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Modeling persistence of volatility in the Moroccan exchange market using a fractionally integrated EGARCH

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Abstract. We have tried in this article to detect, examine, and analyze the persistence in the conditional volatility of the major Moroccan stock market index called MASI, using a fractionally integrated EGARCH model that has the property of capturing long memory along with shocks to the conditional volatility. A GARCH (1,1) and IGARCH models were also estimated for comparative purposes using Akaike, Schwarz and log likelihood information criterion. We used daily returns of MASI index covering the period between 04/01/1993 and 03/02/2017. Our results confirm the presence of a strong persistence in the volatility of the Moroccan index which is inconsistent with the weak efficiency form of Fama's efficient markets hypothesis. The findings of this study could be of particular use to investors and academics interested in the forecasting of daily volatility in the Moroccan context. This paper broadens previous long memory estimation research by applying a FIEGARCH specification enabling it, not only to account for persistence, but also, to measure the leverage effect. Moreover, we believe that, to the best of our knowledge, this paper is the first to model the volatility of the Moroccan stock market using a FIEGARCH approach.

Keywords. Volatility, Persistence, Long memory, FIEGARCH, MASI.

JEL. G11, G17, C53, C58.

1. Introduction

Volatility modeling continues, nowadays, to occupy a central place in financial economics, numerous studies and papers are being exclusively devoted to the study of this complex phenomenon. Among the hardest challenges one may face in the modeling of volatility are the stylized fact, or volatility patterns.

These stylized facts describe the features that observed volatility exhibits, which means that any model that seeks to represent volatility is forced to incorporate these behaviors in order to produce quality estimates and forecasts. As these patterns tend to have a gradual importance in the process of representing volatility, in a way that some patterns are more essential than others, the long memory is by far the most determining pattern in volatility time series. It derives its eminent place from the idea of making volatility predictable.

The stylized facts, and more precisely, the persistence of volatility is amajor threat to Fama's (1970) famous efficient markets theory, since it goes a gainst its main principle, which is that prices incorporate all available information making

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price forecasts impossible. However, a long memory process would imply that the present prices could be determined based on past prices considering that such processes entail significant autocorrelations rates in the price series for relatively long periods.

For markets that prove to be marked with a long memory process, almost no modeling of volatility could be done without accounting for this process. Moreover, financial econometrics is continuously enriched by new frameworks and models aimed at the sole purpose of modeling this persistence.

Being aware of the importance of such studies in the understanding of financial markets, and having in mind the lack of similar research in the Moroccan context, we propose in this article to perform an econometric study with the purpose of modeling the long memory in the volatility of the Moroccan financial market. The insight we intend to make lies in the application and use of a FIEGARCH parameterization, allowing not only to account for persistence, but also to incorporate the leverage effect. This approach represents an extension to the previous body of literature, relying almost exclusively on simple FIGARCH models.

The rest of the paper is organized as follows. First, we expose a brief review of the literature covering the main papers on the subject, from theories and mathematical formulation to real applications. Second, we present a data and methodology section encompassing the data description and proprieties, along with the methodology followed. We follow up with a results and discussion section in which we draft the main results and conclusions. And end with a conclusion section.

2. Literature review

Before we can go on with the literature review, we find it convenient to start with a definition of long memory so that the following text may become easily understood.

A long memory process could be defined as a slow hyperbolically decaying rate of autocorrelations, which means that the autocorrelations rates take too long to dissipate. Therefore, a current shock to the volatility would have long-lasting effects. Unlike the case of short memory processes in which autocorrelations disappear at an exponential rate leaving shocks to have a limited time impact.

The first paper to ever address the issue was made by Hurst (1951). He was an engineer in charge of the construction of reservoirs for the Nile river water. He had to study the flow of the river through documenting its levels on different days for a time frame. This operation led to the discovery of the long memory property of the Nile river flow.

After the discovery made by Hurst, and especially with the long memory measure baptized as the Hurst exponent, many scientists from different scientific backgrounds started to work on the issue, and to document similar patterns in their respective fields of expertise such as climatology, geology, physics and other natural sciences.

Nevertheless, the credit for the first application of the long memory in economics and finance goes to Mandelbrot & Ness (1968). They were the pioneers of projecting Hurst's discovery to stocks markets through the application of the Hurst exponent to the measurement of markets degrees of persistence.

