists. In this case, a simple calculation reveals that the pair of optimal PSB bias  $\eta_P^*$  and the dissonant incumbent's equilibrium choice  $x^*$  becomes  $(\eta_P^*, x^*) = (1, b)$ . Then, when a politically pressured PSB chooses its

bias  $\eta_P$ , the coexistence of the commercial media helps to hold politicians accountable if the pair of  $(\eta_C, \eta_P^*)$  satisfies inequality (1).



**Figure 3.** Examples of boundary line (3) (the blue line) and optimal PSB bias  $\eta_P^*$  (the black bold line) when the commercial media is pro-incumbent

#### 5. Social welfare

This section examines changes in social welfare when commercial media bias varies. We consider the following three situations, namely, when only the commercial media exists (situation C), when the commercial media coexists with a PSB that is free from political power and is mandated to choose zero bias (situation  $CP^*$ ), and when the commercial media coexists with a PSB that is politically pressured and chooses bias  $\eta_P^*$  at the beginning of the game (situation  $CP^{**}$ ).

Subsection 5.1 explains the additional settings. Subsection 5.2 (Subsection 5.3) compares social welfare in situation C with that in situation  $CP^*$  (situation  $CP^{**}$ ) and discusses the significance of the PSB's existence. When we do not specify whether the commercial media is anti- or pro-incumbent, the statement holds in both cases.

#### 5.1. Additional settings

We derive each period's welfare from the voter's benefit in each period. Remember that at the end of each period, the voter obtains benefit 1 (benefit 0) when the officeholder chooses x = a (x = b). We assume without loss of generality that the voter does not discount the future. Then, we derive social welfare from the sum of the expected welfare in each period calculated at the beginning of the game, denoted by SW.

We represent equilibrium social welfare in the three situations C,  $CP^*$ , and  $CP^{**}$  by  $SW^C$ ,  $SW^*$ , and  $SW^{**}$ , respectively. Then, we obtain the following proposition.

**Proposition 5.** *In situations C, CP*\*, and  $CP^{**}$ , we obtain the following.

- 1. Social welfare when x = a becomes  $SW^C|_{x=a} = SW^*|_{x=a} = SW^*|_{x=a} = SW^*|_{x=a} = \frac{3}{2}$ .
- 2. Consider social welfare when x = b. Then, we obtain the following. (a) In situation C,  $SW^C|_{x=b} = 1 + \frac{1}{2}(\pi \frac{1}{2})(1 \eta_C)$ .

(b) In situation 
$$CP^*$$
,  $SW^*|_{x=b} = 1 + \frac{1}{2}(\pi - \frac{1}{2})\{1 - \frac{1}{2}\eta_C + (\eta_C)^2 f\}$ .

(b) In situation 
$$CP^*$$
,  $SW^*|_{x=b} = 1 + \frac{1}{2}(\pi - \frac{1}{2})\{1 - \frac{1}{2}\eta_C + (\eta_C)^2 f\}$ .  
(c) In situation  $CP^{**}$ ,  $SW^{**}|_{x=b} = 1 + \frac{1}{2}(\pi - \frac{1}{2})\{1 - \frac{1}{2}(\eta_C + \eta_P^*) + (\eta_C - \eta_P^*)^2 f\}$ .

Result 1 of this proposition reveals that  $SW|_{x=a}$  does not vary in the three situations C,  $CP^*$ , and  $CP^{**}$ , and it does not relate to biases  $\eta_C$  and  $\eta_P$ . This is primarily because when x = a, the dissonant incumbent chooses the action that the congruent incumbent chooses, and the expected probability of the incumbent being congruent becomes the same as that of the challenger being congruent. Conversely, result 2 reveals that  $SW|_{x=b}$  varies in terms of the situation and bias, primarily because when x = b, the reelection probability of the congruent incumbent and that of the dissonant incumbent differ, and the expected probability of the period 2 politician being congruent and choosing x = a depends on the biases. Specifically, results 2(a) and 2(c) show that in situation C or  $CP^{**}$ , when x = b, social welfare reduces with an increase in commercial media bias  $\eta_C$ . Furthermore, results 2(b) and 2(c) show that when x = b, social welfare in situation  $CP^*$  is greater than in situation  $CP^{**}$ .

Moreover, a simple calculation indicates that when x = a (i.e., political accountability works), social welfare is always greater than when x = b (i.e., political accountability does not work). Thus, Propositions 3 and 5 show that especially in situation  $CP^{**}$ , when Case 1 or 3 applies to the real world, even if commercial media bias  $\eta_C$  becomes high enough, political accountability works with a smaller or no PSB bias, and social welfare SW\*\* takes a high value, that is,  $\frac{3}{2}$ . In other words, in situation  $CP^{**}$ , an increase in commercial media bias  $\eta_C$  is not always bad for social welfare (the change of  $SW^*$  in situation  $CP^*$  is discussed in the following subsection).

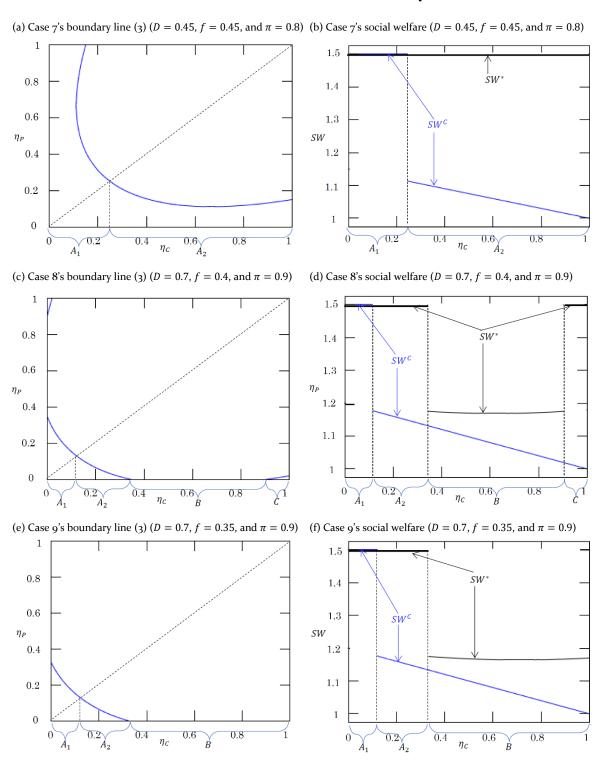
#### 5.2. Comparison of social welfare in situations *CP*\* and *C*

This subsection compares social welfare  $SW^*$  in situation  $CP^*$  with social welfare  $SW^{C}$  in situation C. We obtain the following corollary regarding their relationship.

