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Intertemporal modeling of the current account

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Abstract. This paper explores the long-run current account to GDP ratio in the present value model framework (PVMCA). Firstly, we use Euler equation at macro level to identify a general equation of the per capita current account to GDP. Secondly, through the overlapping generations model we determine the equation of per-capita CA using relevant variables, and discuss the empirical validity of the PVMCA via the quasi-elasticity of CA-to-GDP with respect to the per capita growth rate of output and consumption. We show that the elasticities of CA-to-GDP to per-capita output and ant to per-capita consumption growths interact in opposite paths, meaning that a higher growth rate of consumption tomorrow involves more saving yesterday and brings up a positive current account balance.

Keywords. Current account, Consumption, Intertemporal Model, Per-capita GDP, Quasi-elasticity.

JEL. E10, E20, F40.

1. Introduction

The theoretical intertemporal model is relying on the society behavior as a consumer and a producer in achieving the required adjustments leading to a long-run equilibrium of the economy. Many of empirical papers adopt a simple version of the PVMCA by assuming that the change in the net output is the only determinant, this leads to rejecting the intertemporal model validity (Otto, 1992). The theoretical and empirical PVMCA is improved by adding the global interest rate, the return rate of global equity markets, and the real exchange rate (Hoffmann, 2013, Souki & Enders, 2008, Kano, 2008), but these papers did not consider the per capita macroeconomic variables.

Due to the result that the CA-to-GDP ratio can only be negative and that the positive case appears to be unstable, Obstfeld & Rogoff (1996) and Cerrato *et al.* (2015) introduce the overlapping generations to overcome this limitation. We start to explain that the limitation of the PVMCA at the macro level could be escaped partially by considering the relevant per capita variables by making some hypotheses about the dynamic interaction between the CA-to-GDP ratio relatively to the per capita growth rates of GDP and consumption.

We suggest modeling the per capita PVMCA to highlighting that the population size matters in analyzing the CA dynamic. Except for the findings

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of Hoffmann (2013) about the permanent global shocks on the Chinese current account, most of the papers support that the domestic shocks on the current account dominate. Such results should be normalized by analyzing at per capita level. The most information about such dynamic is inherent in the per capita real GDP, per capita real consumption, and the return rate on domestic and foreign assets.

Section 2 addresses briefly some basics of the PVMCA to model the long-run equation. Section 3 deals in detail with the importance of overlapping generations and the per capita dimension of the relevant variables. Section 4 focuses on how to reveal a testable model through the quasi-elasticity of the current account-GDP ratio with respect to the per capita growth rates of output and consumption. We conclude in Section 5.

2. Long-run current account model

The most used utility function in the PVMCA framework depends on the infinite time horizon, generalizing the utility function for a lifetime as $s = [t, T]$ as follows:

$$U_t = \lim_{T\to\infty}(u(C_t) + \beta u(C_{t+1}) + \beta^2 u(C_{t+2}) + \cdots) = \sum_{s=t}^{\infty}(1+\delta)^{t-s}u(C_s) \quad (1)$$

where β is a positive subjective discount (or subjective time preference) rate, because it is related to the consumer state of mind indicating his/her future credence compared to the current values. It can be measured by $\beta = 1/(1+\delta)$ where δ represents a discount rate$(0 < \delta < 1)$. From the identity of current account CA_t at real values, defined as the net accumulation of foreign assets:

$$CA_t \coloneqq B_{t+1} - B_t = Y_t + \tau B_t - C_t - G_t - I_t \quad (2a)$$

The sequential constraint serving to maximize the utility, through the investment and consumption processes, by supposing the return rate on foreign assets τ with $0 < \tau < 1$, will be as follows:

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+\tau}\right)^{s-t}(C_s + I_s + G_s) + \lim_{T\to\infty}(1+\tau)^{-T}B_{t+T+1} = (1+\tau)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+\tau}\right)^{s-t}Y_s \quad (2b)$$

We rewrite the constraint (2b) to regulate in the case of surplus the hypotheses of the relationship between the return rate on foreign asset holdings, output growth rate, and the consumption growth rate. By assuming the constancy of growth rates in the steady state, all output and its components except consumption grow at the same rate of g_Y, we get the following:

$$\frac{1+\tau}{\tau-g_C}C_t + \lim_{T\to\infty}(1+\tau)^{-T}B_{t+T+1} = (1+\tau)B_t + \frac{1+\tau}{\tau-g_Y}Y_t - \frac{1+\tau}{\tau-g_I}I_t - \frac{1+\tau}{\tau-g_G}G_t \quad (2c)$$

Under the assumption that $\tau > g_C$, the consumption function can be as:

$$C_t = \begin{cases} (\tau - g_C)\left[B_t + \frac{1}{\tau - g_Y}Y_t - \frac{1}{\tau - g_I}I_t - \frac{1}{\tau - g_G}G_t - \lim_{T\to\infty}(1+\tau)^{-T-1}B_{t+T+1}\right] \\ (\tau - g_C)\left[B_t + \frac{1}{\tau - g_Y}Y_t\left(1 - \gamma - g_K(K_t/Y_t)\right) - \lim_{T\to\infty}(1+\tau)^{-T-1}B_{t+T+1}\right] \end{cases} \quad (2d)$$

