

## A note on computing the standard errors of estimate of composite index

By Sudhanshu K. MISHRA <sup>†</sup>

**Abstract.** This short note proposes working out of the standard errors of estimate of composite indices when they are constructed by using intrinsically derived weights. It illustrates the proposed method, using the jackknife re-sampling technique, by an example that relates to crime of different types in Uttar Pradesh (India). Improvements are suggested through bootstrapping.

**Keywords.** Composite index, Standard error of estimate, Jackknife resampling, Crime data, Uttar Pradesh, India.

**JEL.** C43, C61, C71.

### 1. Introduction

A composite (or synthetic) index is  $Z=Xw$ , where  $Z$  is an array of  $n$  elements,  $X$  is an  $n \times m$  matrix (of  $m$  variables, called the indicator variables or the constituent variables, each one being an array of  $n$  values called replicates, cases or observations) and  $w$  is a row vector of  $m$  elements, often called weights. The weight vector may be extraneous, based on certain concept or criteria. Alternatively it may be determined on the basis of certain criteria pertaining to the properties of  $X$ . In any case, an index value is a weighted mean.

There could be many methods to obtain weights ( $w$ ) intrinsically from  $X$  (standardized to have zero mean and unit Std. deviation). For example, one may set up the criterion as  $C = \sum_{j=1}^m r^2(Z, x_j)$  or the sum of the squared values (squared Euclidean norm) of the coefficients of correlation between the composite index,  $Z$ , and the constituent variables,  $x_j$ , for all  $j$ .  $C$  is minimized. Composite indices using such weights are called Principal Component scores. This is the most popular method that has a history of over 50 years of its use. Instead of using the Euclidean norm, one may also use absolute or Chebyshev norm (Mishra, 2011). Weights based on minimization of the Shapley value norms that best equalize the mean expected marginal contribution of the constituent variables to the composite index ( $s(Z, x_j)$ ) or  $C = \sum_{j=1}^m s^2(Z, x_j)$  has also been proposed (Mishra, 2016; 2017).

### 2. A need to obtain the standard errors of estimate of composite index values

Were the values of a composite index used purely for descriptive purposes, it was not necessary to raise the issue of their standard errors of estimate. But in practice, composite indices (their values) are used for inferential purposes. They are compared very often and such comparisons are used for inference. This use necessitates obtaining their standard errors of estimate.

<sup>†</sup> North-Eastern Hill University, Shillong, 91 - 793022, New Delhi, India.

918130397754

mishrasknehu@hotmail.com

A mathematical method to obtain such standard errors of estimate may be based on many assumptions that might not be realistic with respect to the data being analysed. They may also be cumbersome to work out and use. This short note suggests a simple method based on jackknife resampling to resolve this problem. Jackknife resampling is a well-established and amply applied method to obtain standard error of estimate of statistically estimated parameters where deductive (mathematically derived) methods are either inapplicable or cumbersome (Efron, 1979, 1981; Wolter, 1985; Efron & Tibshirani, 1993; Shao & Tu, 1995).

### 3. Jackknife resampling method to obtain the standard errors of estimate

Jackknife is a re-sampling method that leaves one observation (case) at a time and thus constructs n sample indices by using the weights obtained from the samples. More elaborately, let  $Y_k(n-1, m)$  be a subset of  $X(n, m)$  such that it excludes the  $k^{\text{th}}$  case. This  $Y_k$  is used to obtain weight vector  $w_k(m)$  and using this weight the composite index  $Z_k = Xw_k$  is constructed. Since  $w_k$  is based on sample,  $Z_k$  inherits its sample nature. This is done for  $k=1,2,\dots,n$  and thus we have  $Z_k(n); k = 1,2,\dots,n$ . From these  $Z_{k=1,\dots,n}$  we may obtain mean and standard deviation such that  $\bar{Z}(n) = \sum_{k=1}^n Z_k(n)$  or stated more elaborately,  $\bar{Z}_i = (1/n) \sum_{k=1}^n Z_{ik}$  where  $i=1,2,\dots,n$  refer to cases and  $k$  refers to the sample of  $n-1$  size drawn from  $X$ . Similarly,  $s^2(Z_i) = (n-1)[(1/n) \sum_{k=1}^n Z_{ik}^2 - \bar{Z}_i^2]$ .

