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## Volatility stylized facts in the Moroccan stock market: Evidence from both aggregate and disaggregate data

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**Abstract.** Financial markets in emerging countries are generating considerable literature, aiming to understand their organization, perspective, and performance. In this context, few studies have expressed interest in the Moroccan financial market and even fewer researches have addressed the issue of the Moroccan financial market volatility. In this paper, we investigate variety of common properties, labelled as “stylized facts. Our results show that global and sectoral indices of Moroccan Stock Market share the majority of stylized facts. In fact, absolute returns correlation coefficients are positive and tends to decay at a much slower pace. Hence, volatility of Moroccan Stock Market captures the properties of volatility clustering and long memory. We also find evidence of volatility asymmetry. Yet, the level is not statistically significant for most of the indices. More interestingly, the Omori law indicates that Moroccan Stock market is relatively stable after financial shocks.

**Keywords.** Asymmetry, Long memory, Multifractality Omori law, Stylized facts, Volatility.  
**JEL.** G11, G17, C53, C58.

### 1. Introduction

The study of volatility dynamics has become a topic of concern to researchers, academics, practitioners and regulators. In fact, volatility modelling is of paramount importance and its forecast is crucial in asset valuation, risk management and monetary policy design (Poon & Granger, 2003). Several reasons have been advanced as to explain the growing interest in this issue. First, it is a key element in assessing market risk. In fact, an adequate estimate of volatility is crucial to determine more finely the probable losses and to compute, infinite, the economic capital likely to cover the market risk. Second, volatility is a key parameter in pricing derivative securities (Ederington & Guan, 2006). Black & Scholes (1973) consider that the price of a call option is a function of the current value of the underlying stock, its volatility, the residual maturity, the strike of the option, and the risk-free rate. Thus, in order to price options, the

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model requires estimating volatility, which is the only unobservable parameter. Third, volatility estimation is essential to building optimal portfolios. In this sense, an adequate measure of volatility enables individuals to compare the risk between different asset classes, in order to determine the optimum allocation that is appropriate for an investor, given his objective and his tolerance for risk. Accordingly, investors and practitioners always keep a close eye on volatility evolution (Bollerslev, Gibson, & Zhou, 2011). Fourth, the excess volatility of the firm is an important element in determining the probabilities of bankruptcy (Daly, 2011). For example, the KMV Credit Monitor model suggests that computing these probabilities requires estimating the volatility of financial assets. Fifth, volatility is a significant factor in determining the bid-ask spread. Indeed, low (high) volatility translates into a narrow (broad) price range.

Sixth, financial crises have dramatically increased volatility spillover and contagion among global financial markets. In this regard, the analysis of financial market volatility is more justified by the fact that market shocks can have a huge impact on the real world (Banque de France, 2003). Thus, decision-makers base their perception and anticipation of the evolution of the economy by using volatility as a barometer of the strength of the system. For example, the Federal Reserve (FED) takes into account the volatility of equities, commodity, bonds and exchange rates to establish its monetary policy. In a similar vein, volatility and market contagion are the main sources of investor loss of confidence and a reduction in capital flows (Baillie & DeGennaro, 1990); hence, an adequate forecast of market volatility is essential for efficient decision-making (Maddala & Rao, 1996). Finally, the Basel agreements have emphasized the development of internal models of risk management. As a result, the estimation of volatility becomes a mandatory exercise for several institutions in order to better assess their own risks and to forecast economic capital.

Volatility of the stock market refers to the fluctuation of the price of the main stock indices over a given period. It is associated to risk, yet it is not the same as risk (Poon & Granger, 2003). It is an unobservable parameter still, but can be estimated albeit there has been some disagreement concerning its estimation, the simplest and most common measure is the weighted average of squared deviations of expected returns over a given period. Figlewski (2004) states that, for small samples, the sample mean return is not a relevant estimator of the true mean. In that sense, one can use the square return as a proxy of the conditional variance. On the other hand, Staudte & Sheather (1990) and Huber (1996) point out that adjusted absolute mean deviation is a robust estimator especially when the data is contaminated by measurement errors. Furthermore, the interquartile range, which is a measure of the statistical dispersion, could be used to measure fluctuations in financial markets. It should be noted that some authors suggest the use of intraday data for estimating volatility. Indeed, Parkinson (1980) proposed the first advanced estimator. He recommended the use of both the highest and lowest prices of each trading day instead of

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closing prices. In a related move, other authors proposed an extension of the Parkinson estimator suggesting incorporating opening, lowest, highest and closing prices in their formula for computing volatility (See [Roger & Satchell, 1991](#); [Garman & Klass, 1980](#) and [Yang & Zhang, 2000](#)).

Moreover, there are several salient features about financial time series and financial market volatility that are now well documented ([Poon & Granger, 2003](#)). These stylized facts include fat tail distributions, volatility clustering, asymmetry, persistence and mean reversion. That said, the use of mathematical models to improve the ability of forecasting is justified. In that sense, several models having been proposed to account for time-varying variance and for the stylized facts of financial markets. The examination of the volatility of stock market returns by current econometric models has been the subject of a large number of empirical works, the most popular of which to forecast volatility are the GARCH family models. Since their introduction by Bollerslev, the literature of GARCH models has grown considerably, which makes it impossible to provide a complete literature review on this subject. However, we can cite the works of Engle (1991), Bera & Higgins (1993), Bollerslev, Engle & Nelson (1994), Diebold & Jose Lopez (1995), Bauwens *et al.*, (2006), Teräsvirta (2006), Silvennoinen & Teräsvirta (2007), Chou (1988), Day & Lewis (1992), and Lamoureux & Lastrapes (1993), Bailie, Bollerslev, & Mikkelsen (1996). In light of this, this article aims to describe the various stylized facts that have been shown to affect the volatility in developed markets. To our knowledge, this particular area has been neglected in the Moroccan case. Hence, we aim to address the following questions:

(1) Does Moroccan stock market exhibit the main stylized facts such as leptokurtosis, volatility clustering, persistence, long memory and leverage effects? (2) How does Moroccan Stock Market react to financial shocks?

The structure of the research proceeds as follows. Section 2 provides an overview of some stylized facts. This is followed by a review of literature (section.3); whereas data and methodology of this paper are described in Section 4. In section 5, we discuss the main results. Finally, Section 6 concludes this research.

## 2. Stylized facts

Modelling volatility is often guided by facts characterizing financial time series. There are several common features of financial series that are now well documented. The knowledge of such facts may be useful for establishing reliable nonlinear empirical models to forecast volatility. More specifically, the empirical properties we study in this article are the following:

### 2.1. Stationarity and ergodicity of logarithmic returns

Many studies on time series have been based on the assumptions of stationarity and ergodicity. In general, a stochastic process is weakly stationary if its mean and variance are invariant and finite over time and

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the value of the covariance depends only on the lag between the two time periods. With regard to the ergodicity property in the weak sense, the mean and the temporal covariance of a stationary series converge respectively in probability towards the expectation and the theoretical covariance when  $N$  tends to infinity.

### 2.2. Absence of autocorrelations of return and long-term correlation of volatility

In general, the autocorrelation function decays quickly to zero. That is, financial series returns exhibit significant serial correlation for a very short amount of time (Cont, 2001). The lack of autocorrelations in returns provides strong support for market efficiency (Fama, 1991). In this sense, it is not possible to predict future returns. On the other hand, the autocorrelation coefficients of the absolute returns and the square returns are significant, positive and slowly decreasing. However, for some illiquid markets, the correlations of returns over very short time scales may be significant because of the effects of microstructure and the non-synchronization of trading. In addition, the absence of autocorrelations does not seem to occur for low frequency series (weekly and monthly data).

### 2.3. Volatility clustering

It has been observed that financial assets volatility occurs in clusters. In fact, the periods of large fluctuations of returns alternate with periods of variations of the same magnitude; while the small variations are generally followed by small variations (Mandelbrot, 1963; Fama, 1965). Statistically speaking, this grouping by volatility flashes is synonymous of positive autocorrelations of returns. The property of accumulation of volatility implies an ability to improve the forecast of volatility. In this sense, several models have been proposed to capture this stylized fact. The most popular one is the Generalized Autoregressive Conditional Heteroscedasticity model, which relates conditional volatility to past volatility.