We focus on the permanent component of the current account modeling allowing for consumption "tilting". It is usual to consider the domestic GDP net of investment and government expenditures to determine the resources available for current and future consumption. We assume that the capital growth rate $I_t := g_K\left(\frac{K_t}{Y_t}\right)Y_t$ and the capital coefficient $k_Y := \frac{K}{Y}$ are constant and that the government spending is a fraction of the GDP, $G_t := \gamma Y_t$. To guarantee non-negative sign of the consumption, we suppose that the coefficient of the net output is positive $\frac{\tau - g_C}{\tau - g_Y} > 0$. The constraint (2b)requires the well-known condition of the no-cheater-Ponzi-game, meaning that there is no exhaustion of all resources during all periods of life, but there are savings for future generations (Appendix A):[1]

$$\lim_{T\to\infty}(1+\tau)^{-T}B_{t+T+1} \geq 0 \quad (3)$$

Maximizing the utility *function* (1) under the resources condition (3) and (2b) leads to the same Euler consumption equation (Gourinchas & Parker, 2002) for each period $s \geq t$after differentiating U_t on C_t and C_{t+1}.[2] The utility maximization consists on

$$\max \sum_{s=t}^{\infty}(1+\delta)^{t-s}u(C_s)$$

under the sequential constraints $B_{s+1} = (1+\tau)B_s + Y_s - C_s - G_s - I_s$ with $s \geq t$, and a constraint ruling out Ponzi games. The constraint (3) makes right to assume that there is a function, called the value function, which leads to the maximal constrained value of U_t as a function of overall initial resources $W_t \equiv (1+r)B_t + \sum_{s=t}^{\infty}(1+r)^{-s+t}\,Y_s$ by supposing that $I = G = 0$ from (2b). By writing the value function like $J(W_t)$which is differentiable (Stocky & Lucas, 1989), and according to a simple dynamic equation of the initial wealth, we have:

$$W_{t+1} = (1+\tau)W_{t+1} + \sum_{s=t+1}^{\infty}(1+\tau)^{-s+t+1}\,Y_s = (1+\tau)(W_t - C_t)$$

We obtain Euler equation for consumption (For more details see Appendix B):

[1]For more details see Obstfeld & Rogoff (1996), pages 63-66.

[2]According to the modeling task, Euler equation can be used in any levels (micro or macro). For more details see Bertola, Foellmi & Zweimuller (2006, Chapter 3, pages 32 and 49).

$$C_{s+1} = \beta^{\sigma}(1+\tau)^{\sigma} C_s \Leftrightarrow 1 + g_C = (1+\delta)^{-\sigma}(1+\tau)^{\sigma} \quad (4)$$

In the equation(4) and at steady state, the consumption growth rate$g_C$is assumed constant. At the stable growth process, due to Bellman equation, the optimal consumption function (2d)can be modeled as follows:

$$C_t = \frac{\tau - g_C}{1+\tau}\left[(1+\tau)B_t + \frac{1+\tau}{\tau - g_Y} Y_t\big(1 - \gamma - g_K(K_t/Y_t)\big)\right] \quad (5a)$$

The second term inside the square brackets corresponds to the present value, discounted by $(1+\tau)$, of the net resource which grows at rate g_Y. In terms of ratio to GDP, we obtain:[3]

$$\frac{C_t}{Y_t} = (\tau - g_C)\frac{B_t}{Y_t} + \frac{\tau - g_C}{\tau - g_Y}\left(1 - \gamma - g_K k_{Y_t}\right) \quad (5b)$$

From the result $(5b)$, it appears that the average propensity to consume (APC) is supported by financial returns payments from net foreign assets and a fraction of net domestic resources. In the case of a closed economy, we have $B = 0$ and $CA = 0$, then the APC is supported only by the net domestic resources according to the coefficient $\frac{g_C}{g_Y}$.

By relating the optimal consumption equation to the current account identity (2a),we obtain the steady-state current account to GDP ratio:

$$\frac{CA}{Y} = g_C\frac{B}{Y} + \frac{g_C - g_Y}{\tau - g_Y}(1 - \gamma - g_K k_Y) = \sigma(\tau - \delta)\frac{B}{Y} + \frac{g_C - g_Y}{\tau - g_Y}(1 - \gamma - g_K k_Y) \quad (6a)$$

This macro equation shows that there is net saving or dissaving depending on the sign of the RHS of $(6a)$. Its first term indicates a fraction of the financial returns payments on its net foreign asset holdings.[4] It will be positive if the net assets are positive and the return rate on the foreign assets is greater than the discount rate. The second term represents a fraction of current resources and its sign depends on the difference between consumption and GDP growth.