### 4. An illustrative example

By way of giving an example, we use the crime data for 68 districts of Uttar Pradesh (India), presented in Table 1. For purpose of analysis, all crime statistics for a particular district have been divided by population so that the crime rate per lakh population is obtained. From this set, 68 samples of 67 cases (leaving 1 out of  $n=68$ ) have been drawn and for them the sample correlation matrices are computed. For each correlation matrix, eigen values are computed and sample weight vectors are obtained. Those weights are used for computing 68 sample indices. Their mean and standard deviation are computed. The detailed results are presented in Table 2.

**Table 1. District-Wise Statistics for Major Crimes in Uttar Pradesh (India) for the Year 2014**

District	Murder	Rape	Kidnap	Robbery	Theft	Auto Theft	Riots	Crim Brch	Cheating	Griev Hurt	CrueltyHusb	Populn(Lakh)
Agra	178	77	463	218	3512	2824	413	99	382	317	389	36.11
Aligarh	179	112	471	242	2297	1660	436	129	354	285	455	29.90
Allahabad	132	109	330	125	2245	1673	129	124	511	353	355	49.42
Ambed. Ngr	24	20	59	13	86	39	37	19	43	126	41	20.25
Auraiya	34	21	99	14	165	119	20	28	78	7	151	11.79
Azamgarh	68	42	183	73	387	247	203	53	134	313	158	39.51
Badaun	120	71	164	53	331	165	9	27	68	323	62	30.69
Baghpat	83	35	121	45	332	184	56	23	61	9	90	11.64
Bahraich	53	68	202	14	258	126	73	39	93	169	0	23.84
Ballia	36	21	116	28	287	188	113	30	83	188	95	27.52
Balrampur	28	19	67	4	68	33	22	13	27	45	2	16.85
Banda	42	71	151	25	240	119	86	31	65	40	96	15.00
Barabanki	57	35	65	17	113	60	3	64	148	306	36	26.73
Bareilly	132	76	337	80	886	509	153	147	462	181	139	35.99
Basti	31	15	53	11	85	42	20	21	19	86	40	20.69
Bijnor	84	80	184	59	434	181	77	42	203	16	0	31.31
Bulandshr	171	82	291	122	841	483	158	65	17	17	0	29.23
Chandoli	20	26	78	27	156	88	39	39	152	207	113	16.40
Chitrakoot	33	23	46	6	81	45	12	29	40	16	12	8.01
Deoria	42	35	137	13	182	117	81	19	35	209	6	27.30
Etah	65	54	172	63	399	236	110	38	26	322	120	27.88
Etawah	55	30	136	113	506	385	26	54	119	100	104	13.40
Faizabad	32	37	120	25	295	156	28	48	138	208	8	20.88
Fatehpur	55	58	131	21	217	135	42	70	93	273	93	23.06
Firozabad	140	61	284	92	687	453	201	62	175	347	146	20.46
GautamB.Ngr	104	54	210	227	3680	2344	191	106	382	17	198	11.91
Ghaziabad	166	118	482	114	3392	2635	70	176	493	19	575	32.90