**Corollary 1.** When we compare social welfare SW\* in situation CP\* with social welfare  $SW^{C}$  in situation C, we obtain the following.

- When  $\eta_C \leq 1 \frac{D}{2\pi 1}$ , situation  $CP^*$ 's social welfare  $SW^*$  is the same as situation C's social welfare  $SW^C$ , i.e.,  $SW^* = SW^C$ . 2. When  $\eta_C > 1 - \frac{D}{2\pi - 1}$ ,  $SW^* > SW^C$  always holds.

In situation  $CP^*$ , the shape of boundary line (3) indicates only the following three patterns of change in the dissonant incumbent's choice  $x^*$  with an increase in commercial media bias  $\eta_C$ : Case 7 is  $x^* = a$ ; Case 8 is  $x^* = a \rightarrow b \rightarrow$ a; and Case 9 is  $x^* = a \rightarrow b$ . Figures 4 (a), (c), and (e) present the examples of each case, where boundary line (3) is described by the blue solid line. Subsequently, social welfare  $SW^*$  (social welfare  $SW^C$ ) is described by the black (blue) solid line in Figures 4 (b), (d), and (f). Then, Proposition 5 and Corollary 1 show that especially in situation  $CP^*$ , when Case 7 or 8 applies to the real world, even if commercial media bias  $\eta_C$  becomes high enough, social welfare  $SW^*$  takes a high value, that is,  $\frac{3}{2}$ . In other words, in situation  $CP^*$ , increases in commercial media bias  $\eta_C$  do not always damage social welfare.



**Figure 4.** Boundary line (3) and social welfare  $SW^*$  and  $SW^C$ 

Next, we consider the role of PSBs at which they are mandated to choose zero bias. Corollary 1 shows that social welfare in situation  $CP^*$  is greater than or equal to that in situation C. When commercial media bias  $\eta_C$  becomes sufficiently large (i.e.,  $\eta_C > 1 - \frac{D}{2\pi - 1}$ ), situation  $CP^*$  always yields a greater value for social welfare. Thus, if we can mandate PSBs to choose zero bias, their existence improves social welfare. The intuition is as follows. When

commercial media bias  $\eta_C$  is small, political accountability works both in situations  $CP^*$  and C. Meanwhile, when commercial media bias  $\eta_C$  becomes large in situation C, social welfare decreases greatly because the voter only has access to the highly biased commercial media, and thus, political accountability becomes not to work.

#### 5.3. Comparison of social welfare in situations *CP\*\** and *C*

This subsection compares social welfare  $SW^{**}$  in situation  $CP^{**}$  with social welfare  $SW^{C}$  in situation C. We obtain the following corollary concerning their relationship.

**Corollary 2.** When we compare social welfare SW\*\* in situation CP\*\* with social welfare  $SW^{C}$  in situation C, we obtain the following.

- 1. Consider a case in which  $\eta_C \le 1 \frac{D}{2\pi 1}$ . Then, in situation C,  $x^* = a$ .

  (a) When  $x^* = a$  in situation  $CP^{**}$ ,  $SW^{**} = SW^C$ .

  - (b) When  $x^* = b$  in situation  $CP^{**}$ ,  $SW^{**} < SW^C$
- 2. Consider a case in which  $\eta_C > 1 \frac{D}{2\pi 1}$ . Then, in situation C,  $x^* = b$ .

  (a) When  $x^* = a$  in situation  $CP^{**}$ ,  $SW^{**} > SW^C$ .

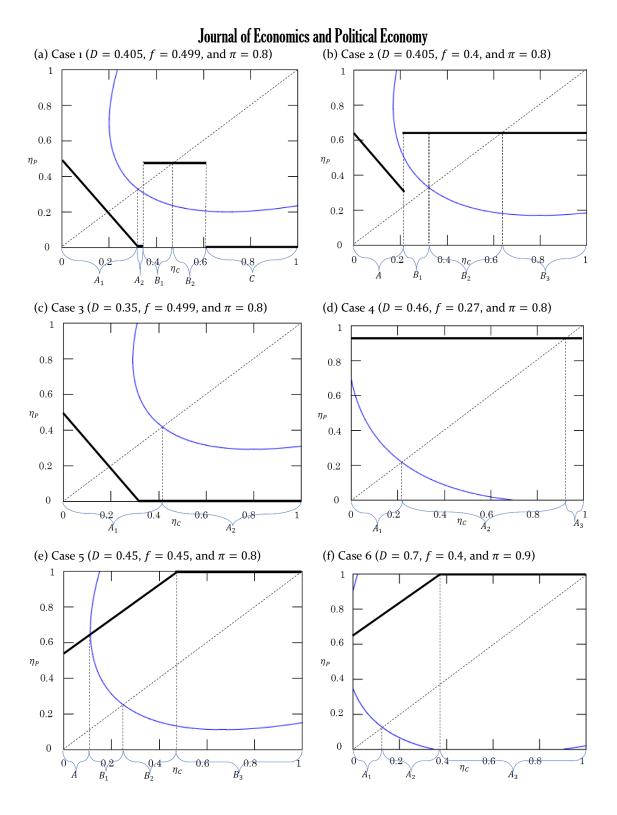
  - (b) When  $x^* = b$  and  $\eta_P^* \ge \eta_C$  in situation  $CP^{**}$ ,  $SW^{**} \le SW^C$ .
  - (c) When  $x^* = b$  and  $\eta_P^* < \eta_C$  in situation  $CP^{**}$ ,  $SW^{**} > SW^C$ .