Considering a "patient" economy where $\beta(1+\tau) > 1$, such economy saves more than "impatient" economy and could tend to realize CA surpluses. It would start from a low level of consumption and save early on. After that in tendency, it is possible that consumption growth will be higher than GDP growth ($g_C > g_Y$), this allows using up all intertemporal resources. In fact, the economy could save a fraction of its current resources, and then the second term of the RHS of (6)will be positive if the return rate on foreign assets is greater than the output growth. By supposing that g_C is positive i.e. $\tau > \delta$, the

[3] This result appears in Obstfeld & Rogoff (1996) at page 118.

[4] Knowing that g_C, δ and τ are between 0 and 1, by using the approximate value around zero of the elements of the logarithm of the equation (4), we find that $g_C \approx \sigma(\tau - \delta)$. Also, from the equation (B2) and assuming logarithmic utility $\sigma = 1$, we get that $g_C \approx \tau - \beta$.

current account to GDP ratio should indicate a current surplus. Besides, with the CA identity(2a), the foreign assets to GDP ratio is as follows:

$$\frac{B_{s+1}}{Y_{s+1}} = \left(\frac{1+g_C}{1+g_Y}\right)\frac{B_s}{Y_s} + \frac{g_C-g_Y}{(1+g_Y)(\tau-g_Y)}\left(1-\gamma-g_K k_{Y_s}\right) \tag{7a}$$

The sequential equation (7a) shows that the foreign-assets-to-GDP ratio path will be unstable if its slope is greater than one i.e. $1+g_C > 1+g_Y$. However, the equation will be stable when $1+g_C < 1+g_Y$ which is close to the rational consumer behavior. But, this last condition makes the coefficient of net output negative in the current account equation (6) if the return rate on foreign assets exceeds the GDP growth.According to Jordà, Schularick & Taylor (2011), the dynamic of CA-to-GDP ratio amplifies the risks of global instability. In the steady-state and by treating the foreign-assets-to-output ratio as exogenous, the equation (7a) becomes:

$$\frac{B}{Y} = \frac{-1}{\tau-g_Y}\left(1-\gamma-g_K k_Y\right) \tag{7b}$$

If the return rate on foreign assets is lesser than the GDP growth i.e. $\tau < g_Y$, the foreign-assets-to-output ratio will be positive. By inserting (7b) into (6a), we obtain:

$$\frac{CA}{Y} = \frac{-g_Y}{\tau-g_Y}\left(1-\gamma-g_K k_Y\right) \tag{7c}$$

There is a rigorous limitation of the PVMCA; because in the steady state, a small open economy can only support debt. Also, there is no motivation pushing to invest the current account surplus through foreign assets, because the home resources allocation will be more fruitful domestically.

This limitation induces to improve the intertemporal model through the overlapping generations which influence the consumption efforts and then the current account (Blanchard, 1985; Weil, 1989; Obstfeld & Rogoff, 1996). If we introduced for the output and consumption the basic identity$g_X \equiv g_x + n$from $x = X/N$,where $n$is the population growth rate and $g_x$is the per capita growth rate of the variable x and g_X is the aggregate growth rate, in the steady-state, by assuming $\tau - \beta = g_C$ and $g_c + n \equiv g_C$,we reach exactly the equation (6c):

$$\frac{CA}{Y} = \left(\frac{g_Y}{g_Y-g_c-n}\right)\left(\frac{g_C-g_y-n}{g_C+\beta-g_y-n}\right)\left(1-\gamma-g_K k_Y\right)$$
$$= \frac{-g_Y}{g_C+\beta-g_Y}\left(1-\gamma-g_K k_Y\right)$$

indicating that this approach does not overcome the limitation of PVMCA.[5]

3. Overlapping generations and the long-run PVMCA

[5]Some empirical papers as Cerrato, Kalyoncu, Naqvi & Tsoukis (2015) use this tautological approach, but it does not resolve the limitation of the PVMCA.

By introducing the overlapping generations in the PVMCA (Weil, 1989; Obstfeld & Rogoff, 1996; Cerrato *et al.,* 2015), we overcome the limitation of equation (6c)by reaching more generalized outcomes. In the steady state, we can now write the equation of per capita current account ca_t to per capita output y_t by determining the equation of the APC, from equation (C8c, Appendix C), as follows

$$\frac{c_t}{y_t} = (1-\beta)\left[(1+\tau)\frac{b_t}{y_t} + \frac{1+\tau}{\tau - g_y}(1-\zeta)\right] \tag{5c}$$

Using the current account identity, we obtain

$$\frac{ca_t}{y_t} := (1+g_y)\frac{b_{t+1}}{y_{t+1}} - \frac{b_t}{y_t} \Rightarrow \frac{ca}{y} = g_y\frac{b}{y}$$

where the last RHS represents the long-run current account to GDP ratio. By using (C2g, Appendix C), we obtain a long-run equation

$$\frac{ca}{y} = \frac{g_y[(1+g_c)-(1+g_y)]}{[(1+n)(1+g_y)-(1+g_c)](\tau-g_y)}(1-\zeta) \tag{8}$$