After Hurst's works, it took several years for researchers to come up with another measure and conceptual frameworks to model this persistence. This was essentially due to the lack of comprehension of the impact of persistence on the determination of future values.

The first paper to follow up Hurst's insight was the implementation of the ARFIMA (Auto-Regressive Fractionally Integrated Moving Average) model by Granger & Joyeux (1980) and Hosking (1981). This linear model was considered at the time as a major breakthrough in the ARMA modeling. It enabled for the first time, the basic ARMA processes to embody the long memory estimation. The advantages, as well as the quality performance of the ARFIMA processes, was a direct incentive for researchers to deepen the analysis in a quest for a better modeling, especially that linear models have started to show their weakness with the evolution of financial markets.

The basis for the modeling of long memory in nonlinear models comes from the development of the ARCH (Auto-Regressive Conditional heteroskedasticity) models by the prize Nobel winner Engel (1982). He presented an innovative way to model conditional volatility using a nonlinear equation. This model was very successful that it knew many extensions in the years following its inception. As a matter of fact, the generalizing of the process made by Bollerslev & Taylor (1986) into a GARCH (Generalized Auto-regressive conditional heteroscedasticity) is considered the most notorious advancement to the process. This generalization of the initial process made it possible to model an important number of financial time series properties. GARCH was the first serious attempt to model long memory using a "sophisticated" nonlinear equation. The long memory was assessed through summing up the ARCH and GARCH parameters so that any closer results to unity was considered as a sign of strong persistence in the data.

The success of the GARCH model coupled with the development of financial markets has led to a large series of extensions to the original model, making them every time, more capable of reflecting financial time series patterns. Among the main newly created models we can give the example of the EGARCH, TGARCH, EGARCH, IGARCH etc. They become so diversified that their number could only be limited to the imagination.

Since the focus in this paper will be on the modeling of long memory we will be only limited to the GARCH family models related to the issue with a certain stress on leverage effect GARCH model of EGARCH(Exponential GARCH).

As the initial GARCH model provided the first measure of long memory, it was limited in a sense that it has a finite persistence. It was only thanks to the integrated GARCH of Bollerlev & Engel (1986) that persistence got to be infinite, allowing, therefore, the shock to last infinitely.

Arguably, and despite the success of the both GARCH and IGARCH, market data have had proven that the two models are still restricted in a way that they only represent extreme cases of persistence, and therefore, not allowing the persistence to be flexible, imposing either an I(0) process for GARCH or an I(1) process for IGARCH. It was only after the introduction of the FIGARCH model, which was inspired by the combination of the properties of GARCH models and the innovation of the fractionally integrated ARMA processes, that persistence could be efficiently modeled. The credits and the merits of this model go to Baillie *et al.,* (1996).

The FIGARCH model was a major discovery in terms of modeling long memory because, in contrast to previous models, it allowed the differentiating parameter d to range between 0 and 1 and not to be restricted to the two extreme values. This model, by definition, nests the GARCH and IGARCH as two special cases.

Later on, even the notorious FIGARH was subject to development. The main extension that was brought to it, was the FIEGARCH by Bollerslev & Mikkelsen (1996). It was meant to further enlarge the initial FIGARCH in order to capture more stylized facts, and more precisely, to capture news impact. In the sake of simplification, we can say that the FIEGARCH nests now the three models of GARCH, IGRACH, and EGARCH as special cases. Or in other words, it is a combination of a FIGARCH and an EGARCH.

After having presented a brief review on the emergence of long memory models, we will now shift the focus towards some of the many papers that made use of the above-expressed models in order to measure long-term dependencies in volatility time series across different markets and conditions.

The presence of persistence and long-range dependencies in time series has motivated the works of Peters (1996), and Huang & Yang (1999), Barkoulas *et al.,*

(2000) and Panas (2001) among others, to prove empirically the existence of this feature in financial markets. These papers have had for a common conclusion that long memory is a consistent feature of financial markets.

Nevertheless, a certain number of papers like Chow *et al.,* (1996) and Grau-Carles, (2005) could not document any empirical evidence regarding the existence of persistence in the financial markets they studied. This conclusion would make judgments on the existence of long memory process market-specific.

The measurement of long-term dependencies among time series has motivated the application of a large set of techniques and mathematical formulations. Nonetheless, the FIGARCH family of models was often said to be more efficient, and can, therefore, outperform any other conditional heteroskedasticity model in forecasting and modeling stocks returns Beine *et al.,* (2002) and Banerjee & Sarkar (2006).