### 2.4. Volatility asymmetry

It is commonly accepted that stock market volatility is asymmetric. Indeed, the effects of positive and negative shocks can have a different impact on volatility. Specifically, volatility tends to be higher when the price change is downward than when it is upward. In this sense, Black (1976) has shown that stock market returns are negatively correlated with changes in volatility. This phenomenon of asymmetry results mainly from two effects: the financial leverage effect and the retroactive effect<sup>2</sup> (Banque de France, 2003). The first effect involves the asymmetric impact of the change in the firm's leverage. Indeed, a decrease in the price of a company's stock, following the arrival of bad information, reduces the value of equity and increases the leverage effect. This increase in leverage results in a weakening of the company's structure and, as a result, excessive volatility

<sup>2</sup>Known as feedback effect

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in stock prices. Conversely, higher returns translate into lower leverage and lower market volatility. On the other hand, the retroactive effect refers to the fact that, in an uncertain environment, anticipated changes in future volatility encourage investors to demand an additional risk premium on their investments to pay more for assets that become riskier, causing prices to fall. In addition, this price variation is more sensitive to the rise than the fall in volatility of the same magnitude, since investors tolerate less risk in the event of an increase in volatility. Wu (2001) shows that the retroactive effect is most remarkable when the covariance between a company's returns and that of the market is stronger.

### 2.5. Mean reversion

Another property that characterizes financial assets is mean reversion. Indeed, shocks tend to fade in the long run. Hence, conditional volatility will always return to its long-run volatility. However, the necessary speed of return is unknown. The so-called half-life is the time taken by the volatility shock to cover half the distance back towards its mean volatility after a deviation from it. It has been suggested that the persistence of shock implied in GARCH class models is spuriously high in the presence of structural breaks, resulting in high volatility persistence (Lamoureux & Lastrapes, 1990). Hence, numerous studies that have been conducted in this direction proved that the Markov Regime-Switching GARCH (MRS-GARCH) model, a relatively new model, could statistically reduce the persistence of shocks on volatility forecasting and allow the clustering to be generated by state varying.

### 2.6. Long memory / multifractality

The concept of long memory, introduced for the first time by Hurst in 1950, was expanded to include meteorology, economics and finance. This concept refers to a very long-lasting impact of changes in volatility on future movements. Indeed, empirical work has shown that the autocorrelation function of absolute logarithmic returns remains for a very long time significantly positive and indicates the existence of dependence between the different logarithmic returns. Evidence shows that it decreases in law in a hyperbolic way:

$$\gamma_a(h) = \text{Corr}[\delta_{\Delta t} X(t), \delta_{\Delta t} X(t+h)] \underset{h \rightarrow +\infty}{\sim} \frac{c}{h^\alpha} \quad (1).$$

In a more practical way, to say that there exists a long memory is therefore to verify if the series of absolute correlations do not absolutely converge:  $\sum_{h=1}^{+\infty} |\gamma_a(h)| = +\infty$ . On the other hand, multifractality can be defined as a condition where there exists a "scaling property in moments of the process" (Mandelbrot *et al.*, 1997).

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### 2.7. Non-normality, fat tail distribution and Extreme Events

As early as the 1960s, Mandelbrot pointed out that the normal distribution on which financial theory is based, is not adequate to model the returns of financial series. Indeed, empirical works have shown that the number of large (either positive or negative) returns is far bigger than what is expected on the basis of modern finance theory. Moreover, shocks have a strong impact on volatility and lead generally to a number of aftershocks. This is again a feature that most financial models are unable to replicate.

## 3. Review of literature

### 3.1. Overview of Casablanca Stock Exchange

The creation of the Casablanca Stock Exchange (CSE) dates back to 1929, under the name of "Office de Compensation des Valeurs mobilières" (Office for Clearing Transferable Securities), later renamed in 1948 as "Office de cotation des valeurs mobilières" (Securities Trading Office). Here, one may note that the organization and functioning of the Casablanca Stock Exchange have remained almost immutable for more than half a century despite the evolution of the Moroccan economy. In fact, the organization of the market hampered its attractiveness at a time when domestic investors were showing a grand interest in investing in the stock market. To mitigate and cope with these shortcomings, a reform took place in 1967 the aim of which was to provide the Moroccan financial market with a stock exchange that is legally and technically well organized. In 1983, the Program of Structural Adjustment (PSA) was started, to be completed ten years later, in order to consolidate the fundamental equilibrium, improve the investment climate, carefully control debt and inflation, and enhance the liberalization of the financial sectors.

During the last few years, Morocco has continued to undergo major reforms to develop its capital market. Major reforms of the stock market were initiated and put forward in 1993 to complete and strengthen the regulatory and technical framework. Along this, the authorities promulgated three founding texts, namely the Dahir providing Law No. 1-93-211 of 21 September 1993 relating on the Stock Exchange, the Dahir providing Law No. 1-93-212 on the Council of Ethics for Securities (CDVM), newly named the Control Authority of the Capital Market (AMMC), and the Dahir providing Law No. 1-93-213 on Undertakings for Collective Investment in Transferable Securities (OPCVM). The main objective of these reforms was to develop a substantial modern market, through the creation in July 1994 of the CDVM whose mission was primarily to ensure the control of transactions and protect savings. The market authorities also created the "Société de Bourse des Valeurs de Casablanca" (Stock Exchange Company of Casablanca "SBVC"), a private company responsible for the managing of the Casablanca Stock Exchange. In November 1996, the authorities established Undertakings for Collective Investment in Transferable Securities (UCITS), financial intermediaries for the management of securities portfolio.

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Four years later, in 1997, another set of reforms was undertaken to complement the fundamental laws of 1993 and to enhance supervision, security and transparency. This was through the creation of the Central Depository of securities (MAROCLEAR), an entity that ensures the dematerialization of securities and their storage, as well as the administration of all events related to the securities lifecycle. On a technical level, the reform initiated the use of an electronic trading system. This transition from an auction to an electronic listing has been generalized to all securities listed on the Casablanca Stock Exchange since August 1998.

With regards to Moroccan stock market performance, CSE is considered Africa's second-largest Stock Exchange after the Johannesburg Stock Exchange. Following the reforms, the period 2002-2007 was marked by a rise in performance of the CSE. Indeed, the market capitalisation increased from 87.2 billion dirhams to 531.75 billion dirhams. In other words, the ratio stock market capitalization to GDP jumped from 20.46 % to 95.1%. Similarly, trading volume experienced growth from 22.4 billion dirhams to 359.7 billion dirhams at a rate of 1600%. However, the stock market fell by 20% in 2007 due to the subprime crisis. In the aftermath, the Casablanca Stock Exchange experienced a volatility of 70 billion dirhams in March 2008. This market fluctuation is considered the most extreme instability in the history of the Moroccan stock market. This confirms that financial crisis has no spared any financial market in the world. Table1. illustrates the continuous fall of market capitalisation and volume trading during the period 2008-2015. It was not until 2016, that market capitalisation reverted to its highest historical value.

**Table 1.** *Casablanca Stock Exchange development*

Year	N of Listed companies	Volume of trading In billion dirhams	Number Of transactions	Market Capitalization In billion dirhams
2019	75	75	147993	626
2018	76	52	187015	582
2017	74	69	290548	625
2016	75	73	188 685	583
2015	75	52	130 477	453
2014	75	50	154 887	484.4
2013	75	62	125 243	451.1
2012	77	61	156984	445
2011	76	103	218970	516
2010	73	120	330 084	579
2009	77	72	285 460	508.9
2008	77	122	470 175	531.75
2007	73	359.7	468 953	586.3
2006	64	166.4	237 997	417.1
2005	54	148.5	160 444	252.3
2004	52	71.8	72 625	206.5
2003	52	53.7	38754	115.5
2002	55	22.4	37949	87.2

**Source:** compiled by the authors based on the CSE annual reports.

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### 3.2. Stylized facts in Moroccan Stock Market: a review of literature

Financial markets in emerging countries are generating considerable literature, which aims to understand their organization, perspective, and performance. In this context, few studies have expressed interest in the Moroccan financial market and even fewer researches have addressed the issue of the Moroccan financial market volatility. In his paper, Bakir (2002) suggested that the Moroccan Stock Market has not yet reached the maturity stage of developed markets. He states that financial liberalization reforms did not result in a significant improvement of the efficiency of the financial market. In this sense and based on Azur classification (1997), El Bakkouchi (2014) considers that Moroccan stock market is in an early stage of development characterized by basic regulatory and institutional frameworks, high volatility, limited number of listed companies and low market capitalization.