In the steady state, requiring the stability condition i.e. $\tau > g_y$ and the veracity of double inequality $\beta(1+\tau) > 1 + g_y > 1$, the equation(8) shows that the long-run factor of the current-account-output ratio could have any sign, and there is no sign presumption as in the equation (6c). We can now derive the effects of the per capita (or aggregate) consumption and output growth rates on the CA-to-GDP ratio. By supposing that the population growth is zero i.e. per capita and aggregate growth rates will be equal, then the current-account-GDP ratios modeled in equations ($6c$) and (8) are equivalent.[6]

When the population growth rate is increasing, the first factor of the denominator in RHS will be positive. Accordingly, there are many economic motives pushing to invest the current account surplus through foreign assets. But, if $\beta(1+\tau) < 1 + g_y$, there will be current account deficit; and the economy borrows from abroad or increases its domestic loans to continue the productive activities and finance new economic projects. But, such borrowing process will support a higher consumption than current resources tolerate.

4. Quasi-elasticities of the long-run current account

Assuming that the first factor of the denominator in RHS of (8)is positive and knowing that the second factor is positive, we can derive the effects of per capita GDP, per capita consumption growth rates, and population growth on the CA-to-GDP ratio. Firstly, we derive the per capita output multiplier:

[6] By using equations (C6, Appendix C) and (8), we can determine a more compatible form with data through the aggregate variables instead of per capita ones.

$$\frac{\partial\left(\frac{ca}{y}\right)}{\partial g_y}=\left[\frac{V_1-(1+n)U_1}{V_1^2}\right]\left(\frac{U_2}{V_2}\right)+\left[\frac{U_2-V_2}{V_2^2}\right]\left(\frac{U_1}{V_1}\right)=\frac{(n-g_c)U_2}{V_1^2V_2}+\frac{(g_c-\tau)U_1}{V_2^2V_1} \quad (9a)$$

where $U_2 \coloneqq (1+g_c)-(1+g_y)$, $U_1 \coloneqq g_y$, $V_1 \coloneqq (1+n)(1+g_y)-(1+g_c)$; and with the stability condition of the output path, we have $V_2 \coloneqq \tau-g_y > 0$.By assuming the current account surplus, we get $V_1>0$ and $U_2>0$. Also, we suppose $U_1>0$. Since $n<g_c$, the first term of the last RHS of the equation (9a) has a negative sign. In addition, if $g_c<\tau$, the sign of the partial derivativeis negative meaning that the increase in per capita GDP growth leads to a decline in per capita current account toper capita output i.e. in CA-to-GDP ratio. An early economic growth, allowing that the financial resources would be more available increasingly through time, may drive to deficits in current account particularly if the return rate on foreign assets exceeds the per capita consumption growth. Similarly, as indicated by Cerrato *et al.*, (2015), a smaller economic growth could mean that there are more resources available early on, thus the tendency for a CA deficit early on shrinks. Equivalently, an economic growth, leading to a saving growth and generating lately less available resources, could drive to negative effects. Considering that the fluctuations in savings, and congruously in investment, reflect the GDP fluctuations, these latter affect the current account (Blanchard & Giavazzi 2002). We cannot state that such effects are minor or not, the empirical exploration can help to identify some direct and reversal implications of consumption and saving behaviors on current account dynamics.

While, if $g_c>\tau$, then the multiplier sign will depend on the interaction between the population growth and the per capita growth rates of consumption and output with U_2V_2 and U_1V_1. We find three negative and five positive terms.[7] Byassuming that return rate on foreign assets is closer to per capita GDP growth rate compared to per capita consumption growth rate, then by adding first negative to third positive terms, and second negative to second positive terms we obtain negative result. While adding third negative to fourth positive terms leads to smaller positive result compared to negative one. The final outcome depends on the effects of the remaining first and fifth positive terms. Due to that, these latter values are the smallest ones, the negative multiplier hypothesis dominates. To corroborate this outcome from the literature, Aizenman & Sun (2010) confirms that, despite the speed or slower growth in the Chinese economy, its surplus current account remains constrained by the limited growth of the partner economies supporting deficits current account, which reversely could slow down China economic growth.

Secondly, we determine the multiplier of per capita consumption growth on $\frac{ca}{y}$:

[7] The negative terms are $-g_c[g_c(\tau-g_y)]$, $g[g_c(\tau-g_c)]$ and $n[g_y(g_y-\tau)]$, respectively. The positive terms are $g_y[g_y(g_c-\tau)]$, $g_c[g_y(\tau-g_y)]$, $n[g_c(\tau-g_y)]$, $n[g_y(g_c-\tau)]$ and $n[g_y^2(g_c-\tau)]$, respectively.

$$\frac{\partial\left(\frac{ca}{y}\right)}{\partial g_c}=\left(\frac{U_1}{V_1^2}\right)\left(\frac{U_2}{V_2}\right)+\left(\frac{V_2}{V_2^2}\right)\left(\frac{U_1}{V_1}\right)=\frac{g_y(g_c-g_y)}{V_1^2V_2}+\frac{g_y(\tau-g_y)}{V_2^2V_1}=\frac{g_yU_2}{V_1^2V_2}+\frac{(\tau-g_y)U_1}{V_2^2V_1} \tag{9b}$$

The sign of this multiplier is positive, meaning that the consumption growth rate has a positive effect on the CA-to-GDP ratio. As g_c increases, the economy becomes more "patient" with a smaller early consumption and higher later economic growth. Equivalently, this economy saves more initially and then holds dynamically foreign asset due to its positive current account.