In the same line of ideas, the superiority of FIGARCH over other models was documented in different sets of markets. For instance, Antonakakis & Darby (2013) have proven that the FIGARCH can be particularly more efficient than other models in the market of exchange rates, while Jin & Frechette (2004) have demonstrated the unique and remarkable performance of a FIGARCH model in predicting *futures* volatility.

Regarding the application and use of the FIEGARCH model, we can cite the paper of Goudarzi (2010) in which they could prove the existence of a long memory process in the volatility of the Indian stock exchange market's main index through a FIEGARCH model. Fakhfekh & Jeribi (2015) employing a FIEGARCH model were able to study the impact of the Tunisian revolution on the features of Tunisia stock exchange index such as asymmetry and long memory.

3. Data and methodology

3.1. Data

Our dataset consists of the daily closing prices of the Moroccan leading financial market index labeled as the Moroccan all shares index or MASI, and spans the period ranging from 04/01/1993 to 03/02/2017. The period between the years of 1993 up to 2002 belonged initially to the IGB[1] index that was later substituted by the new MASI index. The data was collected from the CDG CAPITAL BOURSE's[2] website and totaled 5996 observations. We have chosen the longest period of study that the data allows in order to fully and thoroughly analyze and study the long memory properties of the series. The price series were later standardized into a series of returns to have zero mean and unit variance, following the formula:

$$r_t = \log\left(\frac{X_t}{X_{t-1}}\right) \qquad (1)$$

Where r_t stands for the returns at the moment t, X_t the prices at moment t and X_{t-1} the prices at t-1. Using a daily frequency, from a statistical point of view, helps yield a relatively much more valid statistical analysis because of the large sample. Furthermore, a growing number of financial studies are now favoring high-frequency data over low frequency.

The returns series was later examined for the existence of a unit root employing the Augmented Dickey-Fuller test, the results in table 1 show that the null hypothesis of the presence of a unit root I(1) cannot be accepted and therefore the series returns are stationary at level.

[1] Indice général boursier
[2] www.cdgcapitalbourse.ma

Table 1. *Augmented Dickey-Fuller unit root test results*

Index returns	ADF Value	t-stat at 1%
MASI	-43,11***	-3,43

Note: *** values are statistically significant at levels 1%,5%,and 10%

Later on, the following mean equation was estimated for the MASI return series:

$$r_t = c + r_{t-1} + \varepsilon \quad (2)$$

Where, c is a constant, and ε stands for the error term.

This mean equation was used to test for ARCH effect in the series using the White test. The results as shown in Table 2, imply the rejection the null hypothesis of homoskedasticity and therefore, the acceptance of the alternative hypothesis that the series is marked by an ARCH effect.

Table 2. *ARCH effect test results*

Dependent variable	White statistic
MASI returns	433,01***

Note: *** values are statistically significant at levels 1%, 5% and 10%

3.2. Methodology

In this article, we will be extending the fractionally integrated generalized autoregressive conditional heteroskedasticity first developed by Baillie & Bollerslev (1996) FIGARCH, to a fractionally integrated EGARCH of Bollerslev & Mikkelsen (1996), in order to account for asymmetry often observed in financial markets around the world. The application of this model will enable us, to measure, model and estimate persistence in volatility of MASI as well as capturing asymmetric leverage effect.

The methodological approach in this article starts with a descriptive statistics subsection in order to provide key data distribution parameters, which will help us understand its properties, and better calibrate our models. In the following subsection, we will be trying to enrich the preliminary tests with more advanced assessments to detect the existence of persistence in the volatility of MASI and analyze its characteristics, before lastly running our estimation model FIEGARCH along with the comparison-oriented GARCH(1,1) and Integrated GARCH(1,1).

Regarding the estimation process and the presentation of our FIEGARCH model, we propose in the following paragraphs a brief mathematical description of the model as well as the attributes of its estimation.

Before proceeding to the presentation of the FIEGARCH model we start first by exposing the mathematical formulas of GARCH and IGARCH since they were the departure models that led to the FIEGARCH parameterization, and considering that they represent two special cases of FIEGARCH.

In general ARCH models are based on the principle of using lagged squared returns to model conditional variance. They represent a weighted average of past squared returns, with the weights being calculated based on historical volatility, in contrast to the exponentially weighted moving average (EWMA) model.