For research that included Moroccan market, Limam (2003) investigated long memory properties of stock market index returns for fourteen developed and developing countries. Using parametric and semi parametric estimation procedures of Geweke-Porter-Hudak (GPH) and modified rescaled range statistics (R/S), he found evidence of long memory in the studied Arab countries, except Jordan. In contrast, developed markets and some emerging markets have short memory. The author concludes that fractional integration dynamics in stock returns is strongly linked to the level of development in stock markets. Likewise, Assaf (2006) provides empirical evidence of the long memory behaviour in return volatility of the stock markets of Egypt, Jordan, Morocco, and Turkey, but long memory for return only in the stock returns for Egypt and Morocco. In the same vein, (Alagidede, 2011) apply FIGARCH model to a sample of African stock market data with varied commencement and ending dates from 10 January 1995 to 16 November 2006. The author shows evidence of long memory in the equity markets of Egypt, Kenya, Morocco, Nigeria, Tunisia and South Africa. Similarly, Anoruo & Gil-alana (2011) examines the existence of mean reversion in the stock market prices in ten African countries by means of long-range dependence techniques. The results failed to find evidence of mean reversion for all of the stock market price series in all cases. However, the author found evidence of long memory in the returns and volatility for Egypt, Morocco, Tunisia and Nigeria.

Boubaker & Makram (2012) explore heavy tails and double long memory in three North African stock indices, namely TUNINDEX (Tunisia), MASI (Morocco) and EGX30 (Egypt). He demonstrates that an  $\alpha$ -stable distribution better explains the behaviour of return than the normal distribution. The results also show evidence of long memory in both returns and volatility. In that sense, the authors demonstrate that long memory dynamics in the returns and volatility is better modeled by the joint ARIMA-FIGARCH model. For the case of Moroccan stock market, the best model for capturing the dual long-memory property in the returns and volatility of MASI is the ARIMA (1,0.047,2)-FIGARCH (1,0.257,1)



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specification. The author concludes that the double long-memory model can provide a better explanation for long-memory dynamics in both returns and volatility.

Still, a challenging question is to assess whether the identified long memory is real or spurious. For this purpose, Assaf (2015) examines the presence of long memory in returns and volatility of the MENA equity markets, including Morocco. The author breaks the full period of study to two subsample periods, using unit root tests that allow for structural breaks and employing the Bai & Perron (1998, 2003a, 2003b) to test for multiple breaks in the mean returns. The results indicate that the volatility measures represented by absolute and squared returns show evidence of long memory for the full and subsample periods, while the returns show a weak evidence of long memory. However, the returns and volatility measures display less evidence of long memory in the after-crisis period as opposed to the before-crisis period. The authors attribute these findings to financial and economic conditions that took place in the MENA region after the crisis. In the same vein, Geoffrey, Tah, & Darrat, (2017) state that when structural breaks are ignored, the results indicate the existence of long memory components in stock returns and variance in the majority of the African markets. However, once structural breaks are introduced in the testing models, the long memory evidence significantly dissipates and the results support instead short memory behavior across markets.

As far as multifractality is concerned, Benbachir & El Alaoui (2011) employed the Multifractal Detrended Fluctuation Analysis (MF-DFA) method to study the multifractal behaviour of the Casablanca Stock Exchange. The results showed the existence of two principal sources of multifractality: The long-range temporal correlations and the fat-tail distribution. They also documented that MASI exhibits a richer multifractal feature than MADEX. Similarly, Lahmiri (2017) attempted to investigate the existence of fractality and chaos in returns and volatility of family business companies listed on CSE, and also in returns and volatility of the CSE market index. The author concluded that most family business stocks and market index exhibit long memory in volatility. Concerning chaos, the results revealed that only volatility of market index was chaotic.

Turning to volatility asymmetry, Brooks (2007) used APARCH model to estimate the volatility in a range of Middle East and African equity markets including Moroccan Stock Market Index from January 1995 to December 2005. The findings reveal that the majority of markets exhibit standard leverage effect. Furthermore, Al-Hajieh (2015) investigate the asymmetric property of stock market volatility for 17 Islamic indices. He shows, by using EGARCH and GJR-GARCH models, that conditional variance exhibits long persistence of volatility for all countries. The EGARCH and GJR-GARCH results confirm that the conditional variance is an asymmetric function of the past residuals; however, this is not statistically significant in the cases of Tunisia, Morocco, Lebanon, Bahrain and Oman.

In contrast to earlier findings, Coffie (2017) found evidence of reverse volatility asymmetry in both Morocco and BVRM stock markets. In fact, the

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GJR estimates imply that positive instead of negative shocks will have a higher effect on future volatility. This is an interesting result controverting the widely accepted theory of volatility asymmetry (i.e. bad news induces a higher return volatility than positive news). The author attributes these results to higher trading volume associated with price rising stocks. In fact, there is an excessive demand in both markets for such stocks relatively to their contrarian counterparts and this leads to the arousal of higher volatility for positive returns than negative returns.

### 4. Data and methodology

#### 4.1. Data

Our data consists of daily closing price series for the global indices: MASI and MADEX along with daily prices of 20 sectoral indices of the Moroccan equity stock market. All the series were collected from the Casablanca Stock Exchange Website. The selection of sectoral indices is primarily based upon data accessibility and the historic length. Hence, some sectors are excluded from our analysis. Depending on the index considered, the series span between 2503 and 4041 trading days. Figure 1 presents the historical price of our series. At first glance, it appears that Insurance, Food Producer & Processors and Banks indices are having an upward trend, while Real Estate, Leisures & Hotels and Paper & Forestry among other sectors have downward trends. The daily return series is expressed in logarithmic difference between the two successive prices acquiring the continuous compounding return:  $R_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$  Where  $\ln$  is the natural logarithm,  $P_t$  is current closing price and  $P_{t-1}$  is previous closing price.

#### 4.2. Methodology

Our empirical investigation of the stylized facts begins with the descriptive statistics as to specify the distributional properties of our financial series. This is followed by formal evaluation of stationarity of logarithmic returns, using Advanced Dickey Fuller, Philips Perron and KPSS tests. Then, we analyse the autocorrelations of returns and absolute returns. A possible significant positive autocorrelation of nonlinear functions of returns is a quantitative sign of volatility clustering. Moreover, we fit the average autocorrelation function with an exponential and power decay as to investigate the long memory property. A natural extension of this analysis is to estimate the fractional differencing parameter ( $d$ ) of an ARFIMA model. In that sense, we use in section 5.6 the partition function as to determine the fractal character of the series. Furthermore, we employ GARCH methodology as to determine the speed at which the return revert to its long run mean (Half Life). On the other hand, we uncover the asymmetric behaviour of our data by using exponential GARCH model. This specification considers that a negative shock leads to a high

conditional variance with respect to positive shocks. Furthermore, Finally, the Omori law techniques are used to describe the dynamics of aftershocks.

## 5. Results and discussion

### 5.1. Descriptive statistics

By inspecting the descriptive statistics listed in Table 2, it is possible to specify the distributional properties of our financial series. The average daily returns are small relative to the standard deviation for all the indices<sup>3</sup>; they vary between  $-5.20e-05$  and  $4.55e-04$ . Annualizing the daily log returns for each series implies average annual returns ranging between of approximately  $-1.88\%$  and  $18.06\%$  for the “EEE” and “AGRO” Indices respectively. Standard deviation is small for the rest of the indices. For instance, volatility is about 0.007 for the global indices. More interestingly, some sectors with relatively higher return yields to small risk than other indices, for instance “AGRO”, “BANK”, and “DISTR”. This finding is inconsistent with the risk-return trade off advanced by portfolio theory. On the other hand, the returns are substantially negatively (positively) skewed for fourteen (eight) series implying that there is a high probability to earn superior (inferior) returns than the mean return. Finally, all the returns distributions are superior to three. These are indicative of a heavy-tailed non-normal distribution. This assumption, which is in line with most finding, is further confirmed by Q-Q plot and Jarques-Bera test statistics<sup>4</sup>.