The previous findings, that $\partial(ca/y)/\partial g_y<0$ and $\partial(ca/y)/\partial g_c>0$, indicate that there is no parallel fluctuation between per capita consumption and GDP growth rates. Irrespective to their signs, the two multipliers would have different coefficients, and consequently the dynamic paths of per capita real GDP and per capita real consumption are not homogeneous.

Lastly, we have to determine the effect sign of the population growth rate on per capita current account:

$$\frac{\partial\left(\frac{ca}{y}\right)}{\partial n}=\left[\frac{-(1+g_y)U_1}{V_1^2}\right]\left(\frac{U_2}{V_2}\right)=\frac{-g_y(1+g_y)U_2}{V_1^2V_2} \tag{9c}$$

The population growth multiplier has a negative sign. This result is expected because a rise in population growth rate expands the proportion of dependent children, dependent overage parents, and young savers. This outcome is exhibited in many empirical works as Karras (2009).The new young population takes advantage from the economic efforts of the previous generations, and would lately boost the output growth. In such case, we reach the similar outcome discussed on the GDP growth multiplier. This means that the dynamic interaction between new population through the overlapping generation, consumption and saving could generate lately less available resources, and drive to negative effects on CA-to-GDP ratio growth.

In light of the above outcomes, we can build theoretical models by focusing on a limited number of random variables leading to an optimal level of foreign assets (Sachs, 1982). We can derive an estimable model by linearizing the equation (8)as follows:

$$\frac{ca_t}{y_t}=\beta_0+\beta_1g_{y_t}+\beta_2g_{c_t}+\beta_3g_{n_t},\quad \beta_1<0,\ \beta_2>0,\ \beta_3<0 \tag{10}$$

where the parameters β_i with $i=1,2,3$ are initially the partial derivatives of equations (9). Also, the intercept β_0 will be estimated using the multipliers of the partial derivatives and the sample means of the related variables. Other regressors are highlighted in some previous literature but without offering theoretical consensus as openness index, budget-balance-to-GDP ratio, M2-to-GDP ratio.[8] The GDP and consumption multipliers of equations (9a) and (9b)

[8]As Sheffrin & Woo (2000), Lee & Chinn (2002), Blanchard & Giavazzi (2002), Chinn & Prasad (2003), Corcetti & Müller (2006) and Cerrato *et al.* (2015).

provide a testable restriction between the two partial derivatives named β_1 and β_2, respectively; we can write that:

$$\frac{\frac{ca}{y}}{g_y} = \beta_1 + \beta_2 \text{ or } \frac{\frac{ca}{y}}{g_c} = \beta_1 + \beta_2 \tag{11}$$

This restriction allows testing empirically the validity of the long-run PVMCA. We show by using appropriate elasticities that such restriction could be expressed as

$$\frac{g_y}{g_c} E\left(\frac{ca}{y}, g_c\right) + E\left(\frac{ca}{y}, g_y\right) = 1 \text{ or } \frac{g_c}{g_y} E\left(\frac{ca}{y}, g_y\right) + E\left(\frac{ca}{y}, g_c\right) = 1 \tag{12}$$

then, by using the restriction (11) thelong-run quasi-elasticities of the current account to GDP respecting to per capita output and consumption growth rates should add up to one. According to the opposite signs of each multiplier and then the related elasticities interact in opposite paths. This interaction means that a higher growth rate of consumption tomorrow i.e., later on, involves more saving yesterday i.e. earlier and bring up a positive current account balance. According to Yang, Zhang & Shaojie (2010), such interaction happens in the Chinese economy and leading to surplus current account path. Whereas, a higher output growth tomorrow implies fewer resources yesterday and bringing up a negative current account balance. In such case, the economy should build precautionary saving to face any negative fluctuation mostly in economic growth rate (Sandri, 2011). The issue lies in which among the two dynamic multipliers and their corresponding paths overcomes the other.

5. Conclusion

The stability of the long-run per capita CA-to-GDP ratio requires a positive difference between the return rate on foreign assets and the output growth rate. However, the CA-to-GDP ratio sign depends on the dynamic interactions between population, consumption, and output growth rates. By considering the overlapping generations in the PVMCA framework, via per capita macro-level instead of aggregate macro-level variables, there is no need of the sign presumption of CA-to-GDP.

With the stability condition, and by postulating a “patient” economy, which saves more than an “impatient” economy, the economy can tend to realize surpluses in its current account. It would start with a low level of consumption and save early on. After that, in tendency, the per capita earnings are expected to happen later in life, and the consumption growth could be higher that GDP growth allowing to use up all intertemporal resources. In such perspective, the individual will be more inclined to reduce his/her saving efforts during both the first and last period of his/her economic and social life. The incitation mechanism will work when an increase in population growth leads to reversing the sign difference between per capita consumption and per capita output growth rates, and generates a dynamic surplus in the current account through foreign assets.