## Journal of Economic and Social Thought

Ghazipur	64	24	128	32	260	160	113	36	93	273	67	30.49
Gonda	45	37	139	18	180	117	48	36	135	237	12	27.66
Gorakhpur	113	92	299	89	804	594	193	87	266	363	223	37.85
Hamirpur	31	29	68	18	83	34	34	21	31	52	42	10.42
Hardoi	79	39	177	13	275	145	36	45	95	418	140	33.97
Hathras	72	60	146	56	339	232	112	50	89	101	103	13.33
Jalaun	36	12	64	12	117	61	11	20	42	11	41	14.56
Jaunpur	55	50	133	58	297	191	107	38	70	416	49	39.11
Jhansi	68	28	112	22	316	192	65	32	17	20	119	17.47
Kannauj	39	22	100	8	147	69	10	31	74	2	89	13.85
Kanpur Deh	46	20	157	24	121	58	40	24	68	140	144	15.84
Kanpur Ngr	137	78	442	139	1536	1136	233	189	546	238	594	41.37
Kaushambi	40	14	92	13	154	82	23	34	74	123	86	12.95
Khiri	79	49	214	33	611	423	68	38	118	281	135	32.00
Kushi Ngr	36	22	141	7	136	76	119	38	63	261	50	28.92
Lalitpur	27	25	34	15	59	22	26	11	42	10	19	9.77
Lucknow	132	68	377	60	3192	2086	174	511	923	34	1083	36.81
Maharajngj	33	32	103	14	61	41	41	15	52	150	68	21.67
Mahoba	27	22	58	18	57	27	26	18	30	3	22	7.09
Mainpuri	57	23	142	34	476	367	85	38	75	171	74	15.93
Mathura	86	73	191	89	1159	846	190	70	267	245	267	20.70
Mau	28	25	88	23	226	120	93	25	101	227	81	18.49
Meerut	226	93	438	341	3550	2635	294	109	431	24	520	30.02
Mirzapur	25	19	12	5	45	25	1	22	0	89	28	21.15
Moradabad	90	89	190	66	788	507	78	59	221	51	403	37.50
Muzaffarngr	137	37	242	95	800	469	136	66	145	13	168	35.42
Pilibhit	54	69	128	29	200	108	13	42	123	2	179	16.44
Pratapgarh	72	68	248	53	431	262	200	71	185	306	184	27.27
Raibareilly	50	48	113	15	195	93	60	25	67	144	38	28.72
Rampur	54	34	100	25	344	149	34	59	122	0	121	19.22
Saharanpur	92	73	273	110	793	459	205	48	214	20	194	28.48
St. Kabirngr	28	23	39	10	59	25	22	14	30	146	0	14.25
Shahjahanpur	88	83	210	62	281	122	42	47	118	282	127	25.49
Shrawasti	15	5	40	5	49	17	16	5	65	165	0	11.75
Sidharthngr	17	20	58	16	62	34	25	13	15	1	46	20.39
Sitapur	91	74	216	40	384	179	65	49	78	125	180	36.17
Sonbhadra	32	26	20	2	77	34	3	22	8	152	13	14.63
St. Ravidasngr	17	19	34	8	73	47	1	15	28	55	41	13.52
Sultanpur	53	46	157	56	331	179	73	33	88	206	68	31.91
Unnao	77	69	268	44	334	141	46	53	135	35	218	27.00
Varanasi	52	47	166	46	706	467	79	154	393	209	275	31.48