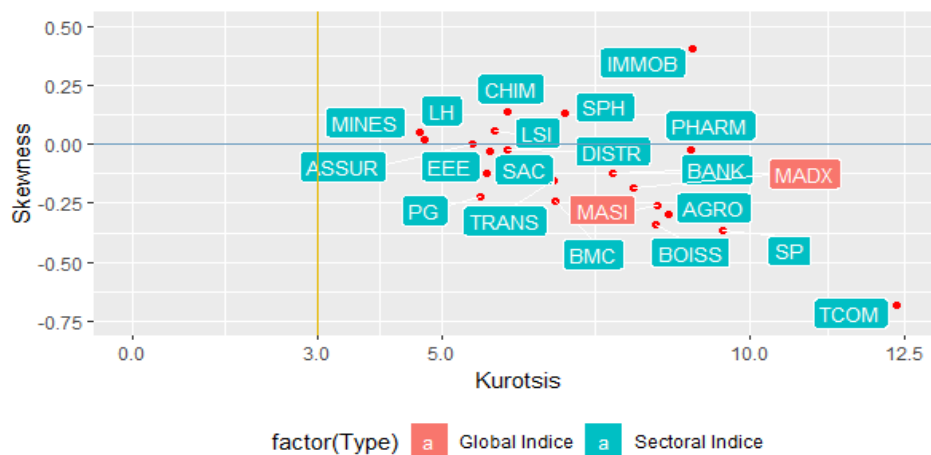


Figure 1. Skewness VS Kurtosis for sectoral and global indices

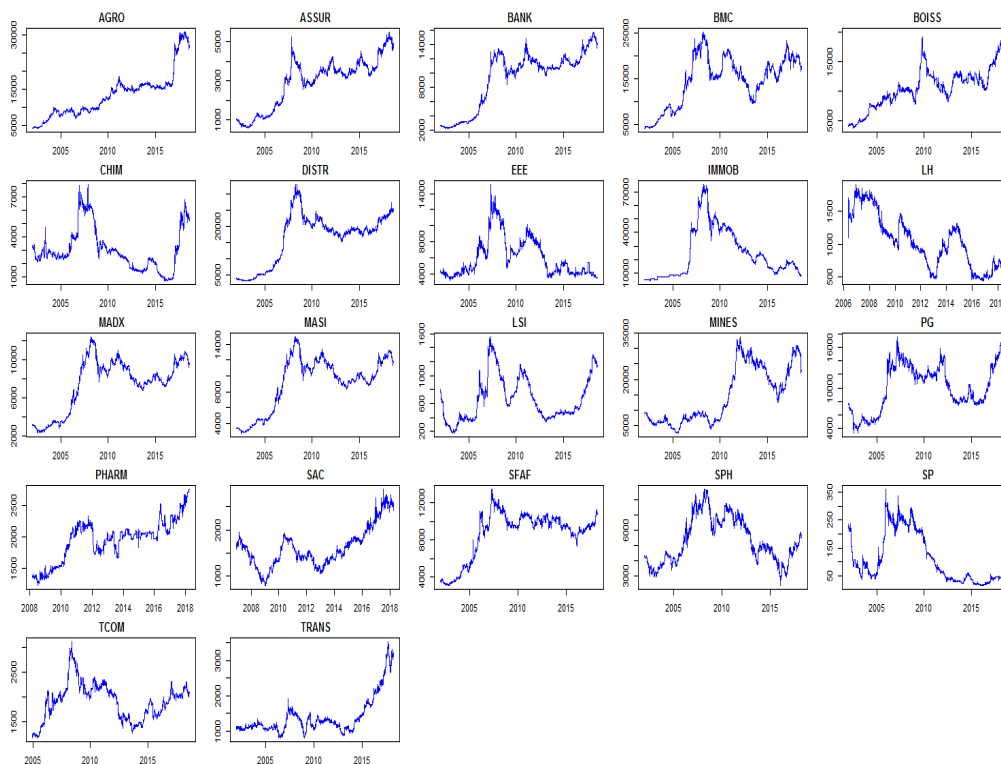
<sup>3</sup>The null-hypothesis hypothesis that the mean is zero is not rejected when using t-test

<sup>4</sup>Results can be sent upon request.

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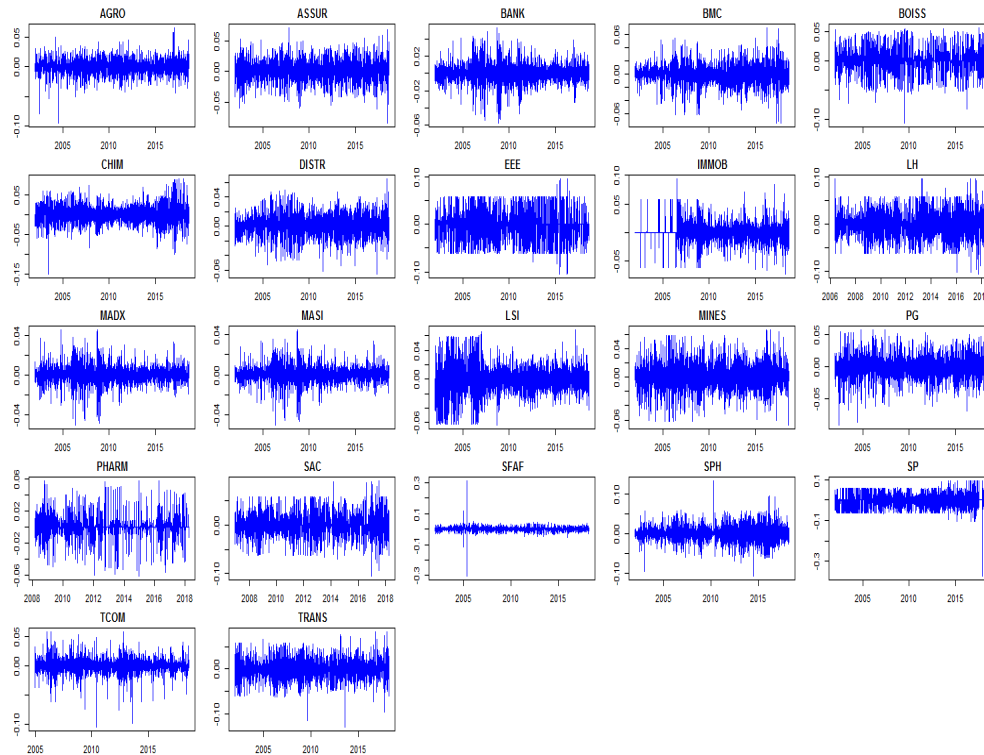
**Table 2.** Descriptive statistics

Sectors	Tickers	Start	#Daily Obs	Min	Mean	Max	SD
MADEX	MADX	03/01/2002	4140	-0.05	2.6e-04	0.05	0.008
Moroccan All Shares	MASI	03/01/2002	4140	-0.05	2.9e-04	0.04	0.007
Food Producer & Processors	AGRO	03/01/2002	4140	-0.09	4.50e-04	0.07	0.010
Insurance	ASSUR	03/01/2002	4140	-0.08	3.7e-04	0.07	0.014
Banks	BANK	03/01/2002	4140	-0.06	4.0e-04	0.05	0.010
Construction & Building Materials	BMC	03/01/2002	4140	-0.07	3.5e-04	0.07	0.013
Beverages	BOISS	03/01/2002	4140	-0.11	3.3e-04	0.06	0.015
Chemicals	CHIM	03/01/2002	4140	-0.15	1.1e-04	0.09	0.021
Distributors	DISTR	03/01/2002	4140	-0.07	4.4e-04	0.06	0.012
Electrical & Electronic Equipment	EEE	03/01/2002	4140	-0.10	-5.2e-05	0.09	0.023
Real Estate	IMMOB	03/01/2002	4140	-0.07	1.0e-04	0.09	0.015
Leisures and Hotels	LH	15/05/2006	3050	-0.10	-1.1e-04	0.09	0.024
Software & Computer Services	LSI	03/01/2002	4140	-0.06	8.1e-05	0.07	0.017
Mining	MINES	03/01/2002	4139	-0.07	2.3e-04	0.07	0.017
Oil & Gas	PG	03/01/2002	4139	-0.09	1.3e-04	0.06	0.016
Pharmaceutical Industry	PHARM	06/03/2008	2503	-0.06	2.8e-04	0.06	0.013
Utilities	SAC	12/12/2006	2813	-0.10	1.3e-04	0.09	0.020
Inves Companies & Other Finance	SFAF	07/01/2002	4029	-0.30	2.7e-04	0.31	0.012
Holding Companies	SPH	07/01/2002	4041	-0.11	6.8e-05	0.13	0.017
Paper & Forestry	SP	03/01/2002	4139	-0.37	-4.8e-04	0.01	0.030
Telecommunications	TCOM	16/12/2004	3401	-0.10	1.7e-04	0.06	0.011
Transport	TRANS	07/01/2002	4040	-0.13	2.6e-04	0.08	0.018



**Figure 2.** Historical price of global and sectoral indices

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**Figure 3.** Returns of global and sectoral indices

### 5.2. Stationarity of logarithmic returns

A Figureical plot is presented in Figure 2. As it can be seen, the daily returns of sectoral and global indices are stationary. Augmented Dickey Fuller, Philips Perron as well as KPSS confirm this result (See Table 3).