To start, GARCH model of Engel (1982) and Bollerslev (1986) can be presented as follows:

$$h_t = \alpha_0 + \sum_{i=0}^{q} \alpha\, u_{t-1}^2 + \sum_{i=0}^{p} \beta\, ht_{t-1}^2 + \varepsilon_t \quad (3)$$

With u^2_{t-1} standing for lagged squared residuals called ARCH parameter, h_{t-1} as lagged conditional volatility known as GARCH parameter, and ε_t being the residual parameter. For this model α_0; α_i and α_j must be positive i.e α_0 ;α_i ; $\alpha_j > 1$, which is also known as the non negativity constraint. The sum of α_1 and β must be inferior to unity, and the closer the sum is to 1 the stronger the persistence is. Nevertheless, if this condition is violated, the process is said to have infinite shocks in the variance which can be modeled via an IGARCH process.

GARCH model allows lagged shocks (u^2_{t-1}, u^2_{t-2},…,u^2_{t-q}) to impose a finite impact of q periods on the conditional variance h_t. as well as it allows lagged conditional variance terms (h^2_{t-1}, h^2_{t-2},…, h^2_{t-p}) to have amemory longer than (p). The longer is the memory of the shock the bigger is β_1.

The IGARCH model of Bollerslev & Engel (1986) is presented in the following form:

$$h_t = \alpha_0 + \sum_{i=0}^{q} \alpha\, u_{t-1}^2 + \sum_{i=0}^{p} \beta\, ht_{t-1}^2 + \varepsilon_t \tag{4}$$

The IGARCH specification is similar to a GARCH model. However, the IGARCH has a constraint implying α+β = 1.

This suggests an infinite persistence in the conditional volatility due to shocks in the squared returns. This model can also be expressed in terms of an ARMA (p,q) process as:

$$\Phi(L)(1-L)\varepsilon_t^2 = \omega + [1-\beta(L)]v_{t;} \tag{5}$$

With $\Phi(L)$ and $\beta(L)$ being ARCH and GARCH polynomials. For this model, persistence of volatility that can be defined as the slow rate of decay in the autocorrelation function of a time series is equal to unity. (1- L) is the differencing operator that can be actually expressed as $(1\text{-}L)^d$ with d=1.

By replacing the first difference operator (1-L) in the above model with the fractional differencing operator $(1\text{- }L)^d$, where d is a fraction 0 < d < 1, the FIGARCH model can be implemented.

Therefore, and Following Baillie Bollerslev & Mikkelson (1996), the FIGARCH(p,d,q) class of models may be presented as follows:

$$h_t = \Phi(L)(1-L)^d\varepsilon_t^2 = \omega + [1-\beta(L)]v_{t;} \tag{6}$$

The FIEGARCH of Bollerslev and Mikkelson(1996) can be presented as:

$$\Phi(L)(1-L)^d(\ln\sigma_t^2 - \omega) = \psi(L) + g(z_{t-1}) \tag{7}$$

In terms of conditional volatility the FIEGARCH (1,d,1) can be expressed by the following equation:

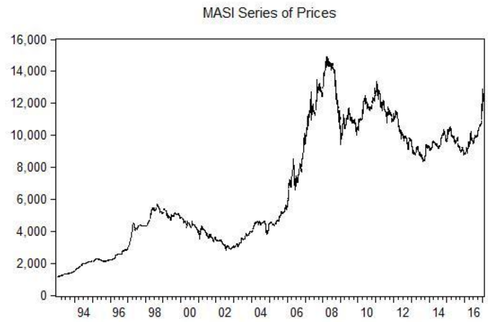
$$h_t = (1-L)^d(ln\sigma_t^2 - \omega) = \Phi_1 h_{t-1} + g(z_{t-1}) + \psi_1 g(z_{t-2}) \tag{8}$$

With Φ_1 and ψ_1 standing for polynomials in the lag operator,$(1\text{-}L)^d$ is the fractional difference operator that oscillates between 0 and 1 giving the model more flexibility in the capturing of persistence in time series. The asymmetry feature is then accounted for by $ln\sigma_t^2$. g(.) is the news impact function that governs the way by which past returns impact current volatility. And at last $z_t$stands for normalized innovations $z_t = \frac{\varepsilon_t}{\sigma_t}$.