Figure 5 illustrates the examples of Cases 1-6 presented in Section 4, in which the range of  $\eta_C$ , [0,1], is divided into some ranges. Using the ranges in Figure 5,  $SW^{**}$  ( $SW^{C}$ ) is described by the black (blue) solid line in Figure 6.

Results 1 and 2(a) of Corollary 2 are clear from Proposition 5. The reason for results 2(b) and 2(c) is as follows. When the bias is higher, the selection effect, which makes the dissonant incumbent politician harder to reelect, is less powerful. Thus, if PSB bias  $\eta_P^*$  is larger (smaller) than commercial media bias  $\eta_C$ , social welfare in situation  $CP^{**}$  becomes smaller (greater) than that in situation C.

Now, we consider the role of PSBs at which they are politically pressured. When the commercial media is anti-incumbent (i.e., Cases 1-4) and its bias becomes high enough, PSB bias becomes lower than commercial media bias, and PSB's existence, even if it is politically pressured, always improves social welfare. In other words, when the commercial media is anti-incumbent, even if PSBs are politically pressured, their existence can improve social welfare in the changing media environment. Conversely, when the commercial media is pro-incumbent (i.e., Cases 5 and 6), PSB bias  $\eta_P^*$  is larger than commercial media bias  $\eta_C$ , and the existence of the PSB does not improve social welfare.

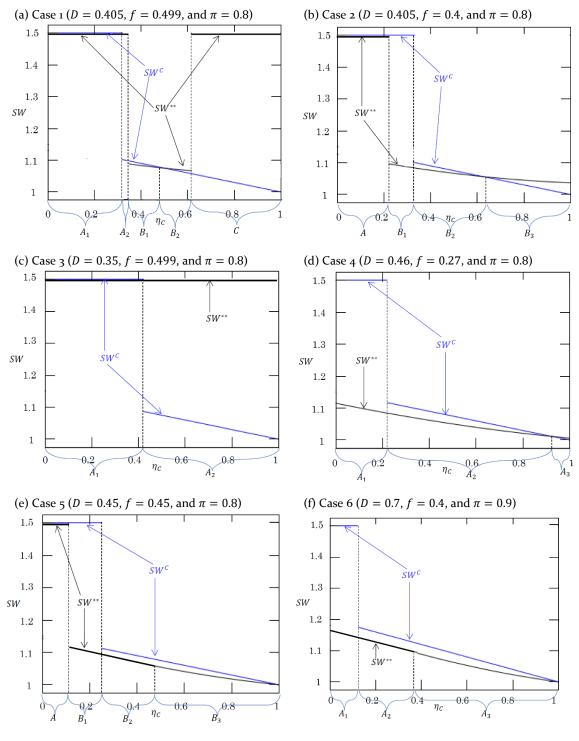
Next, we compare social welfare  $SW^*$  in situation  $CP^*$  and social welfare  $SW^{**}$  in situation  $CP^{**}$ . Proposition 5 and Corollary 2 show that when  $x^* = b$ ,  $SW^*$  is larger than  $SW^{**}$ . Furthermore, given  $\eta_C$ , the case where  $x^* = b$  in situation  $CP^*$  and  $x^* = a$  in situation  $CP^{**}$  never occurs. Thus, a policy that mandates PSBs to choose zero bias improves social welfare. In addition, even if we cannot mandate PSBs to choose zero bias, PSBs have a role improving social welfare with high commercial media bias  $\eta_C$  when the commercial media is anti-incumbent.



**Figure 5.** Examples of boundary line (3) (the blue line) and optimal PSB bias  $\eta_P^*$  (the black bold line)

Finally, we examine a case in which only a politically pressured PSB exists. In this case, social welfare, denoted by  $SW^P$ , becomes  $SW^P|_{x=a}=\frac{3}{2}$  and  $SW^P|_{x=b}=1+\frac{1}{2}(\pi-\frac{1}{2})(1-\eta_P)$ . Moreover, as presented in Subsection 4.2, in this case, the pair of optimal PSB bias  $\eta_P^*$  and the dissonant incumbent's

equilibrium choice  $x^*$  becomes  $(\eta_P^*, x^*) = (1, b)$ ; subsequently, social welfare becomes  $SW^P|_{x=b,\eta_P=1}=1$ , which equals  $SW^C|_{x=b,\eta_C=1}=1$ . A simple calculation reveals that in this case, social welfare always becomes smaller than or equal to that in situation C,  $CP^*$ , or  $CP^{**}$ . Therefore, the coexistence of both the PSB and the commercial media outlet is needed to improve social welfare. Note that if the PSB is mandated to choose zero bias, social welfare under a situation in which only a PSB exists (i.e.,  $SW^P = \frac{3}{2}$ ) becomes greater than or equal to that in situation C,  $CP^*$ , or  $CP^{**}$ .



**Figure 6.** Examples of social welfare  $SW^{**}$  and  $SW^{C}$  **S. Ihara, JEPE, 11(1-2), 2024, p.1-27.** 

#### 6. Conclusion

This study examined a situation in which a PSB can be politically pressured and the commercial media outlet offers more biased news, investigating the effect of their bias on political accountability and social welfare. Furthermore, we discussed the significance of PSBs in the changing media environment.

The main results are as follows. First, we explored the relationship between biases and political accountability. Then, we revealed the pairs of PSB bias and commercial media bias with which political accountability works even if biases exist. In addition, we found that when only the commercial media exists, political accountability does not work when commercial media bias is high. However, when the PSB coexists with the commercial media, depending on the level of PSB bias, political accountability can work even if commercial media bias increases.