**Table 3.** Tests of Stationarity

Indices	ADF	PP	KPSS
AGRO	-14.952***	-4198.257***	0.103***
ASSUR	-14.611***	-4250.232***	0.138***
BANK	-15.477***	-3304.018***	0.434***
BMC	-14.458***	-3847.695***	0.399***
BOISS	-15.197***	-4010.844***	0.102***
CHIM	-14.877***	-3827.335***	0.225***
DISTR	-14.887***	-4321.892***	0.645***
EEE	-16.149***	-3623.444***	0.154***
IMMOB	-13.255***	-3370.082***	1.055***
LH	-14.736***	-3068.897***	0.101***
MADX	-14.408***	-2861.51***	0.336***
MASI	-14.346***	-2863.157***	0.456***
LSI	-13.907***	-3636.617***	0.254***
MINES	-14.279***	-3572.408***	0.167***
PG	-14.484***	-3661.186***	0.096***
PHARM	-14.561***	-2434.891***	0.047***
SAC	-15.107***	-2715.189***	0.227***
SFAF	-15.422***	-4687.235***	0.352***
SPH	-15.177***	-4558.539***	0.026***
SP	-15.821***	-3943.132***	0.096***
TCOM	-13.937***	-2969.679***	0.212***
TRANS	-16.728***	-3955.80***	0.200***

Notes: \*\*\*, \*\*, \*: significant at respectively 1%, 5% and 10% level.

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### 5.3. Absence of autocorrelation of returns mean reversion and volatility clustering

#### 5.3.1. Analysis of the ACF

Figures 4 and Figure 5 display the autocorrelation in both returns and absolute returns. As expected, the linear autocorrelation of returns displays very little structure. In fact, the coefficient of correlation is significantly equal to zero after the first lag. Following Taylor<sup>5</sup>, we found that 92.95% of the autocorrelation coefficient estimates lie between -0.05 and 0.05. In contrast to the lack of dependence in returns, absolute returns<sup>6</sup> correlation coefficients are usually positive and tend to decay at a much slower pace. This suggests that while the signs of future returns are not predictable, their magnitudes are. This is also a sign of existence of volatility clustering, meaning that prolonged periods of low volatility are followed by periods of high volatility (Andersen & Bollerslev, 1997). Moreover, Figure.6 indicates the long run autocorrelation function of return volatility for all indices. As we can see from this figure, return volatility of 'Holding companies' sector is more autocorrelated than any other indices. Furthermore, the autocorrelation function decreases rapidly for approximately the first 20 lags and more slowly for the following lags.

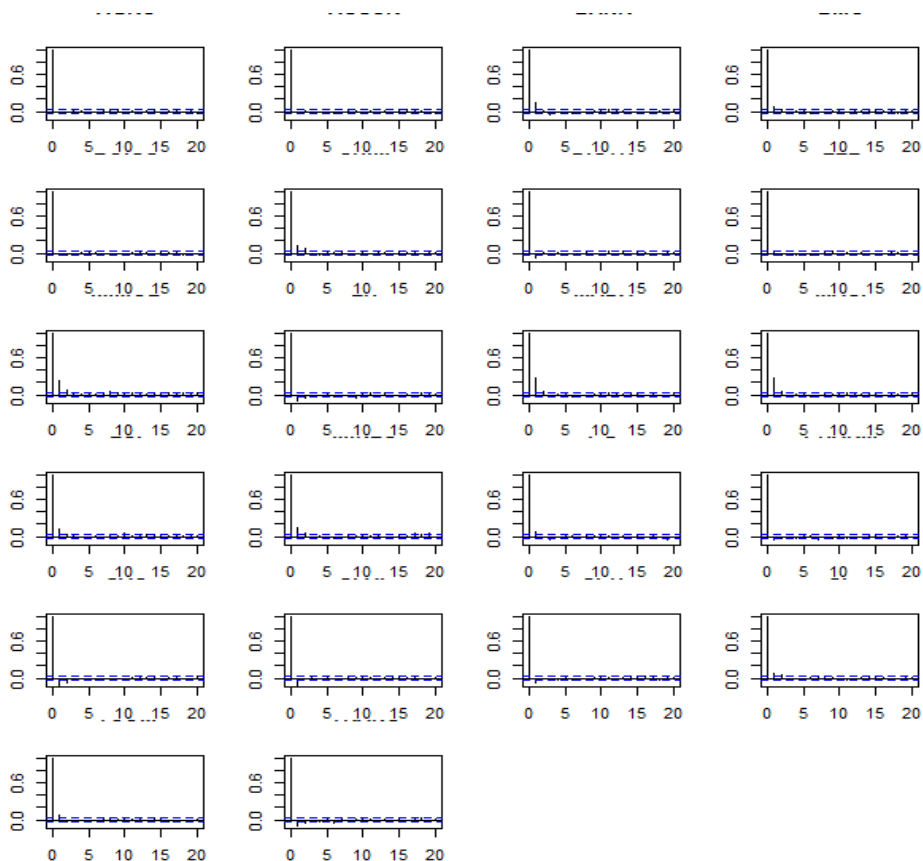
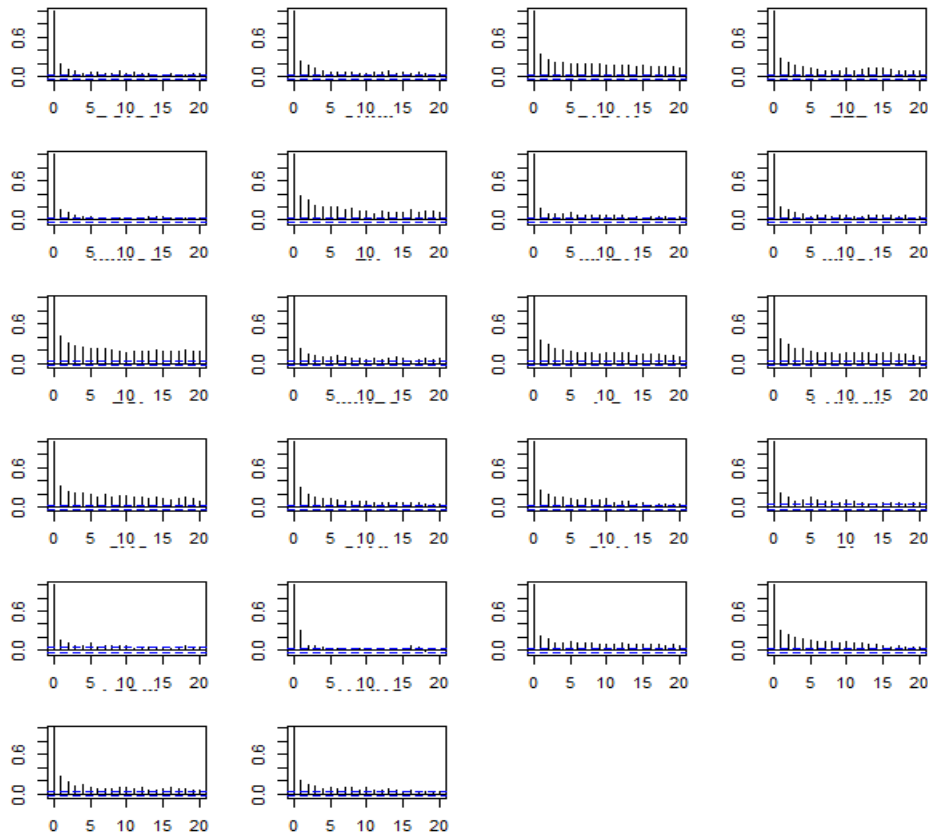


Figure 4. ACF of returns of global and sectoral indices

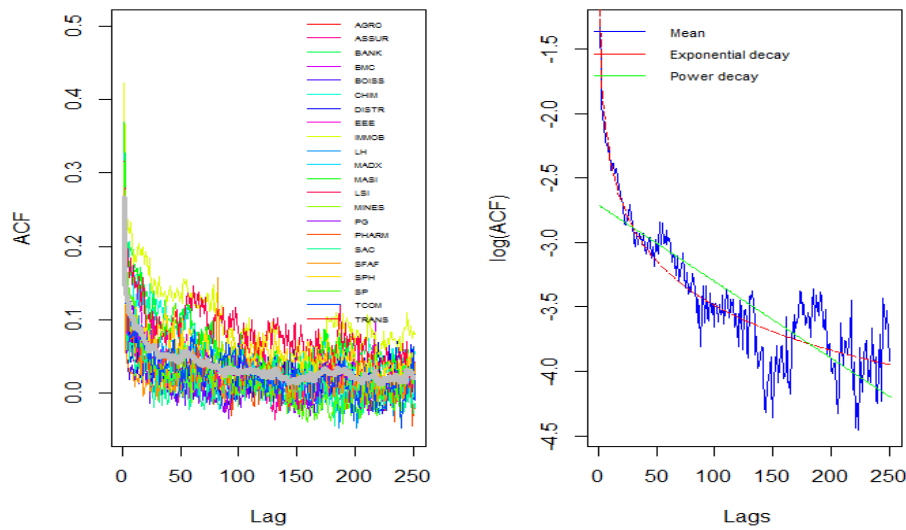
<sup>5</sup>Taylor found 90% of the autocorrelation estimates to lie between -0.05 and 0.05

<sup>6</sup> Alternatively, any other measure of the extent of fluctuations as squared returns. However, the autocorrelation in absolute return is higher than the autocorrelation in the squared returns.