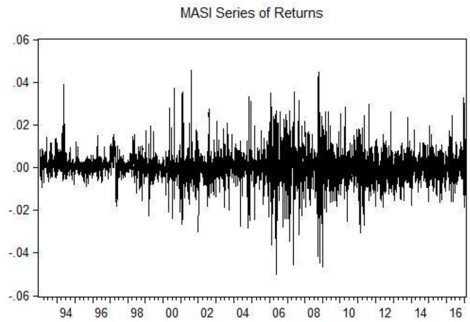
Regarding the estimation process of our models, we have decided to assume a normal distribution for the error term after having run benchmark tests previously. According to the Baillie *et al.,* (1996) methodology, the models were also estimated employing the maximum likelihood function. Furthermore, the Broyden Fletcher Goldfarb Shanno (BFGS) algorithm was used for the optimization of our unconstraint nonlinear models.

4. Empirical application

4.1. Descriptive statistics:



Graph 1. *MASI's series evolution of prices*



Graph 2. *MASI's series evolution of returns*

The observation of the graphs confirms the stationary tests that were conducted previously in this paper, since the graph of the evolution of prices is one of a deterministic nonstationary process with a trend, in contrast to the graph of returns that shows values oscillating around the mean which describes a stationary process with no trend. Furthermore, we can notice at the left graph how the evolution of prices was moving in a continuous upward trend until reaching a peak of approximately 15000, before making a spectacular decline as a consequence of the world financial crisis.

The returns graph demonstrates some important volatility behaviors, firstly it enables us to clearly define periods of high and low volatility, high periods (which are of the more importance) are the ones coinciding with the adoption of the euro as a common currency in the eurozone between 1999 up to 2003, the financial crisis from 2007 up to 2009, and last the Arab unrest in 2011. Secondly and lastly, the returns series displays a clear pattern of volatility clusters since periods of high volatility tend to be followed by periods of high volatility and periods of low volatility are commonly followed by periods of low volatility in a persistent way.

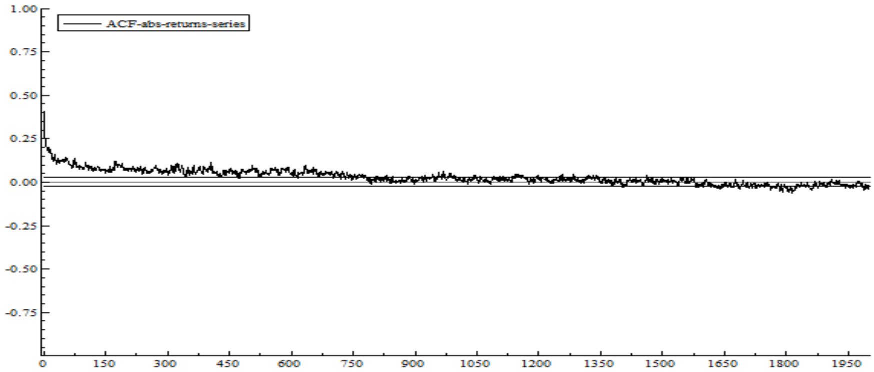
To provide further evidence on this volatility clustering behavior, we proceeded to apply the Ljung Box test statistic Q(12) to the series of absolute returns in order to assess the null hypothesis of a white noise process.

Table 3. *Ljung Box Q statistic test applied to absolute returns series*

Dependent variable	Ljung Box Q(12)
MASI absolute returns series	433,01***

Note: ***: values are statistically significant at levels 1%, 5% and 10%

Following the estimated value of Q(12) in Table 3, the null hypothesis of no serial correlation i.e white noise was rejected and we could deduce that the absolute returns series is affected by autocorrelation.



Graph 3. *Plot of the autocorrelation function of the MASI absolute returns series*

The horizontal bands represent Bartlett's formula for MA(q) 95% confidence intervals.

The persistence of volatility can be particularly seen in the plot of the sample of the Autocorrelation function of the absolute returns series in Graph 3. Absolute returns have the property to provide estimates of the returns' variance at every moment t. Consequently; the autocorrelation function (ACF) of absolute returns should display positive and statistically significant autocorrelations. These properties are drawn from the fact that financial markets are generally featured by volatility clustering, and in many cases long memory processes.

The absolute returns were chosen instead of r_t, because, according to the efficient markets hypothesis, the latter cannot be forecasted.