Next, we focused on a case in which a PSB is politically pressured and examined the bias selected by the PSB and political accountability. When the commercial media is anti-incumbent, we identified four patterns of change in political accountability owing to an increase in commercial media bias. In one of them, we found that when the commercial media bias surpasses a certain level, political accountability becomes not to work with a larger PSB bias; when the commercial media bias becomes sufficiently large, political accountability works again with a smaller or no PSB bias. This result is noteworthy in that we revealed that when the PSB exists (even if it is politically pressured), an increase in commercial media bias does not necessarily lead to a situation in which political accountability does not work.

Moreover, when the commercial media is pro-incumbent, we found two patterns of change in political accountability owing to an increase in commercial media bias. Then, we identified that when the commercial media bias increases, PSB bias also increases and political accountability becomes not to work, which result contrasts the result at which the commercial media is anti-incumbent.

We furthermore examined social welfare when commercial media bias varies, revealing that even if commercial media bias increases, in some cases, political accountability works, and social welfare takes a high value. In addition, we discussed the significance of PSBs in the changing media environment. First, when political pressure on PSBs is banned and PSBs are mandated to choose zero bias, we revealed that social welfare when the PSB coexists with the commercial media is always greater than or equal to social welfare when only the commercial media exists. This indicates that in this case, the existence of PSBs improves social welfare in the changing media environment.

Next, when PSBs are politically pressured, we demonstrated the following points. When the commercial media is anti-incumbent, the social welfare in a situation where the commercial media bias is relatively small and only the commercial media exists is greater than or equal to the social welfare when the PSB coexists with the commercial media. Conversely, when the commercial media bias becomes relatively large, PSB's existence can improve social welfare more than when only the commercial media exists, even if PSBs are politically pressured. Meanwhile, if the commercial media is proincumbent, the existence of the PSB does not improve social welfare.

In various advanced countries, PSBs coexist with the commercial media, and commercial media bias is likely to increase in the changing media environment. It varies according to the country and era, depending on whether the commercial media is pro- or anti-incumbent and whether PSBs are politically pressured. Thus, the importance of this study is that by considering various cases, we examined the significance of PSBs, specifically from the perspective of political accountability.

PSBs have various objectives other than ensuring political accountability.[21] For example, reports during times of disaster, educational programs, or socially valuable programs are characteristic PSB contents. However, PSBs need license fees or specific taxes to offer programming. Thus, future research should consider other factors when considering the significance of PSBs. However, our study focused on the role of political accountability as considering many factors simultaneously may complicate the discussion.

#### Acknowledgement

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#### Appendix

#### Proofs of all propositions and corollaries

#### **Proof of Proposition 1**

First, we consider the voter's election choice when he believes that the dissonant incumbent chooses x = b. We define  $\pi_a^m$  ( $\pi_b^m$ ) as the media m = P, C viewer's expectation about the probability that the incumbent is congruent when media m offers signal a (signal b). Then, we obtain the following:

$$\pi_{a}^{P} \mid_{x=b} = \frac{\frac{1}{2} \{\pi + (1-\pi)\eta_{P}\}}{\frac{1}{2} \{\pi + (1-\pi)\eta_{P}\} + \frac{1}{2} \{(1-\pi) + \pi\eta_{P}\}} \text{ for } \eta_{P} \in [0,1];$$

$$\pi_{b}^{P} \mid_{x=b} = \frac{\frac{1}{2} (1-\pi)(1-\eta_{P})}{\frac{1}{2} (1-\pi)(1-\eta_{P}) + \frac{1}{2} \pi(1-\eta_{P})} \text{ for } \eta_{P} \in [0,1].$$

In the case of an anti-incumbent commercial media,

$$\pi_a^C \mid_{x=b} = \frac{\frac{1}{2}\pi(1-\eta_C)}{\frac{1}{2}\pi(1-\eta_C) + \frac{1}{2}(1-\pi)(1-\eta_C)} \quad \text{for } \eta_C \in [0,1);$$

$$\pi_b^C \mid_{x=b} = \frac{\frac{1}{2}(\pi\eta_C + 1 - \pi)}{\frac{1}{2}(\pi\eta_C + 1 - \pi) + \frac{1}{2}\{(1-\pi)\eta_C + \pi\}} \quad \text{for } \eta_C \in [0,1].$$

In the case of a pro-incumbent commercial media,

$$\pi_a^C \mid_{x=b} = \frac{\frac{1}{2} \{ \pi + (1-\pi)\eta_C \}}{\frac{1}{2} \{ \pi + (1-\pi)\eta_C \} + \frac{1}{2} \{ (1-\pi) + \pi\eta_C \}} \text{ for } \eta_C \in [0,1];$$

$$\pi_b^C \mid_{x=b} = \frac{\frac{1}{2} (1-\pi)(1-\eta_C)}{\frac{1}{2} (1-\pi)(1-\eta_C) + \frac{1}{2} \pi(1-\eta_C)} \text{ for } \eta_C \in [0,1).$$

A simple calculation reveals that  $\pi_a^P \mid_{x=b} \geq \frac{1}{2}$  and  $\pi_b^P \mid_{x=b} < \frac{1}{2}$ . In the case of an anti-incumbent commercial media,  $\pi_a^C \mid_{x=b} > \frac{1}{2}$  and  $\pi_b^C \mid_{x=b} \leq \frac{1}{2}$ , and in the case of a pro-incumbent commercial media,  $\pi_a^C \mid_{x=b} \geq \frac{1}{2}$  and  $\pi_b^C \mid_{x=b} < \frac{1}{2}$ . For simplicity, we have assumed that if the expected profit given by the incumbent and that given by the challenger are the same, the voter who observed message a (message b) votes for the incumbent (the challenger). Thus, we see that the voter reelects the incumbent if he observes message a and elects the challenger if he observes message b.

Next, we consider the voter's election choice when he believes that the dissonant incumbent chooses x = a. In this case, we obtain

$$\pi_a^P \mid_{x=a} = \pi_b^P \mid_{x=a} = \pi_a^C \mid_{x=a} = \pi_b^C \mid_{x=a} = \frac{1}{2}$$

Thus, we see that irrespective of the voter's belief regarding the incumbent's choice x, the voter reelects the incumbent when observing message a and elects the challenger when observing message b.