In the PVMCA framework, the long-run economic growth rate multiplier has a negative effect on CA-to-GDP ratio. Thus, an economic growth, leading to a saving growth and generating lately less available resources, could drive to negative effects on the current account. However, the consumption growth rate multiplier would positively affect the long-run CA-t-GDP ratio. As there is a "tilt" factor, exercised by the representative consumer towards foreign assets, the economy becomes more "patient" with a smaller early consumption and higher later economic growth. Such economy saves more initially, and then dynamically holds foreign assets due to its positive current account. The output and consumption multipliers provide a testable restriction stating that the long-run quasi-elasticities of the CA-to-GDP with respect to per capita output and consumption growth rates should add up to one. According to the opposite signs of each multiplier, the related elasticities interact in opposite paths, meaning that a higher growth rate of consumption tomorrow involves more saving yesterday and brings up a positive current account balance.

Appendices

Appendix A. Ponzi falsehood condition

If the present value of what the economy consumes and invests exceeds the present value of its output i.e. $\lim_{T\to\infty}(1+\tau)^{-T}B_{t+T+1} < 0$, then the economy continues to borrow and pays the increased interests on the abroad debt instead of converting their real resources to foreign borrowers. This process can be done by reducing $(C+I)$ to less than $(Y-G)$. While if the present value of the output exceeds the present value of what the economy consumes and invests i.e. $\lim_{T\to\infty}(1+\tau)^{-T}B_{t+T+1} > 0$, then the economy does not utilize their resources completely. This implies that the economy will be in excess resources state, which could be invested in foreign financial markets. Besides, from the available resources the economy can increase its utilities by improving slightly the consumption level. When the economy is close to $\lim_{T\to\infty}(1+\tau)^{-T}B_{t+T+1} = 0$, the present value of the output will be equal to the present value of what the economy consumes and invests.

Appendix B. Bellman equation

The dynamic programming is based on a recursive equation involving the value function named Bellman equation (1957) which describes inter-temporally the maximizing path of the utility from consumption. The optimal consumption path from the standpoint of time t should maximize U_{t+1} under the constraint of future wealth W_{t+1} which is generated from present consumption decision C_t. Bellman equation can be as

$$J(W_t) = \max_{C_t}\{u(C_t) + \beta J[(1+\tau)(W_t - C_t)]\}$$

Then, the necessary first order condition (FOC) is: $u'(C_t) - (1+\tau)\beta J'(W_{t+1}) = 0$.

To transform this condition into a familiar expression, we apply the envelope theorem, by considering that the change in the wealth corresponds to the change in the optimal utility. We assume that an increase in wealth at any time has the same effect on the lifetime utility regardless that the wealth is allocated for consumption or saving. By using that $C = C(W)$, we can easily show that $J'(W) = u'(C)$ at each time during the maximizing consumption path. This leads to the same consumption Euler equation: $u'(C_t) = \beta(1+\tau)u'(C_{t+1})$. With isoelastic utility function, we have to find the best guesswork of the value function using Bellman's equation, and we reach the optimal consumption function (Obstfeld & Rogoff 1996). By using the dynamic programming, we obtain that

$$u'(C_s) = \beta(1+\tau)u'(C_{s+1}) = \frac{1+\tau}{1+\delta}u'(C_{s+1}) \tag{B1}$$

With

$$u(C) := \begin{cases} C^{1-1/\sigma}/(1-1/\sigma) & if\ \sigma \neq 1, \sigma > 0 \\ \ln(C_t) & if\ \sigma = 1 \end{cases} \tag{B2}$$

where the positive parameter σ stands for the elasticity of intertemporal substitution.It corresponds to the degree of response of consumption growth to changes in return rate τ on saving. It is defined by $\sigma = -\frac{u'(C)}{C\,u''(C)}$ where $u'(C)$ is determined from the well-known Euler equation for consumption.Knowing that the utility function u_t has a constant relative risk aversion (CRRA), as measured by Arrow-Pratt (1965, 1964), and we have$u'''(C) > 0$. This result indicates that there is a positive motivation for precautionary saving, as measured by Kimball (1990) by the relative prudence $p(C) = -\frac{C\,u'''(C)}{u''(C)} = 1 + \sigma^{-1}$. If $\sigma = 1$, the utility function is logarithmic, a relative risk aversion $a(C) = 1$ and a relative prudence $p(C) = 2$. The utility function is as follows:

$$U_t = \sum_{s=t}^{\infty}(1+\delta)^{t-s}\frac{C_s^{1-1/\sigma}}{1-1/\sigma} = \sum_{s=t}^{\infty}\beta^{s-t}\frac{C_s^{1-1/\sigma}}{1-1/\sigma} \tag{B3}$$