**Note:** Data for a few districts were not available. Those districts have been not included in the analysis.

**Table 2. Estimated composite Indices, Standard Deviation and Intervals with them**

District	$\bar{Z}$	$\bar{Z} - 3s$	$\bar{Z} - 2s$	$\bar{Z} - s$	$\bar{Z}$	$\bar{Z} + s$	$\bar{Z} + 2s$	$\bar{Z} + 3s$	s
Agra	10.817	9.683	10.061	10.439	10.817	11.196	11.574	11.952	0.3781
Aligarh	14.886	12.814	13.505	14.196	14.887	15.577	16.268	16.959	0.6909
Allahabad	1.723	1.555	1.611	1.667	1.724	1.780	1.836	1.892	0.0561
Ambed. Ngr	-6.203	-7.107	-6.806	-6.505	-6.204	-5.903	-5.602	-5.301	0.3009
Auraiya	0.350	-0.193	-0.011	0.170	0.351	0.532	0.713	0.895	0.1812
Azamgarh	-3.399	-3.964	-3.776	-3.588	-3.400	-3.212	-3.024	-2.835	0.1881
Badaun	-3.280	-3.898	-3.692	-3.486	-3.280	-3.074	-2.868	-2.662	0.2060
Baghpat	4.840	3.897	4.212	4.526	4.840	5.154	5.468	5.782	0.3141
Bahraich	-2.244	-2.526	-2.432	-2.338	-2.244	-2.150	-2.056	-1.962	0.0940
Ballia	-4.596	-5.365	-5.109	-4.853	-4.597	-4.340	-4.084	-3.828	0.2562
Balrampur	-6.127	-7.044	-6.739	-6.434	-6.128	-5.823	-5.517	-5.212	0.3054
Banda	2.306	1.562	1.810	2.058	2.306	2.554	2.802	3.050	0.2479
Barabanki	-5.244	-6.040	-5.775	-5.510	-5.244	-4.979	-4.714	-4.449	0.2652
Bareilly	2.706	2.037	2.260	2.483	2.706	2.929	3.153	3.376	0.2232
Basti	-6.644	-7.633	-7.303	-6.974	-6.644	-6.315	-5.985	-5.656	0.3295
Bijnor	-1.921	-2.536	-2.332	-2.127	-1.922	-1.717	-1.512	-1.307	0.2049
Bulandshr	2.398	1.545	1.829	2.113	2.397	2.682	2.966	3.250	0.2843
Chandoli	-2.470	-3.194	-2.953	-2.711	-2.470	-2.229	-1.988	-1.747	0.2410
Chitrakoot	-0.877	-1.396	-1.223	-1.050	-0.877	-0.704	-0.531	-0.358	0.1731
Deoria	-5.495	-6.350	-6.065	-5.781	-5.496	-5.211	-4.927	-4.642	0.2847
Etah	-2.620	-3.300	-3.073	-2.846	-2.620	-2.393	-2.167	-1.940	0.2266
Etawah	5.128	4.270	4.556	4.842	5.128	5.414	5.701	5.987	0.2862
Faizabad	-3.480	-4.019	-3.839	-3.660	-3.480	-3.300	-3.121	-2.941	0.1797
Fatehpur	-2.358	-2.979	-2.772	-2.565	-2.357	-2.150	-1.942	-1.735	0.2074
Firozabad	7.461	6.099	6.553	7.008	7.462	7.916	8.370	8.825	0.4543
Gautamb.Ngr	36.333	31.376	33.029	34.682	36.334	37.987	39.640	41.293	1.6528
Ghaziabad	12.491	11.014	11.507	12.000	12.493	12.987	13.480	13.973	0.4932
Ghazipur	-4.597	-5.368	-5.111	-4.854	-4.597	-4.340	-4.083	-3.826	0.2570
Gonda	-4.849	-5.564	-5.326	-5.088	-4.850	-4.612	-4.373	-4.135	0.2381
Gorakhpur	0.875	0.540	0.651	0.763	0.875	0.987	1.099	1.210	0.1118
Hamirpur	-1.429	-1.584	-1.532	-1.481	-1.429	-1.378	-1.326	-1.275	0.0515
Hardoi	-4.738	-5.791	-5.440	-5.089	-4.738	-4.387	-4.036	-3.684	0.3510
Hathras	6.696	5.352	5.800	6.248	6.697	7.145	7.593	8.042	0.4483
Jalaun	-4.672	-5.510	-5.231	-4.952	-4.672	-4.393	-4.113	-3.834	0.2794
Jaunpur	-5.468	-6.356	-6.060	-5.764	-5.468	-5.172	-4.876	-4.580	0.2961
Jhansi	-1.154	-1.655	-1.488	-1.321	-1.154	-0.987	-0.820	-0.653	0.1670
Kannauj	-1.969	-2.582	-2.377	-2.173	-1.969	-1.764	-1.560	-1.355	0.2044
Kanpur Deh	-1.273	-1.930	-1.711	-1.492	-1.273	-1.054	-0.835	-0.616	0.2190
Kanpur Ngr	5.872	5.105	5.361	5.617	5.873	6.129	6.385	6.641	0.2561
Kaushambi	-1.904	-2.480	-2.288	-2.096	-1.903	-1.711	-1.519	-1.327	0.1922
Khiri	-2.775	-3.425	-3.208	-2.992	-2.775	-2.559	-2.342	-2.125	0.2166
Kushi Ngr	-5.366	-6.257	-5.960	-5.663	-5.366	-5.069	-4.772	-4.475	0.2971
Lalitpur	-3.004	-3.559	-3.374	-3.190	-3.005	-2.820	-2.635	-2.451	0.1847
Lucknow	15.151	12.485	13.375	14.266	15.156	16.046	16.937	17.827	0.8903
Maharajngj	-5.166	-5.987	-5.713	-5.440	-5.166	-4.892	-4.619	-4.345	0.2737
Mahoba	0.488	-0.186	0.039	0.263	0.488	0.712	0.937	1.162	0.2246
Mainpuri	0.604	0.179	0.321	0.463	0.605	0.747	0.889	1.031	0.1420
Mathura	8.065	6.846	7.253	7.659	8.066	8.473	8.879	9.286	0.4066