Further, given the voter's strategy, we consider the dissonant incumbent's equilibrium choice of x. We denote the dissonant incumbent's reelection probability when she chooses x = a (x = b) as  $\rho_a^D$  ( $\rho_b^D$ ). When the commercial media is anti-incumbent, we obtain the following equation:

$$\rho_a^D = \left\{ \frac{1}{2} + (\eta_C - \eta_P) f \right\} \left\{ \pi + (1 - \pi) \eta_P \right\} + \left\{ \frac{1}{2} - (\eta_C - \eta_P) f \right\} \pi (1 - \eta_C). \tag{6}$$

Note that  $\pi + (1 - \pi)\eta_P (\pi(1 - \eta_C))$  is the probability that the voter who views the PSB (the commercial media) observes signal a. Similarly, we derive the following equation:

$$\rho_b^D = \left\{ \frac{1}{2} + (\eta_C - \eta_P) f \right\} (1 - \pi + \pi \eta_P) + \left\{ \frac{1}{2} - (\eta_C - \eta_P) f \right\} (1 - \pi) (1 - \eta_C). \tag{7}$$

When the commercial media is pro-incumbent, we derive the following equations:

$$\rho_a^D = \left\{ \frac{1}{2} + (\eta_C - \eta_P) f \right\} \left\{ \pi + (1 - \pi) \eta_P \right\} + \left\{ \frac{1}{2} - (\eta_C - \eta_P) f \right\} \left\{ \pi + (1 - \pi) \eta_C \right\}$$
 (8)

and

$$\rho_b^D = \left\{ \frac{1}{2} + (\eta_C - \eta_P) f \right\} (1 - \pi + \pi \eta_P) + \left\{ \frac{1}{2} - (\eta_C - \eta_P) f \right\} (1 - \pi + \pi \eta_C). \tag{9}$$

We have assumed that the incumbent gains benefit 1 if she is reelected and gains 0 if she is defeated. Then, when she chooses x=a, the dissonant incumbent's expected payoff becomes  $\rho_a^D-D$ , where D is the emotional damage from not being able to take her preferred action, and when she chooses x=b, it becomes  $\rho_b^D$ . Thus, when  $\rho_a^D-D \geq \rho_b^D$ , the dissonant incumbent selects x=a; when  $\rho_a^D-D < \rho_b^D$ , she selects x=b. In the case of both anti- and pro-incumbent commercial media, inequality  $\rho_a^D-D \geq \rho_b^D$  is rearranged by  $D \leq (2\pi-1)[\{\frac{1}{2}+(\eta_C-\eta_P)f\}(1-\eta_P)+\{\frac{1}{2}-(\eta_C-\eta_P)f\}(1-\eta_C)]$ . Thus, we obtain the statements of this proposition.

#### **Proof of Proposition 2**

First, we consider the voter's choice. Then, we obtain the same  $\pi_a^C \mid_{x=a}$ ,  $\pi_b^C \mid_{x=a}$ ,  $\pi_a^C \mid_{x=b}$ , and  $\pi_b^C \mid_{x=b}$  as those in the proof of Proposition 1. Thus, we obtain result 1.

Next, given the voter's strategy, we consider the dissonant incumbent's equilibrium choice of x. When the commercial media is anti-incumbent, the dissonant incumbent's reelection probability when she chooses x=a (x=b) becomes  $\rho_a^D=\pi(1-\eta_c)$  ( $\rho_b^D=(1-\pi)(1-\eta_c)$ ). When the commercial media is pro-incumbent,  $\rho_a^D=\pi+(1-\pi)\eta_C$  and  $\rho_b^D=1-\pi+\pi\eta_C$ . When  $\rho_a^D-D\geq\rho_b^D$ , the dissonant incumbent selects x=a. In the case of both anti- and pro-incumbent commercial media, inequality  $\rho_a^D-D\geq\rho_b^D$  is rearranged by  $\eta_C\leq 1-\frac{D}{2\pi-1}$ . Thus, we obtain the statements of this proposition.

#### **Proof of Proposition 3**

First, we derive equations (4) and (5). We denote the congruent incumbent's reelection probability as  $\rho^C$ . Note that  $\rho^C = \rho_a^D \neq \rho_b^D$ , where  $\rho_a^D$  and  $\rho_b^D$  are defined in the proof of Proposition 1. Then, we can obtain  $\eta_P^*|_{x=a}$  ( $\eta_P^*|_{x=b}$ ) by calculating  $\arg\max_{\eta_P}\frac{1}{2}\rho^C+\frac{1}{2}(\rho_a^D-\frac{1}{2}D)$  ( $\arg\max_{\eta_P}\frac{1}{2}\rho^C+\frac{1}{2}\rho_b^D$ ). Using equations (6) and (7), we obtain equations (4) and (5).

Next, we prove the following lemma.

**Lemma 1.** When the commercial media is anti-incumbent and given bias  $\eta_C$ , only the following situations exist regarding the pair of optimal PSB bias  $\eta_P^*$  and the dissonant incumbent's equilibrium choice  $x^*$ .

- 1. When both  $\eta_P^*|_{x=a}$  and  $\eta_P^*|_{x=b}$  satisfy inequality (1) (inequality (2)),  $(\eta_P^*, x^*) = (\eta_P^*|_{x=a}, a) ((\eta_P^*, x^*) = (\eta_P^*|_{x=b}, b))$ .
- 2. When  $\eta_P^*|_{x=a}$  satisfies inequality (1) and  $\eta_P^*|_{x=b}$  satisfies inequality (2),  $(\eta_P^*, x^*) = (\eta_P^*|_{x=a}, a)$  ( $(\eta_P^*, x^*) = (\eta_P^*|_{x=b}, b)$ ) if the incumbent's expected payoff when  $(\eta_P, x) = (\eta_P^*|_{x=a}, a)$  is greater than (is smaller than or equal to) that when  $(\eta_P, x) = (\eta_P^*|_{x=b}, b)$ .