Appendix C. Per capita PVMCA analysis

We suppose that an individual born on date v, living eternally and on any time t he/shemaximizes U_t^vdefined as follows:

$$U_t^v = \sum_{s=t}^{\infty}(1+\delta)^{t-s}\ln(c_s^v) = \sum_{s=t}^{\infty}\beta^{s-t}\ln(c_s^v)$$

where c_s^vrepresents the individual consumption in time s. Assuming that the number of individuals in the economy is N_t and growing with positive growth rate n:

$$N_t = (1+n)N_{t-1} = (1+n)^t \qquad t \geq 0 \qquad (t=0, N_0 = 1)$$

We also suppose that the successive generations would transmit wealth dynamically, through inheritance or bequest, for instance, to face the economic life. The main assumption is that there is no financial wealth or assets holding at birth i.e. $b_v^{P,v} = 0$, where P stands for the parent. The budget constraint for the individual v at time $t \geq v$ is defined by (Obstfeld & Rogoff 1996, page 182):

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+\tau}\right)^{s-t} c_s^v = (1+\tau)b_t^{P,v} + \sum_{s=t}^{\infty}\left(\frac{1}{1+\tau}\right)^{s-t}(y_s - z_s) \qquad \text{(C1)}$$

where z is the government economic activity. The dynamic equation that governs individual asset accumulation is

$$b_{t+1}^{P,v} = (1+\tau)b_t^{P,v} + y_t - z_t - c_t^v \qquad \text{(C2)}$$

When we maximize the individual utility subject to the budget constraint, according to the equation of the initial dynamic wealth and supposing $z = 0$, we obtain the wealth functions w_t^v and w_{t+1}^v as follows:

$$w_t^v = (1+\tau)b_t^{P,v} + \sum_{s=t}^{\infty}\left(\frac{1}{1+\tau}\right)^{s-t} y_s \qquad \text{(C3)}$$

$$w_{t+1}^v = (1+\tau)b_{t+1}^{P,v} + \sum_{s=t+1}^{\infty}\left(\frac{1}{1+\tau}\right)^{s-t-1} y_s \qquad \text{(C4)}$$

$$= (1+\tau)b_{t+1}^{P,v} - (1+\tau)y_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+\tau}\right)^{s-t-1} y_s = (1+\tau)(w_t^v - c_t)$$

Using the consumption Bellman equation $J(w_t^v)$ and with the FOC, we obtain

$$u'(c_s^v) = \beta(1+\tau)u'(c_{s+1}^v) = \frac{1+\tau}{1+\delta}u'(c_{s+1}^v) \qquad \text{(C5)}$$

This equation is similar to Euler equation. By the logarithmic utility function, we have

$$c_{s+1}^v = \beta(1+\tau)c_s^v \Leftrightarrow 1 + g_c = \beta(1+\tau) \qquad \text{(C6)}$$

Inserting this result in the individual budget constraint, we get

$$c_t^v = (1-\beta)\left[(1+\tau)b_t^{P,v} + \sum_{s=t}^{\infty}\left(\frac{1}{1+\tau}\right)^{s-t}(y_s - z_s)\right] \qquad \text{(C7)}$$

Focusing on the aggregate consumption behavior, we have to sum the consumptions of all age-groups (vintages) born since $t = 0$; for age-group $v = 0$ born at $t = 0$, the number of population members is $N_0 = 1$. At $t = 1$, the number is N_1; with a constant population growth rate, we have $N_1 - N_0 = (1+n) - 1 = n$ as members of the age-group $v = 1$. Similarly, we determine the members' number of the second, third cohort, and so on. For any age-group

$v > 0$, the population number is $n(1+n)^{v-1}$. Hence, the aggregated consumption per capita on date t, as macro weighted average consumption, is

$$c_t = \frac{1c_{t,0}+nc_{t,1}+n(1+n)c_{t,2}+\cdots+n(1+n)^{t-1}c_{t,t}}{(1+n)^t} = \frac{c_{t,0}+n\sum_{s=1}^{t}(1+n)^{s-1}c_{t,s}}{N_t} \tag{C8}$$

We can apply such aggregation to any other individual variable to obtain an aggregate per capita variable, which is just the macro variable divided by total population. We deduce, from the RHS of the previous equation, an expression for b_{t+1}^P and knowing that $b_{t+1}^{P,t+1}=0$, we get

$$b_{t+1}^P := \frac{b_{t+1}^{P,0}+n\sum_{s=1}^{t+1}(1+n)^{s-1}b_{t+1}^{P,s}}{(1+n)^{t+1}} \Rightarrow (1+n)b_{t+1}^P = \frac{b_{t+1}^{P,0}+n\sum_{s=1}^{t}(1+n)^{s-1}b_{t+1}^{P,s}}{N_t} \tag{C2b}$$

where $b_t^P \coloneqq \frac{B_t^P}{N_t}$ represents the average per capita value at time t of the net financial assets that the individuals own from time $t-1$. From the equation (C2a) and the last expression of (C2a), we can write that

$$(1+n)b_{t+1}^P = (1+\tau)b_t^P + y_t - z_t - c_t \Rightarrow b_{t+1}^P = \frac{(1+\tau)b_t^P+y_t-z_t-c_t}{1+n} \tag{C2c}$$