## Journal of Economic and Social Thought

Mau	-3.229	-4.001	-3.744	-3.486	-3.229	-2.972	-2.715	-2.457	0.2573
Meerut	17.570	15.327	16.075	16.823	17.571	18.318	19.066	19.814	0.7479
Mirzapur	-7.819	-8.890	-8.533	-8.177	-7.820	-7.463	-7.106	-6.749	0.3569
Moradabad	-0.571	-1.038	-0.882	-0.727	-0.571	-0.416	-0.261	-0.105	0.1554
Muzaffarngr	-0.542	-1.104	-0.917	-0.730	-0.542	-0.355	-0.167	0.020	0.1874
Pilibhit	1.784	1.020	1.275	1.530	1.785	2.040	2.295	2.550	0.2549
Pratapgarh	1.288	0.755	0.933	1.110	1.288	1.466	1.643	1.821	0.1776
Raibareilly	-5.115	-5.815	-5.582	-5.349	-5.116	-4.882	-4.649	-4.416	0.2332
Rampur	-1.152	-1.669	-1.497	-1.324	-1.152	-0.979	-0.807	-0.634	0.1725
Saharanpur	3.040	2.282	2.535	2.787	3.039	3.291	3.544	3.796	0.2523
St. Kabirngr	-5.947	-6.797	-6.514	-6.231	-5.948	-5.664	-5.381	-5.098	0.2833
Shahjahanpur	-0.422	-0.896	-0.738	-0.580	-0.422	-0.264	-0.106	0.052	0.1580
Shrawasti	-6.900	-8.234	-7.789	-7.345	-6.901	-6.457	-6.012	-5.568	0.4443
Sidharthngr	-6.580	-7.650	-7.294	-6.937	-6.581	-6.225	-5.869	-5.513	0.3562
Sitapur	-2.928	-3.435	-3.266	-3.097	-2.928	-2.759	-2.590	-2.421	0.1691
Sonbhadra	-6.465	-7.375	-7.071	-6.768	-6.465	-6.162	-5.858	-5.555	0.3033
St.Ravidasngr	-6.058	-6.926	-6.637	-6.348	-6.059	-5.770	-5.480	-5.191	0.2892
Sultanpur	-4.285	-4.918	-4.707	-4.496	-4.286	-4.075	-3.864	-3.653	0.2108
Unnao	0.032	-0.421	-0.270	-0.119	0.032	0.183	0.334	0.485	0.1511
Varanasi	0.427	-0.009	0.136	0.282	0.427	0.573	0.718	0.864	0.1454

$Z_c$  =PC Index;  $\bar{Z}$  = Jackknife Mean;  $S$  =Jackknife Std. Deviation;  $\bar{Z} \pm tS$  =ts deviation from from Jackknife mean