The graph represents up to 1950 lags of the autocorrelations of absolute returns, and as one can notice, peaks of autocorrelations are still significant on a very long time range, since they are outside of the 95% no serial correlation confidence interval, implying long-term dependencies in the autocorrelations i.e long memory property.

At last, we propose to make use of an additional important long memory metric which is Hurst's exponent.

For a given time series X_t= 1, 2, …., T of the mean $\bar{X}_t$, the R/S statistic can be formulated as

$$\frac{R}{S} = \frac{1}{[\frac{1}{T}\Sigma_{j=1}^{T}(X_i - \bar{X}_t)^2]^{\frac{1}{2}}}\left[max_{1\leq k\leq T}\sum_{j=1}^{k}(X_j - \bar{X}_t) - min_{1\leq k\leq T}\sum_{j=1}^{k}(X_j - \bar{X}_t)\right] \quad (9)$$

The R/S statistic can be defined as a statistical tool which allows the analysis of a time series data in order to attempt to find repeated patterns in the data.

The Hurst exponent denoted by H can be derived from the R/S statistic according to the following formula:

$$H = \frac{\log R/S}{\log T} \quad (10)$$

The statistic H can take values ranging between 0 and 1. 0 being the anti-dependence, 0.5 denoting a random walk process while a 1 is a synonym of strong persistence in the time series. It is noteworthy to specify that the H in contrast to previous long memory metrics, does not allow statistical significance tests, which can be a considerable drawback.

For our Time series, the calculus of the Hurst exponent yields the following result: H = 0.71

The results being comprised between 0.5 and 1, suggest the presence of strong serial dependency in the time series. Therefore, we can clearly add the Hurst exponent to the other statistics which have all indicated the presence of a long memory process in the Moroccan financial market data. These conclusions motivate our choice of a FIEGARCH model.

Table 4. *Descriptive statistics*

Descriptive statistics	MASI Returns
Mean	0.000392
Median	0.000290
Max	0.045547
Min	-0.050167
Standard deviation	0.006767
Skewness	-0.086517
Kurtosis	10.30330
Jarque-Bera	1330.91
P-value	0.000000

Note: ***: values are statistically significant at levels 1%, 5%,and 10%

The descriptive statistics of the returns series exhibit a positive and a very small mean value in comparison to the more important standard deviation value. The skewness statistic implies that the returns series is skewed to the left revealing a non symmetric behavior, while the high value of the kurtosis indicates the existence of thick tails.

Lastly, the *Jarque-Bera* statistic confirms the previous shape-related results implying the rejection of the null hypothesis of a normal distribution in favor of a fat-tailed distribution of returns, which is very common in financial time series.

4.2 Model estimation

As all the preliminary tests are run, we can now step forward, and start estimating our principal model FIEGARCH, as well as the benchmark models that are GARCH(1,1) and IGARCH.

Table 5. *Model estimation results*

Model parameters	GARCH(1,1)	IGARCH(1,1)	FIEGARCH
Constant in mean	0.000262***	0.000293***	0.0002545***
Constant in variance	1.38E-06***	1.527633***	0.0374628
Alpha	0.284154***	0.091386***	-0.926871***
Beta	0.727950***	0.908614***	0.962261***
Leverage	------	------	0.645537***
d	------	1	0.690718***
AIC	-7.674530	-7.676671	-7.676739
SIC	-7.658943	-7.663319	-7.667100
Log Likelihood	23099.33	23013.82	23016.927

Note: ***: values are statistically significant at levels 1%, 5%,and 10%

Before going into the analysis of the estimation results, we can also test the persistence of volatility by summing up the ARCH and GARCH parameters and see how close they are to 1. Since the GARCH model has an Alpha+Beta< 1 restriction, results have atendency to show a number close to the value of 1 as it is the case of our example. And since we cannot statistically decide whether they are equal, or smaller than unity, we will apply the *Wald* test, which has the following null and alternative hypothesis:

$$\begin{cases} H0: \text{alpha} + \text{Beta} = 1 \\ H1: \text{alpha} + \text{Beta} < 1 \end{cases}$$

Table 6. *Wald test results*

Wald test x^2	P value
0.925314	0.339

The results of the Wald test suggest the acceptance of the null hypothesis implying an infinite persistence in the volatility of MASI. These results favor the estimation of an IGARCH model.