**Figure 5.** ACF of absolute returns of global and sectoral indices

In the following, we will adjust the mean autocorrelation function of absolute returns with an exponential decay and a power decay functions. The exponential decay describes a short memory process; it obeys to the following relationship:  $|\rho_k| = Ae^{-\gamma k}$ , where A denotes the average level of autocorrelation and  $\gamma$  is the speed at which the autocorrelation decrease. The power decay refers to long-memory process, it ascertains that the autocorrelation function decreases according to the following relationship  $|\rho_k| = Ck^{-\beta}$ . As is noticeable from Figure 6 and according to the regression output, the power decay fit better the decrease of the autocorrelation function.



**Figure 6.** Long run autocorrelation function of return volatility for all indices

**Note.** On the left, the thin (wide) line represents individual (average) autocorrelation coefficient for return volatility for all indices- On the right: The average autocorrelation for each indice in blue and the fit from both exponential decay (in red), and power decay model (in green).

5.3.2. Mean reversion: GARCH estimation of volatility

Mean reversion of stock returns volatility is examined by means of ARCH and GARCH terms in the General Autoregressive Conditional Heteroskedasticity model (Elyasiani, Mansur, & Odusami, 2011). The GARCH model is a more general form taking into consideration the variance positivity assumption with a limited number of parameters (See Bollerslev, 1986). This generalization is similar to extending a Moving Average (MA) process to an Autoregressive- Moving Average model (ARMA). Indeed, the GARCH model (p, q) consists of estimating the actual conditional variance as a function of p previous square error terms and q past conditional variances. The model is expressed as follows:

$$h_t = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j h_{t-j} \quad (2)$$

With  $\alpha_i$  and  $\beta_j$  being positive parameters, which guarantee that the variance is obviously positive. For the mean reversion pattern to hold, the sum of ARCH and GARCH terms must be less than one (Carrol & Connor, 2011; Elyasiani et al., 2011). Moreover, the half-life computed for each stock enables us to determine the speed of the mean reversion model of stock returns volatility. We estimate conditional volatility using a GARCH (1.1) model with normal distributions. Estimation results are provided in Table 3. According to the results, all the parameters of the variance equation appear to be significantly different from zero at any traditional level of significance. Indeed, the values of the minimum variances represented by  $\alpha_0$  are very small and close to zero, while ARCH effect ( $\alpha_1$ ), which



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reflects the impact of past shock on volatility, shows positive values, ranging between 0.093 for “DISTR” and 0.432 for “SFAF”. It is also worth noting that the GARCH terms of the models ( $\beta_1$ ) seem to be significantly high. The parameters are superior to 0.53, except for “SFAF”. Hence, for all indices but ‘SFAF’, the volatility is due to the GARCH effect since the GARCH terms are far superior to ARCH effect parameters. This effect is more pronounced for “LSI”. This is a further indication that the market has a long memory and that volatility is more sensitive to its lagged values than to recent market shocks. In addition, the persistence is measured by the sum of GARCH and ARCH terms. The closer the sum is to one, the longer is the persistence of the volatility shock. In the context of Moroccan stock market, the persistence is expectedly high for all sectoral and global indices. The longest persistence of shock prevails in the cases of stock returns of “Construction & Building Materials” (0.982) and “Software & Computer Services sectors” (0.980). Next, it should be highlighted that the volatility process is mean reverting, but holds different speeds of reversion across sectors. Indeed, the speed of mean reversion, expressed by the so-called half-life, is slowest for “Investment Companies & Other Finance” and for “Construction & Building Materials” ( $HL > 30$ ), and relatively slow for indices of “Banks”, “Leisures & Hotels” and “Holding Companies sectors” as well as global indices “MASI” and “MADX” ( $10 < HL < 20$ ). It is rather slow for stock returns of: “Insurance”, “Chemicals”, “Distributors”, “Beverages”, “Electrical & Electronic Equipment”, “Real Estate”, “Mining”, “Utilities”, “Telecommunications and Transport sectors” ( $4 < HL < 10$ ). On the other hand, speed appears to be relatively fast in the case of stock returns of “Pharmaceutical Industry”, “Investment Companies & Other Finance”, “Paper & Forestry” and “Food Producer & Processors sectors”. ( $1 < HL < 4$ ).

### 5.4. Volatility asymmetry

Asymmetric volatility models consider that bad news may not have the same effect on conditional variance as good news. Indeed, Nelson (1991) argues that the volatility of the US equity market is higher after negative shocks than after positive shocks of the same magnitude. Several GARCH models take into account the leverage effect. The best-known asymmetric model is the Exponential GARCH model (EGARCH). Nelson (1991) introduced for the first time the EGRACH model. The author specified the conditional variance in logarithmic form in order to avoid positivity constraints (Geweke, 1986; Taylor & Diebold, 1986). In addition, this specification considers that a negative shock leads to a high conditional variance with respect to positive shocks. Bollerslev & Mikkelsen (1996) reformulated the EGARCH model in the following form:

$$\ln(h_t) = \alpha_0 + \sum_{i=1}^p \alpha_i \left[ \frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}} - \sqrt{\frac{\pi}{2}} \right] + \sum_{j=1}^q \left[ \gamma_j \frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}} \right] + \sum_{j=1}^p \beta_j \ln(h_{t-j}) \quad (3)$$

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The logarithm specification implies that the asymmetric effect, captured by the parameter  $\gamma$ , is exponential rather than quadratic. The sign of  $\gamma$  is expected to be negative so that the total effect of a negative shock is  $(\alpha - \gamma)|\varepsilon_{t-i}|$  and the total effect of a positive shock is  $(\alpha + \gamma)|\varepsilon_{t-i}|$ . In that case, negative changes increase volatility more than positive changes of the same size.

**Table 4.** GARCH estimation

	$\omega$	$\alpha$	$\beta$	$\alpha + \beta$	HL	Uncond Volatility
AGRO	2.63e-05***	0.109***	0.651***	0.760	2.52	1.10e-04
ASSUR	2.20e-05***	0.127***	0.766***	0.892	6.06	2.04e-04
BANK	3.69e-06***	0.135***	0.823***	0.958	16.15	8.87e-05
BMC	3.70e-06***	0.11***	0.872***	0.982	38.16	2.05e-04
BOISS	3.42e-05***	0.107***	0.734***	0.841	4.00	2.15e-04
CHIM	3.29e-05***	0.179***	0.745***	0.924	8.77	4.35e-04
DISTR	1.19e-05***	0.093***	0.825***	0.918	8.10	1.45e-04
EEE	7.02e-05***	0.123***	0.740***	0.863	4.70	5.11e-04
IMMOB	3.54e-05***	0.335***	0.530***	0.865	4.78	2.61e-04
LH	2.86e-05***	0.100***	0.852***	0.952	14.09	5.98e-04
MADX	4.36e-06***	0.223***	0.717***	0.940	11.20	7.25e-05
MASI	3.84e-06***	0.223***	0.714***	0.937	10.65	6.09e-05
LSI	5.67e-06***	0.093***	0.886***	0.980	34.31	2.82e-04
MINES	4.01e-05***	0.186***	0.678***	0.863	4.70	2.94e-04
PG	3.36e-05***	0.155***	0.709***	0.864	4.74	2.47e-04
PHARM	4.56e-05***	0.160***	0.559***	0.719	2.10	1.62e-04
SAC	5.80e-05***	0.113***	0.745***	0.858	4.53	4.08e-04
SFAF	6.73e-05	0.432	0.171	0.604	1.37	1.70e-04
SPH	1.85e-05	0.114	0.824	0.938	10.83	2.98e-04
SP	1.90e-04	0.176	0.61	0.787	2.89	8.89e-04
TCOM	2.28e-05	0.203	0.64	0.842	4.03	1.45e-04
TRANS	4.89e-05	0.127	0.719	0.846	4.14	3.18e-04