Also, the equation of the aggregate per capita consumption is simply related to b_t^P; we get

$$c_t = (1-\beta)\left[(1+\tau)b_t^P + \sum_{s=t}^{\infty}\left(\frac{1}{1+\tau}\right)^{s-t}(y_s - z_s)\right] \tag{C8b}$$

Now, from the equations (C2c) and (C8b), we determine the dynamic equation that governs aggregated private assets accumulation:

$$b_{t+1}^P = \frac{\beta(1+\tau)}{1+n}b_t^P + \left[\frac{(y_t-z_t)-(1-\beta)\sum_{s=t}^{\infty}(1+\tau)^{-s+t}(y_s-z_s)}{1+n}\right] \tag{C2d}$$

By assuming that $z_t := \zeta y_t$ and $b_t^P \coloneqq b_t$, and concentrating on the steady-state balanced growth path, we can rewrite the aggregate per capita consumption(C8b) under the hypothesis $\frac{1+g_y}{1+\tau} < 1$:

$$c_t = (1-\beta)\left[(1+\tau)b_t + \frac{1+\tau}{\tau-g_y}y_t(1-\zeta)\right] \tag{C8c}$$

Therefore, the relationship that governs the private dynamic assets accumulation would be as follows

$$b_{t+1} = \left[\frac{\beta(1+\tau)}{1+n}\right]b_t + \left[\frac{\beta(1+\tau)-(1+g_y)}{(1+n)(\tau-g_y)}\right]y_t(1-\zeta) \tag{C2e}$$

The coefficient $\beta(1+\tau)$ can be interpreted as inclination or tilt of an individual's consumption path. In the framework of small-open-economy hypothesis and according to the outcome(C6), if $\beta(1+\tau) > 1+n$, then the individual can during his age-period accumulate financial assets over time. The per capita aggregated assets would continue to increase in tandem with the positive world real economic growth, and even though the consumption growth rate is greater than the population growth rate i.e. despite the instability of the dynamic equation of per capita foreign financial assets. While if $\beta(1+\tau) < 1+n$, the new age-group members, even though with no inheritance or bequest, they come in the economic activities suitably more rapidly that the per capita macro foreign assets reach a stable steady-state. Besides, whenever the consumption growth rate is positive, then the population growth rate should be positive, and per capita aggregated foreign assets path converges and will be stable if $g_c < n$. Since there is positive real economic growth, we can convert the equation (C2e) to stationary form by dividing both sides by y_{t+1}, we find

$$\frac{b_{t+1}}{y_{t+1}} = \left[\frac{\beta(1+\tau)}{(1+n)(1+g_y)}\right]\frac{b_t}{y_t} + \left[\frac{\beta(1+\tau)-(1+g_y)}{(1+n)(1+g_y)(\tau-g_y)}\right](1-\zeta) \tag{C2f}$$

where $\frac{b_t}{y_t}$ represents net foreign assets to GDP ratio. When $\beta(1+\tau) > (1+g_y)$, the economy will have positive net foreign assets. The slope of the equation $(C2f)$ shows that a rise in the per capita real output growth rate g_y lowers the aggregate long-run net-foreign-asset-to-GDP ratio. It seems that per capita income increases along with his/her life horizon or that earnings are expected to happen later in life, this belief makes the individual more inclined to reduce his/her saving efforts during both the first and last period of his/her economic life. We can intuitively understand this result by the fact that faster GDP growth incites all age-groups to save less.

Also, the equation $(C2f)$ shows that the path of net-foreign-asset-to-GDP ratio becomes unstable becomes unstable if $\beta(1+\tau) > (1+n)(1+g_y)$. While if $\frac{1+g_c}{1+n} < 1+g_y$ i.e. the growth of the average propensity to consume for each generation is less than $(1+n)$, the previous dynamic path will be stable. This case is close to the rational behavior, which does not push the individual to replicate the pattern consumption of his/her generation even if the banking system incites the families to borrow more. Normally, the "tilt" factor should be reduced when the individual expects that his/her consumption growth exceeds his/her income. But, if the slope of the equation $(C2f)$ is less than one, and considering that the process $\frac{b_t}{y_t}$ is stationary, we obtain the corresponding long-run equation

$$\frac{b}{y} = \frac{(1+g_c)-(1+g_y)}{[(1+n)(1+g_y)-(1+g_c)](\tau-g_y)}(1-\zeta) \tag{C2g}$$

This exhibits that the coefficient of the net output depends on the sign of the difference between per capita growth rates of consumption and GDP. When such difference is positive, it corresponds to the consumption "tilt" factor, which could be in fact amplified through the borrowing from banks. The equation (C2g) indicates that in the economy the members of each age-group could have loans when $\frac{1+\tau}{1+\delta} > 1+g_y > 1$, because they hold foreign assets and take advantage of profitability in international financial markets in particular when the return rate is greater than the expected discount rate.

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