Since the IGARCH Model have an infinite persistence of volatility assumption, it was predictable that the model was going to be remarkably significant. Consequently, and judging by the previous results we could safely assume that the d parameter in FIEGARCH is going to be also statistically significant. Accordingly, the only task that is left, will be measuring the differentiating parameter and see how it ranges in the 0 to 1 interval. The parameter d will provide us with an empiric answer regarding the degree of persistence in the MASI volatility.

As for the analysis of the estimation results, an apparent fact is that the three models are notably significant which features their validity. The FIEGARCH model entails a persistence degree of 0.69, this involves strong serial dependence on a long-term scale in the volatility function. The leverage parameter being positive emphasizes the existence of positive shocks in the conditional volatility equation. Judging by the AIC, SIC and log likelihood information criterions, we can safely state that the FIEGARCH model is not only able to capture this strong temporal interdependence, but it is also the best model to estimate and forecast the variance of MASI by capturing other stylized facts such as leverage effect and ARCH behavior, hence, outperforming the two other models.

5. Conclusion

We have tried in this article to investigate and assess the persistence in the volatility of MASI. The persistence or long memory in volatility has a crucial impact on its modeling and is of a paramount importance to risk managers.

The main merit of this paper is to empirically demonstrate the existence of a strong persistence in the volatility of MASI which is inconsistent with the EMH. Hence, proving that the volatility of the MASI is actually predictable and can be, therefore, forecasted based on its past values.

To further highlight the validity and the implications of our model, we proposed a comparison with two other competing models, which are GARCH (1,1) and IGARCH.

The results prove a certain superiority of FIEGARCH model, which can be explained by the fact that it nests both models as special cases in addition to the capturing of the persistence in volatility. Our results could be consequently of great use to financial risk managers, hedge managers as well as academics interested in the modeling of volatility in the Moroccan financial market.

References

Antonakakis, N., & Darby, J. (2013). Forecasting volatility in developing countries' nominal exchange returns. *Applied Financial Economics*, 23(21), 1675-1691. doi. 10.1080/09603107.2013.844323

Baillie, R.T., Bollerslev, T., & Mikkelsen, H.O. (1996). Fractionally integrated generalized autoregressive conditional heteroskedasticity, *Journal of Econometrics*, 74(1), 3–30. doi. 10.1016/S0304-4076(95)01749-6

Banerjee, A., & Sarkar, S. (2006). Modeling daily volatility of the Indian stock market using intra-day data. Calcutta: Indian Institute of Management. *Working Paper* No.WPSNO.588. [Retieved from].

Barkoulas, J.T., Baum, C.F., & Travlos, N. (2000). Long memory in the Greek stock market. *Applied Financial Economics*, 10(2), 177-184. doi. 10.1080/096031000331815

Beine, M., Laurent, S., & Lecourt, C. (2002). Accounting for conditional leptokurtosis and closing days effects in FIGARCH models of daily exchange rates. *Applied Financial Economics*, 12(8), 589-600. doi. 10.1080/09603100010014041

Bentes, S.R., Menezes, R., & Mendes, D.A. (2008), Long memory and volatility clustering: is the empirical evidence consistent across stock markets? *Physica A*, 387(15), 3826-3830. doi. 10.1016/j.physa.2008.01.046

Bollerslev, T., & Engle, R.F. (1993), Common persistence in conditional variances: definition and representations, *Econometrica*, 61(5), 167-186. doi. 10.2307/2951782

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31(3), 307-327. doi. 10.1016/0304-4076(86)90063-1

Brockwell, P.J., & Davis, R.A. (1987). *Time Series: Theory and Methods*, Springer-Verlag, New York.

Chambers, M.J. (1998). Long memory and aggregation in macroeconomic time series, *International Economics Review*, 39(4), 1053-1072. doi. 10.2307/2527352

Chou, R.Y. (1988), Volatility persistence and stock valuations: some empirical evidence using GARCH, *Journal of Applied Econometrics*, 3(4), 279-294. doi. 10.1002/jae.3950030404

Chow, K.V., Pan, M.-S. & Sakano, R. (1996). On the long-term or short-term dependence in stock prices: Evidence from international stock markets. *Review of Quantitative Finance and Accounting*, 6(2), 181-194. doi. 10.1007/BF00367503

Engle, R.F., & Bollerslev, T. (1986). Modelling the persistence of conditional variances. *Econometric Reviews*, 5(1), 1-50. doi. 10.1080/07474938608800095

Engle, R.F., & Gonzalez-Rivera, G. (1991). Semiparametric ARCH models, *Journal of Business Economy and Statistics*, 9(4), 345-360.