Notes: \*\*\*, \*\*, \*: significant at respectively 1%, 5% and 10% level

Despite the interesting conclusions of the GARCH model, most empirical studies argue that returns are negatively correlated with variations in volatility. In that sense, we will integrate asymmetric evolution into the dynamics of volatility using EGARCH (1, 1) model with Gaussian distribution. As discussed earlier, the model does not place any restrictions on the estimated parameters to ensure non-negativity of the conditional variance. In fact, the constant parameters are negative for most of the indices. Moreover, the regression results show that  $\alpha_0, \alpha_1$  (ARCH effect) and  $\beta_1$  (GARCH effect) are highly significant at 1%, 5% and 10%. More importantly, the results show that there is evidence of the asymmetry and leverage effect for the stock returns of the Insurance, Construction & Building Materials, Electrical & Electronic Equipment, Leisure and Hotels, Software & Computer Services, Mining, Pharmaceutical Industry, Holding Companies, Telecommunications sectors, and the global indices: MADX and MASI. However, the level is statistically significant only in the case of Insurance, Pharmaceutical Industry, Holding Companies, Telecommunications. Moreover, the magnitude of asymmetry and leverage

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effect is largest in the case of the Gas & Water followed by Fixed Line Telecom and then by the Electricity, Personal Goods and Oil & Gas sectors.

**Table 5.** EGARCH estimation

	Omega	Alpha	Gamma	Beta
AGRO	-2.177***	0.224***	0.01	0.759***
ASSUR	-1.461***	0.29***	-0.026***	0.826***
BANK	-0.471***	0.273***	0.015**	0.949***
BMC	-0.401***	0.251***	-0.008	0.952***
BOISS	-2.132***	0.273***	0.004	0.740***
CHIM	-1.189***	0.335***	0.047***	0.846***
DISTR	-0.707***	0.178***	0.009	0.918***
EEE	-1.630***	0.265***	-0.015	0.778***
IMMOB	-2.124***	0.411***	0.016	0.747***
LH	-0.589***	0.205***	-0.01	0.919***
MADX	-0.697***	0.375***	-0.008	0.928***
MASI	-0.700***	0.367***	-0.008	0.929***
LSI	-0.197***	0.198***	-0.009	0.975***
MINES	-1.429***	0.366***	-0.011	0.824***
PG	-1.153***	0.289***	0.000	0.860***
PHARM	-2.042***	0.242***	-0.038***	0.760***
SAC	-1.525***	0.239***	0.014	0.800***
SFAF	-5.598***	0.571***	0.185***	0.372***
SPH	-0.863***	0.205***	-0.042***	0.892***
SP	-2.376***	0.305***	0.009	0.659***
TCOM	-0.891***	0.257***	-0.023***	0.898***
TRANS	-1.252***	0.237***	0.052***	0.841***

**Notes:** \*\*\*, \*\*, \*: significant at respectively 1%, 5% and 10% level.

### 5.5. Long memory

Introduced at first by Hurst (1951), long memory can be defined as the presence of dependencies in a time series between distant observations in the past and distant observations in the future. In this section, we will investigate the property of long memory. As discussed above, a stationary process presents a long-memory behavior if its autocorrelations remain significantly positive even for very long lags, that is:  $\sum_{k=-\infty}^{+\infty} |\gamma_x(k)| = \infty$  (4).

Many models could be used to investigate the existence of long memory, for instance, Autoregressive Fractionally Integrated Moving Average Model. Known as the ARFIMA, the model is an extension to ARMA when  $d$  takes non-integer values. An ARFIMA processes is written as follows:  $\phi(L)(1-L)^d(y_t - \mu) = \theta(L)\varepsilon_t$  (5), where  $p$  and  $q$  denote respectively the number of autoregressive parameters and the order of the moving average.  $L$  is a lag parameter and  $d$  is the fractional integration parameter. For  $d = 0$ , the ARFIMA processes can be expressed as a simple ARMA, and thus the process is short memory; for  $d < 0$  the process has negative dependence between distant observations exhibiting anti-persistence, and for  $d > 0$  the processes exhibit long memory.

In order to estimate the fractional differencing parameter, several non-parametric and semi-parametric methods are used including: Maximum Likelihood, GPH and Sperio methods (See Nielsen & Frederikson, 2008).

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Table 7 presents the estimation of the fractional difference parameter ( $d$ ). Results show a strong evidence of long memory in the volatility dynamics of the Moroccan stock market. However, three estimators over four (two estimators over four) suggest the existence of anti-persistent process in the volatility of PHARM and SPH (BOISS).

**Table 6.** Estimation of fractional difference parameter

	Hurst	ML	GPH	Sperio
AGRO	3.26e-02**	5.78e-03**	7.62e-02**	3.87e-02**
ASSUR	1.75e-02**	4.58e-05**	2.25e-01**	1.39e-01**
BANK	2.03e-02**	7.83e-02**	1.27e-01	1.11e-01
BMC	5.33e-02**	5.36e-02**	9.55e-02**	9.30e-02**
BOISS	-1.35e-02**	4.58e-05**	-1.91e-02**	2.42e-02**
CHIM	9.07e-02**	8.87e-02**	1.46e-01**	1.10e-01**
DISTR	9.58e-03**	4.58e-05**	2.42e-01**	2.04e-01**
EEE	3.98e-02**	4.58e-05**	1.53e-01**	4.71e-02**
IMMOB	1.15e-01**	1.68e-01**	2.53e-01**	2.02e-01**
LH	5.25e-02**	4.58e-05**	3.35e-02**	1.38e-02**
MADX	7.10e-02**	1.90e-01**	1.16e-01**	9.19e-02**
MASI	8.17e-02**	1.97e-01**	1.10e-01**	9.16e-02**
LSI	8.63e-02**	9.49e-02**	2.41e-01**	2.07e-01**
MINES	7.25e-02**	1.02e-01**	2.66e-01**	1.98e-01**
PG	1.98e-02**	4.63e-02**	1.05e-01**	6.20e-02**
PHARM	-4.43e-02**	4.58e-05**	-1.20e-01**	-1.15e-01**
SAC	1.49e-01**	4.58e-05**	7.80e-02**	3.25e-02**
SFAF	1.34e-02**	4.58e-05**	1.06e-01**	9.54e-02**
SPH	-2.50e-02**	4.58e-05**	-3.73e-01**	-2.71e-01**
SP	7.55e-02**	5.63e-02**	8.49e-02**	8.01e-02**
TCOM	4.03e-03**	2.97e-02**	7.73e-03**	2.96e-02**
TRANS	5.04e-02**	4.58e-05**	1.36e-01**	9.90e-02**

Notes: \*\*\*, \*\*, \*: significant at respectively 1%, 5% and 10% level.

### 5.6. Multifractality

In order to determine the fractal character of the series of indices, we use the traditional approach used in the physics literature, which is the partition function. For this purpose, we divide the price series  $P(t)$  into  $N$  intervals of length  $\Delta t$ . This function can be defined for a  $\Delta t$  and a chosen value of  $q$  as follows:

$$S_q(T, \Delta t) = \sum_{i=0}^{N-1} \left| \ln \left( \frac{P(i\Delta t + \Delta t)}{P(i\Delta t)} \right) \right|^q \quad (6)$$

Partition plots are created by plotting values of  $\log_{10} S_q(T, \Delta t)$  against  $\log_{10}(\Delta t)$ . In our case, we compute the partition function for different moments  $q$ :  $q = \{1, 2, 3, 4, 5\}$ . (See Figure 6).

As regards to the scale behavior, linearity in the partition plots if found is an indicator of the moment scaling. In fact, the scaling property is presented as:  $E(|X(t)|^q) = c(q)t^{\tau(q)+1}$  (7). We clearly notice that the exact scale invariance is true for all indices, but it begins to disappear beyond the estimated integral time of 3 days for the majority of the indices.