**Proof:** First, we derive a contradiction by assuming that a case exists where  $\eta_P^*|_{x=a}$  satisfies inequality (2) and  $\eta_P^*|_{x=b}$  satisfies inequality (1). In this case,  $\eta_P^*|_{x=b} \neq \eta_P^*|_{x=a}$ ; then,  $\eta_P^*|_{x=b} > \eta_P^*|_{x=a}$ . Now, under the condition of x=b (x=a) and given  $\eta_C$ , the incumbent's expected payoff function of  $\eta_P$  is quadratic, concave downward, and

highest at  $\eta_P = \eta_P^*|_{x=b}$  ( $\eta_P = \eta_P^*|_{x=a}$ ). However, in this case, as  $\eta_P^*|_{x=a}$  ( $\eta_P^*|_{x=b}$ ) satisfies inequality (2) (inequality (1)), within the range that satisfies inequality (2) (inequality (1)), the choice of x = b (x = a) leads to a greater expected payoff than that of x = a (x = b), which is a contradiction.

Results 1 and 2 are easily derived because under the condition of x = b (x = a), the incumbent's expected payoff function of  $\eta_P$  is quadratic, concave downward, and highest at  $\eta_P = \eta_P^*|_{x=b}$  ( $\eta_P = \eta_P^*|_{x=a}$ ), and because within the range that satisfies inequality (1) (inequality (2)), the choice of x = a (x = b) leads to a greater expected payoff than that of x = b (x = a).

We see from Lemma 1 that given  $\eta_C$ , the candidates of  $(\eta_P^*, x^*)$  are  $(\eta_P^*|_{x=a}, a)$  and  $(\eta_P^*|_{x=b}, b)$ . We can confirm the existence of Cases 1–4 in Proposition 3 through the numerical examples presented in this study. Thus, proving that neither the case in which  $(\eta_P^*, x^*)$  changes from  $(\eta_P^*|_{x=b}, b)$  to  $(\eta_P^*|_{x=a}, a)$  nor the case in which  $(\eta_P^*, x^*)$  changes from  $(\eta_P^*|_{x=a}, a)$  to  $(\eta_P^*|_{x=b}, b)$ , to  $(\eta_P^*|_{x=a}, a)$ , and to  $(\eta_P^*|_{x=b}, b)$  occurs, we obtain the statements of this proposition.

First, we consider the case of  $\frac{1}{4f} < 1$ . In this case,  $\eta_P^*|_{x=a} < 1$  and  $\eta_P^*|_{x=b} < 1$  always hold. Given  $\eta_C$ , when we represent the incumbent's expected payoff  $\frac{1}{2}\rho^C + \frac{1}{2}\rho_D^D$  when  $(\eta_P, x) = (\frac{1}{4f}, b)$  by function  $f(\eta_C)$ , we obtain the following:

$$\begin{split} f(\eta_C) &\equiv \frac{1}{2} \left\{ \frac{1}{2} + \left( \eta_C - \frac{1}{4f} \right) f \right\} \left\{ \pi + (1 - \pi) \frac{1}{4f} \right\} + \frac{1}{2} \left\{ \frac{1}{2} - \left( \eta_C - \frac{1}{4f} \right) f \right\} \pi (1 - \eta_C) \\ &\quad + \frac{1}{2} \left\{ \frac{1}{2} + \left( \eta_C - \frac{1}{4f} \right) f \right\} \left( 1 - \pi + \pi \frac{1}{4f} \right) + \frac{1}{2} \left\{ \frac{1}{2} - \left( \eta_C - \frac{1}{4f} \right) f \right\} (1 - \pi) (1 - \eta_C). \end{split}$$

Given  $\eta_C$ , when we represent the incumbent's expected payoff  $\frac{1}{2}\rho^C + \frac{1}{2}(\rho_a^D - D)$  when  $(\eta_P, x) = (-\frac{2\pi - 1}{2 - 2\pi}\eta_C + \frac{1}{4f}, b)$  by function  $g(\eta_C)$ , we obtain the following:

$$\begin{split} g(\eta_C) &\equiv \Big\{ \frac{1}{2} + \Big( \eta_C + \frac{2\pi - 1}{2 - 2\pi} \eta_C - \frac{1}{4f} \Big) f \Big\} \Big\{ \pi + (1 - \pi) \left( -\frac{2\pi - 1}{2 - 2\pi} \eta_C + \frac{1}{4f} \right) \Big\} \\ &\quad + \Big\{ \frac{1}{2} - \Big( \eta_C + \frac{2\pi - 1}{2 - 2\pi} \eta_C - \frac{1}{4f} \Big) f \Big\} \pi (1 - \eta_C) - \frac{1}{2} D. \end{split}$$

Given  $\eta_C$ , when we represent  $\frac{1}{2}\rho^C + \frac{1}{2}(\rho_a^D - D)$  when  $(\eta_P, x) = (0, a)$  by function  $h(\eta_C)$ , we obtain the following:

$$h(\eta_C) \equiv \left(\frac{1}{2} + \eta_C f\right) \pi + \left(\frac{1}{2} - \eta_C f\right) \pi (1 - \eta_C) - \frac{1}{2} D.$$

All functions  $f(\eta_C)$ ,  $g(\eta_C)$ , and  $h(\eta_C)$  are quadratic and concave upward in  $\eta_C$ . Since the coefficient of  $(\eta_C)^2$  of function  $f(\eta_C)$  is  $\frac{f}{2}$ , that of  $h(\eta_C)$  is  $f\pi$ , that of  $g(\eta_C)$  is  $\frac{f}{4(1-\pi)}$ , and  $\frac{f}{2} < f\pi < \frac{f}{4(1-\pi)}$ , the order of the width of the parabola of the three functions is  $f(\eta_C)$ ,  $h(\eta_C)$ , and  $g(\eta_C)$ . In addition, a simple calculation also yields  $g(\eta_C) \ge h(\eta_C)$ , where the two functions touch each other and  $g(\eta_C) = h(\eta_C)$  at  $\eta_C$  that satisfies  $\eta_C = \frac{2-2\pi}{2\pi-1} \frac{1}{4f}$ . Moreover, the axis of functions  $f(\eta_C)$  and  $h(\eta_C)$  is the same, which is  $\eta_C = \frac{1}{4f}$  and is greater than  $\frac{1}{2}$  because  $f < \frac{1}{2}$ .