Engle, R.F., & Mustafa, C. (1992). Implied ARCH models from option prices, *Journal of Econometrics*, 52(1-2), 289-311. doi. 10.1016/0304-4076(92)90074-2

Engle, R.F., (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of U.K. inflation. *Econometrica*, 50(4), 987-1008. doi. 10.2307/1912773

Fakhfekh, M., Jeribi, A., & Hachicha, N. (2015). Long memory properties on Tunisian sectorial stock market index volatilities: evidence from FIEGARCH model, International. *Journal of Sustainable Economy*, 7(4), 280-305. doi. 10.1504/IJSE.2015.072201

Fama, E.F. (1970). Efficient capital markets: A review of the theory and empirical work, *Journal of Finance*, 25(2), 383-417. doi. 10.2307/2325486

Gil-Alana, L.Z., Shittu, O.I., & Yaya, O.S. (2014). On the persistence and volatility in European, American and Asian stocks bull and bear markets, *Journal of International Money Finance*, 40, 149-162. doi. 10.1016/j.jimonfin.2012.12.002

Goudarzi, H. (2010). Modeling long memory in the Indian stock market using fractionally integrated egarch model, *International Journal of Trade, Economics and Finance*, 1(3), 231-237.

Granger, C.W.J., &. Joyeux, R. (1980). An introduction to long memory time series models and fractional differencing. *Journal of Time Series Analysis*, 1(1), 15-39. doi. 10.1111/j.1467-9892.1980.tb00297.x

Grau-Carles, P. (2005). Tests of long memory: A bootstrap approach. *Computational Economics*, 25(1-2), 103-113. doi. 10.1007/s10614-005-6277-6

Hoskin, J.R.M. (1981). Fractional differencing, *Biometrika*, 68(1), 165-176. doi. 10.1093/biomet/68.1.165

Huang, B.N., & Yang, C.W. (1999). An examination of long-term memory using the intraday stock returns. Clarion: Clarion University of Pennsylvania, *Technical Report*, No.99–03.

Hurst, H.E. (1951). Long-term story capacities of reservoirs, Trans. *American Society of Civil Engineering*. 116(1), 770–799.

Jin, H.J., & Frechette, D.L. (2004). Fractional integration in agricultural futures price volatilities. *American Journal of Agricultural Economics*, 86(2), 432-443. doi. 10.1111/j.0092-5853.2004.00589.x

Kang, S.H., Cheong, C., & Yoon, S.M. (2010), Long memory volatility in Chinese stock markets, *Physica A*, 389(7), 1425-1433. doi. 10.1016/j.physa.2009.12.004

Kasman, A., Kasman, S., & Torun, E. (2009). Dual long memory property in returns and volatility: evidence from the CEE countries' stock markets, *Emerging Markets Review*, 10(2), 122-139. doi. 10.1016/j.ememar.2009.02.002

Mandelbrot, B.B., & Van Ness, J.W. (1968), Fractional Brownian motion, fractional noises and applications, *SIAM Review*, 10(4), 422-437. doi. 10.1137/1010093
Pagan, A.R., & Schwert, G.W. (1990), Alternative models for conditional stock volatility, *Journal of Econometrics*, 45(1-2), 267-290. doi. 10.1016/0304-4076(90)90101-X
Panas, E. (2001). Estimating fractal dimension using stable distributions and exploring long memory through ARFIMA models in Athens Stock Exchange. *Applied Financial Economics*, 11(4), 395-402. doi. 10.1080/096031001300313956
Peters, E.E. (1996). *Chaos and Order in the Capital Markets: A New View of Cycles, Prices, and Market Volatility*. New York: John Wiley & Sons. Inc.
Porteba, J., & Summers, L. (1987). The persistence of volatility and stock market fluctuations, *American Economic Review*, 76(5), 1142-1151.
Qiu, T.,Chen, G., Zhong, L.X., & Wu, X.R. (2012), Dynamics of bid-ask spread return and volatility of the Chinese stock market, *Physica A*, 391(8), 2656–2666. doi. 10.1016/j.physa.2011.12.048
Schwert, G.W., & Seguin, P.J. (1990), Heteroskedasticity in stock returns, *Journal of Finance*, 45(4), 1129-1155. doi. 10.1111/j.1540-6261.1990.tb02430.x