**Table 3. Rank Score of UP Districts According to Composite Index and Standard Deviation Intervals**

SL	Distriet	3R	2R	1R	R	R1	R2	R3	SL	Distriet	3R	2R	1R	R	R1	R2	R3
1	Agra	63	63	63	63	63	63	63	35	Jaunpur	10	11	11	11	11	11	11
2	Ahlgarh	66	66	65	65	65	65	65	36	Jhansi	38	39	39	38	38	38	37
3	Allahabad	53	52	52	51	51	51	51	37	Kannauj	32	33	33	33	33	33	33
4	Ambed. Ngr	6	6	6	6	6	6	6	38	Kanpur Deh	36	36	36	37	37	37	39
5	Auraiya	45	45	45	45	45	45	46	39	Kanpur Ngr	59	59	59	59	59	59	59
6	Azamgarh	24	23	23	23	23	23	23	40	Kaushambi	35	35	35	35	35	34	34
7	Badaun	25	25	24	24	24	24	24	41	Khiri	28	28	28	28	28	28	28
8	Baghpat	57	57	57	57	57	57	57	42	Kushi Ngr	12	12	12	12	12	12	12
9	Bahraich	34	32	32	31	30	29	43	43	Lalitpur	26	26	26	26	26	26	26
10	Ballia	20	20	20	19	18	18	18	44	Lucknow	65	65	66	66	66	66	66
11	Balrampur	7	7	7	7	7	7	7	45	Maharajgni	14	14	14	14	14	15	15
12	Banda	54	53	53	53	53	53	53	46	Mahoba	46	46	46	47	47	48	48
13	Barabanki	13	13	13	13	13	13	13	47	Mainpuri	48	48	48	48	48	47	47
14	Bareilly	55	55	55	55	55	55	55	48	Mathura	62	62	62	62	62	62	62
15	Basti	4	3	3	3	3	3	2	49	Mau	23	24	25	25	25	25	25
16	Bijnor	33	34	34	34	34	35	35	50	Meerut	67	67	67	67	67	67	67
17	Bulandshr	52	54	54	54	54	54	54	51	Mirzapur	1	1	1	1	1	1	1
18	Chandoli	30	30	30	30	31	31	31	52	Moradabad	42	42	42	41	41	41	41
19	Chitrakoot	40	40	40	40	40	40	40	53	Muzaffarngr	41	41	41	42	42	42	42
20	Deoria	11	10	10	10	10	10	10	54	Pilibhit	51	51	51	52	52	52	52
21	Etah	29	29	29	29	29	29	30	55	Pratapgarh	50	50	50	50	50	50	50
22	Etawah	58	58	58	58	58	58	58	56	Raibareilly	15	15	15	15	15	14	14
23	Faizabad	22	22	22	22	22	22	22	57	Rampur	37	38	38	39	39	39	38
24	Fatehpur	31	31	31	31	32	32	32	58	Saharanpur	56	56	56	56	56	56	56
25	Firozabad	61	61	61	61	61	61	61	59	St. Kabirngr	9	9	9	9	9	9	9
26	GautamB.Ngr	68	68	68	68	68	68	68	60	Shahjahanpur	43	43	43	43	43	43	43
27	Ghazizabad	64	64	64	64	64	64	64	61	Shrawasti	2	2	2	2	2	2	3
28	Ghazipur	19	19	19	19	20	19	19	62	Sidharthngr	3	4	4	4	4	4	5
29	Gonda	17	17	17	16	16	16	16	63	Sitapur	27	27	27	27	27	27	27
30	Gorakhpur	49	49	49	49	49	49	49	64	Sonbhadra	5	5	5	5	5	5	4
31	Hamirpur	39	37	37	36	36	36	36	65	St.Ravidasngr	8	8	8	8	8	8	8
32	Hardoi	16	16	16	17	18	20	20	66	Sultanpur	21	21	21	21	21	21	21
33	Hathras	60	60	60	60	60	60	60	67	Unnao	44	44	44	44	44	44	44
34	Jalaun	18	18	18	17	17	17	17	68	Varanasi	47	47	46	46	46	46	45

Note: 3R=Rank Score of Z-3s; R3=Rank Score of Z+3s; 2R=Rank Score of Z-2s; R2=Rank Score of Z+2s; 1R=Rank Score of Z-1s; R1=Rank Score of Z+1s; R=0R=R0 = Rank Score of Z-0s = Rank Score of Z+0s = Rank Score of Z