Here, by supposing that there exists a case where the pair of  $(\eta_P^*, x^*)$  changes from  $(\eta_P^*|_{x=b}, b)$  to  $(\eta_P^*|_{x=a}, a)$ , we derive a contradiction. First, the axis of functions  $f(\eta_C)$  and  $h(\eta_C)$  is the same  $\frac{1}{4f}$  and greater than  $\frac{1}{2}$ . When the pair of  $(\eta_P^*, x^*)$  changes from  $(\eta_P^*|_{x=b}, b)$  to  $(\eta_P^*|_{x=a}, a)$ ,  $g(\eta_C)$  or  $h(\eta_C)$  surpasses  $f(\eta_C)$  at the  $\eta_C$  that is greater than  $\frac{1}{4f}(>\frac{1}{2})$ . Thus,  $f(\eta_C)$  also surpasses  $g(\eta_C)$  or  $h(\eta_C)$  with a smaller  $\eta_C$ , which means that the pair of  $(\eta_P^*, x^*)$  changes from  $(\eta_P^*|_{x=a}, a)$  to  $(\eta_P^*|_{x=b}, b)$  and back to  $(\eta_P^*|_{x=a}, a)$ , which is a contradiction. By similarly supposing that a case exists where the pair of

 $(\eta_P^*, x^*)$  changes from  $(\eta_P^*|_{x=a}, a)$  to  $(\eta_P^*|_{x=b}, b)$ , to  $(\eta_P^*|_{x=a}, a)$ , and to  $(\eta_P^*|_{x=b}, b)$  and deriving a contradiction, we can prove that this case also does not occur.

Next, we consider the case of  $\frac{1}{4f} \ge 1$ . When we newly represent the incumbent's expected payoff  $\frac{1}{2}\rho^C + \frac{1}{2}\rho_b^D$  ( $\frac{1}{2}\rho^C + \frac{1}{2}(\rho_a^D - D)$ ) at which  $(\eta_P, x) = (1, b)$  ( $(\eta_P, x) = (1, a)$ ) given  $\eta_C$  by function  $i(\eta_C)$  (function  $j(\eta_C)$ ) and compare the four functions  $g(\eta_C)$ ,  $h(\eta_C)$ ,  $i(\eta_C)$ , and  $j(\eta_C)$ , we can prove the results similarly as in the case of  $\frac{1}{4f} < 1$ . Thus, we obtain the statements of this proposition.

#### **Proof of Proposition 4**

First, we can easily obtain  $\eta_P^*|_{x=a}$  and  $\eta_P^*|_{x=b}$  by using the proof of Proposition 3 and equations (8) and (9).

Next, this proposition's statements are easily derived from the shape of boundary line (3) and the fact that  $\eta_P^* = \eta_C + \frac{1}{4f}$  if  $\eta_C + \frac{1}{4f} \le 1$ ; otherwise,  $\eta_P^* = 1$ .

#### **Proof of Proposition 5**

We denote expected period 1 welfare calculated at the beginning of the game, especially when the dissonant incumbent chooses x = a (x = b) and the voter acts optimally, as  $W_1|_{x=a}$  ( $W_1|_{x=b}$ ). Further, we denote expected period 2 welfare calculated at the beginning of the game, especially when the dissonant incumbent chooses x = a (x = b) and the voter acts optimally, as  $W_2|_{x=a}$   $(W_2|_{x=b})$ . Then, the expected period 1 welfare values at the beginning of the game, which coincide with the expected probability of the period 1 incumbent choosing action a, become  $W_1|_{x=a}=1$  and  $W_1|_{x=b}=\frac{1}{2}$ . Conversely, when Nature determines the incumbent's type as congruent, the expected probability of the period 2 politician choosing action a becomes  $\rho^{c} + (1 - \rho^{c})\frac{1}{2} = \frac{1}{2} + \frac{1}{2}\rho^{c}$ ; when Nature determines the incumbent's type as dissonant and x = a (x = b), the expected probability of the period 2 politician choosing action a becomes  $\rho_a^D \cdot 0 + (1 - \rho_a^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_a^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_a^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_a^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_a^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_a^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_a^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_a^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_a^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_a^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_a^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0 + (1 - \rho_b^D) \frac{1}{2} = \frac{1}{2} - \frac{1}{2} \rho_b^D (\rho_b^D \cdot 0$  $\frac{1}{2}\rho_b^D$ ). Thus, by a simple calculation, the expected period 2 welfare values at the beginning of the game become  $W_2|_{x=a} = \frac{1}{2}(\frac{1}{2} + \frac{1}{2}\rho^C) + \frac{1}{2}(\frac{1}{2} - \frac{1}{2}\rho^D_a) = \frac{1}{2}$  and  $W_2|_{x=b} = \frac{1}{2}(\frac{1}{2} + \frac{1}{2}\rho^C) + \frac{1}{2}(\frac{1}{2} - \frac{1}{2}\rho^D_a) = \frac{1}{2}$  $\frac{1}{2}(\frac{1}{2}+\frac{1}{2}\rho^C)+\frac{1}{2}(\frac{1}{2}-\frac{1}{2}\rho^D_b)=\frac{1}{2}+\frac{1}{4}(\rho^C-\rho^D_b)$ . Then, a simple calculation yields results 1 and 2 of this proposition.

#### **Proof of Corollary 1**

Regarding situation  $CP^*$ , in the range of  $\eta_C$  where  $x^*=a$  holds (this range always contains the range of  $\eta_C \leq 1-\frac{D}{2\pi-1}$ ), Proposition 5 indicates that social welfare is given by  $SW^*|_{x=a}=\frac{3}{2}$ . In the range of  $\eta_C$  where  $x^*=b$  holds (if the range exists), Proposition 5 indicates that social welfare is given by  $SW^*|_{x=b}=1+\frac{1}{2}(\pi-\frac{1}{2})\{1-\frac{1}{2}\eta_C+(\eta_C)^2f\}$ . Conversely, when we consider situation C, Propositions 2 and 5 indicate that when  $\eta_C \leq 1-\frac{D}{2\pi-1}$ ,  $SW^C=SW^C|_{x=a}=\frac{3}{2}$ ; when  $\eta_C>1-\frac{D}{2\pi-1}$ ,  $SW^C=SW^C|_{x=b}=1+\frac{1}{2}(\pi-\frac{1}{2})(1-\eta_C)$ . Thus, we obtain the statements of this corollary.  $\square$ 

#### Proof of Corollary 2

Results 1 and 2 (a) are obtained from Propositions 2, 3, and 5. We now prove results 2(b) and 2(c). In these cases, since in both situations C and  $CP^{**}$ ,  $x^* = b$  holds, we see from Proposition 5 that if  $1 - \frac{1}{2}(\eta_C + \eta_P^*) + (\eta_C - \eta_P^*)^2 f \gtrless 1 - \eta_C$ ,  $SW^{**} \gtrless SW^C$  holds. A simple calculation yields that if  $\eta_C - \eta_P^* + 2(\eta_C - \eta_P^*)^2 f \gtrless 0$ ,  $SW^{**} \gtrless SW^C$  holds. Thus, it immediately follows that if  $\eta_P^* = \eta_C$ ,  $SW^{**} = SW^C$ , and if  $\eta_P^* < \eta_C$ ,  $SW^{**} > SW^C$ . To consider the case of  $\eta_P^* > \eta_C$ , we examine the  $\eta_P^*$  that is obtained as  $\eta_P^* = \eta_C + \alpha$  ( $\alpha \in (0,1]$ ). Then, if  $\alpha$  satisfies  $\eta_C - (\eta_C + \alpha) + 2\{\eta_C - (\eta_C + \alpha)\}^2 f < 0 \iff \alpha < \frac{1}{2f}$ ),

**Journal of Economics and Political Economy**  $SW^{**} < SW^C$  holds. The assumption of  $f < \frac{1}{2}$  reveals that  $\alpha < \frac{1}{2f}$  is satisfied. Thus, we obtain the statements of this corollary.

#### **Endnotes**

- <sup>1</sup> Although fake news has a long history, the number of studies about fake news on social media has increased since the 2016 United States (U.S.) presidential election and the 2016 Brexit referendum (Del Vicario *et al.*, 2017; Lazer *et al.*, 2018). Silverman (2016) reports that in the U.S., during the 3 months preceding the 2016 presidential election, fake election news stories outperformed real news on Facebook.
- <sup>2</sup> Shearer & Mitchell (2021) show that a majority (59%) of Americans who get their news from social media said that they expect it to be largely inaccurate.
- <sup>3</sup> The exception is the U.S., where it is rare for people to name a public news outlet as their main source of news (Mitchell *et al.*, 2018b).
- 4 Previous studies have theoretically unraveled and empirically discovered the effect of media news on political accountability. Strömberg (2016) and Besley (2006) provide detailed reviews on this subject.
- <sup>5</sup> Marwick & Lewis (2017) show that one of the factors that influences the actors to create and spread disinformation, propaganda, and/or fake news is ideology. Today, the Internet makes it easy to spread one's ideology online, and the number of false and/or hyperpartisan news is increasing.
- <sup>6</sup> Although media firms can reveal or hide negative news about an incumbent (Besley & Prat, 2006), we allow them to report false news, as seen in Adachi & Hizen (2014).
- 7 While people dislike misinformation online, Mitchell, Grieco, & Sumida (2018) show that the majority of Americans resist U.S. government actions that might limit their freedom. Thus, our study does not consider restrictions on commercial media bias.
- <sup>8</sup> Silverman (2016) reveals that in the U.S., during the 3 months preceding the 2016 presidential election, among the 20 top-performing fake election stories on Facebook, 17 were clearly pro-Donald Trump or anti-Hillary Clinton. Allcott & Gentzkow (2017) also show similar results.
- 9 Prat & Strömberg (2013) review the literature in detail.
- <sup>10</sup> The theoretical literature on media bias is reviewed by Gentzkow, Shapiro, & Stone (2016), and the empirical literature is reviewed by Puglisi & Snyder (2016).
- Prat (2016) theoretically reviews the literature regarding media capture, and Enikolopov & Petrova (2016) empirically review it.
- <sup>12</sup> Even if we consider a continuum of voters instead of a representative voter, by adding a random number of "noise voters" as presented in the study by Besley & Coate (2003), we obtain the same main result.
- <sup>13</sup> We obtain the same result even if *x* denotes the mixed strategy that is the probability distribution over the pure strategies of the incumbent.
- <sup>14</sup> As news in period 2 does not affect our results, we focus only on the news in period 1.
- <sup>15</sup> For simplicity, we assume that  $\pi$  is common between the two outlets.
- <sup>16</sup> When  $\eta_P$  is 0, the PSB is neutral.
- <sup>17</sup> When  $\eta_C$  is 0, the commercial media outlet is neutral.
- <sup>18</sup> Mitchell *et al.* (2018a) demonstrate that the public all over the world overwhelmingly agrees that the news media should be unbiased.
- <sup>19</sup> Note that Figure 1 describes the case of D=0.45, f=0.45, and  $\pi=0.8.$
- <sup>20</sup> After the PSB determined its bias  $\eta_P$ , it would take some time for the voter to recognize the level of PSB bias. Thus, we assume that the timing of the PSB bias selection occurs at the beginning of the game.
- <sup>21</sup> Armstrong & Weeds (2007) discuss the significance of public service broadcasting in the digital age.

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