**Table 4. Measures of Association Among Rank Scores of Different Interval Limit Values of the Composite Index**

ZR	Kendall's Tau							ZR	Spearman's Rho						
	3R	2R	1R	R	R1	R2	R3		3R	2R	1R	R	R1	R2	R3
3R	1.00000	0.99210	0.99034	0.98507	0.98244	0.97717	0.97191	3R	1.00000	0.99954	0.99943	0.99908	0.99885	0.99824	0.99771
2R	0.99210	1.00000	0.99824	0.99298	0.99034	0.98507	0.97981	2R	0.99954	1.00000	0.99992	0.99969	0.99954	0.99905	0.99859
1R	0.99034	0.99824	1.00000	0.99473	0.99210	0.98683	0.98156	1R	0.99943	0.99992	1.00000	0.99977	0.99962	0.99912	0.99866
R	0.98507	0.99298	0.99473	1.00000	0.99737	0.99210	0.98683	R	0.99908	0.99969	0.99977	1.00000	0.99989	0.99950	0.99916
R1	0.98244	0.99034	0.99210	0.99737	1.00000	0.99473	0.98946	R1	0.99885	0.99954	0.99962	0.99989	1.00000	0.99973	0.99943
R2	0.97717	0.98507	0.98683	0.99210	0.99473	1.00000	0.99473	R2	0.99824	0.99905	0.99912	0.99950	0.99973	1.00000	0.99973
R3	0.97191	0.97981	0.98156	0.98683	0.98946	0.99473	1.00000	R3	0.99771	0.99859	0.99866	0.99916	0.99943	0.99973	1.00000

Ranking of Z values has been an important operation to draw conclusions and to classify the cases according to rank scores is a frequently practised activity. In Table-3 we present the rank score obtained by different districts according to the values of interval limits beginning with -3 standard deviation (error) to +3 standard deviation (error) on the either side of the expected value of Z (mean Z). A scrutiny of the rank scores immediately reveals that rank score change in case of some districts. Accordingly, the measures of association (such as Kendall's Tau or Spearman's Rho as presented in Table-4) among the rank scores of deviation intervals are short of unity which indicates that ranking according to Z may not be always befitting. These results also indicate that one should be cautious in interpreting the results of composite indices when two values are very close to each other since statistically they may be indistinguishable.

## 5. Conclusion

This short note proposes working out of the standard errors of estimate of composite indices when they are constructed by using intrinsically derived weights. It illustrates the proposed method, using the jackknife resampling technique, by an example that relates to crime of different types in Uttar Pradesh (India). Jackknife re-sampling results may further be refined by bootstrapping. It may give further insight as to the nature of variation in the composite index values.

## References

- Efron, B. (1979). Bootstrap methods: Another look at the jackknife. *The Annals of Statistics*, 7(1), 1-26.
- Efron, B. (1981). Nonparametric estimates of standard error: The Jackknife, the bootstrap and other methods. *Biometrika*, 68(3), 589-599. doi. [10.1093/biomet/68.3.589](https://doi.org/10.1093/biomet/68.3.589)
- Efron, B., & Tibshirani, R.J. (1993). *An Introduction to the Bootstrap*, Chapman & Hall, New York.
- Mishra, S.K. (2011). A comparative study of various inclusive indices and the index constructed by the principal component analysis. *IUP Journal of Computational Mathematics*, 4(2), 7-26.
- Mishra, S.K. (2016). A note on construction of a composite index by optimization of Shapley value shares of the constituent variables. *Turkish Econ. Review*, 3(3), 466-472.
- Mishra, S.K. (2017). Almost equi-marginal principle based composite index of globalization: China, India and Pakistan. *Journal of Economic and Social Thought*, 4(3), 335-351.
- Shao, J., & Tu, D. (1995). *The Jackknife and Bootstrap*, Springer Verlag, New York.
- Wolter, K.M. (1985). *Introduction to Variance Estimation*, Springer Verlag, New York.



## Copyrights

Copyright for this article is retained by the author(s), with first publication rights granted to the journal. This is an open-access article distributed under the terms and conditions of the Creative Commons Attribution license (<http://creativecommons.org/licenses/by-nc/4.0>).