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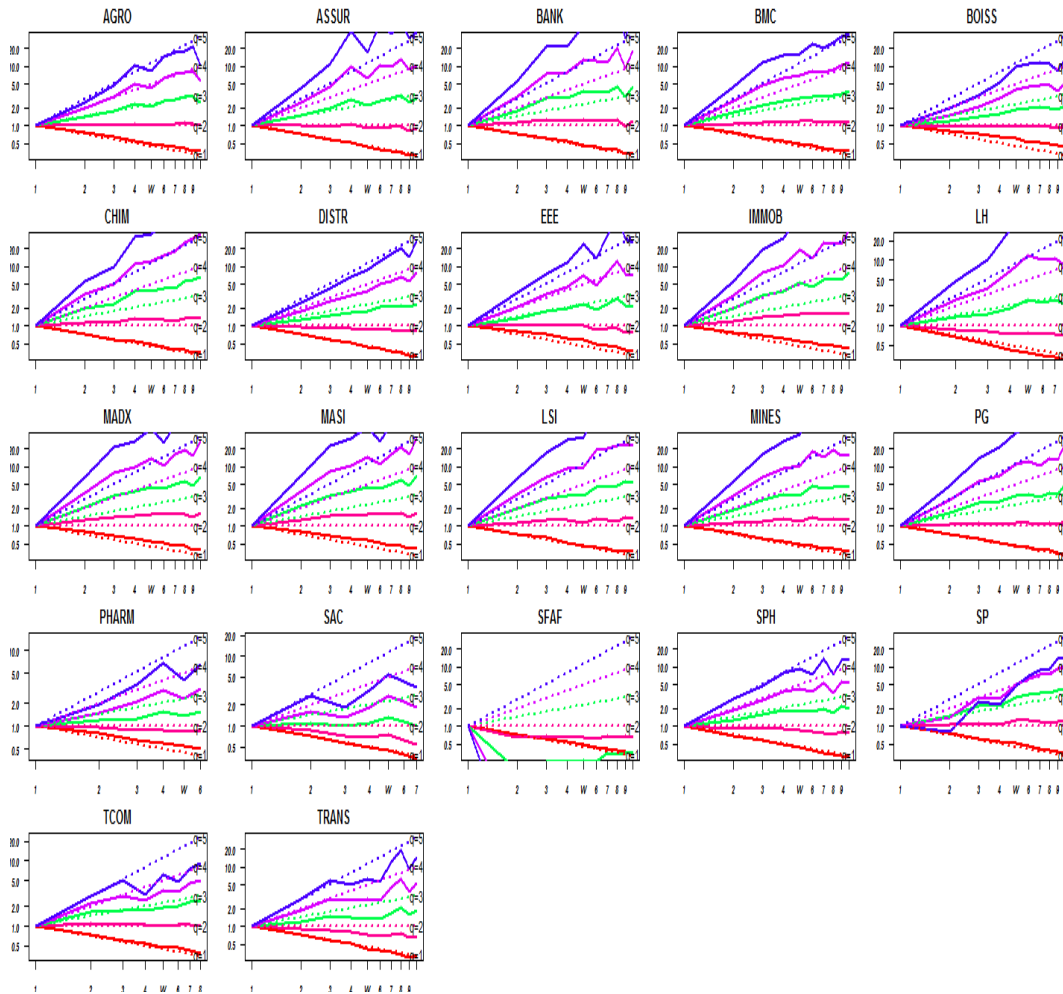


Figure 7. Partition function

### 5.7. Extreme events

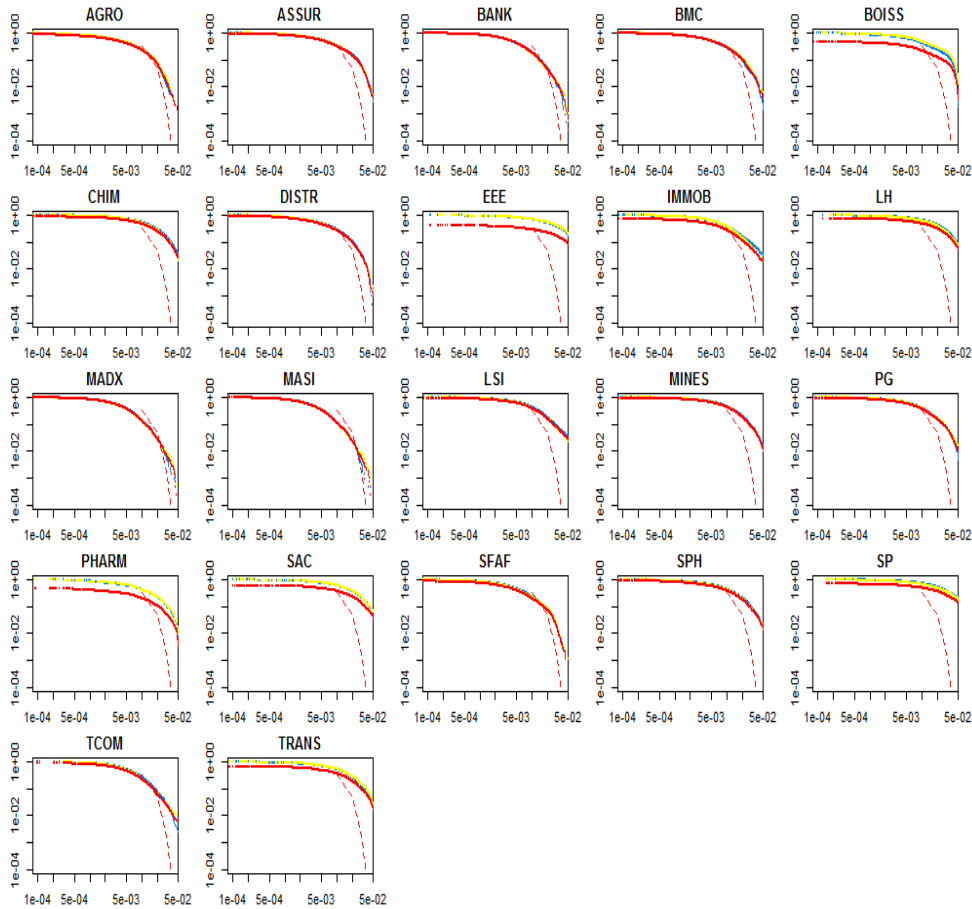
#### 5.7.1. Fat tail

As discussed in section 4.1, the distribution of returns is not Gaussian. In fact, very large fluctuations are much more likely in the stock market than what normal distribution predicts. In the following, we describe the tail behavior of the distribution of returns using the complementary cumulative distribution function  $F(x)$ :

$$F(x) = 1 - \text{prob}(X < x) \quad (8).$$

From Figure 7, we can see that the decay of the *ccdf* is much slower than a Gaussian. This is an evidence of heavy tails. Furthermore, the left tail is clearly heavier than the right one ( $\alpha^+ > \alpha^-$ ) for the cases of MADX and Moroccan All Shares indices as well as the following sectors: Food Producer & Processors, Construction & Building Materials, Beverages, Electrical & Electronic Equipment, Oil & Gas, Pharmaceutical Industry, Utilities and Transport.

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**Figure 8.** Complementary cumulative distribution functions of absolute returns

**Notes:** We present the complementary cumulative distribution function (ccdf) of the absolute value returns of sectoral and global indices (red line). The yellow and blue lines are instead the complementary cumulative distribution functions for negative and positive returns respectively. The red dashed line is the complementary cumulative distribution function of a gaussian with the same variance of the real return distribution.

### 5.7.2. Shocks and aftershocks: Omori law

Omori law describes the dynamics of aftershocks. Introduced at first in seismology, this approach consider that a major earthquake in a region is usually followed by smaller ones, labeled as "aftershocks". Since then, few papers have investigated the behavior of volatility in financial markets after big crashes. For instance, Lillo & Mantegna (2003), Sornette & Helmstetter (2003) Selçuk (2004), Weber *et al.*, (2007) and Petersen *et al.*, (2010), examined the dynamics of volatility during the period around an extreme event, particularly a huge decline. In this section, we examine the dynamics of volatility during the period around an extreme event. In fact, our main goal is to compute the probability of significant correction after a crash. In that sense, Liu & Loewenstein, (2013), states that the probability of another crash may increase after an extreme event. The Omori law is expressed as follows:

$$n(t) = Kt^{-P} \quad (9)$$

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where  $p$  and  $k$  are positive constants and  $n(t)$  denotes the number of aftershocks per unite, that is the amount of time that market volatility exceeds a predetermined threshold. From the form of the formula, we can conclude that the number of aftershocks per unit time decays with power law. The Omori law can be expressed in a more practical way. In fact, the cumulative number of aftershocks,  $N(t)$ , depends on the following relation:

$$N(t) = K \frac{1}{(1-p)} t^{1-p} \quad (10)$$

for  $p \neq 1$

Where  $N(t)$  denotes the number of aftershocks recorded between the moment at which the original shock took place and  $t$ . In calm periods,  $N(t)$  should be linear. Hence,  $p$  should be equal to zero for such periods. To estimate the Omori coefficient, we estimate the parameter of the equation by OLS method.

$$\log(N(t)) = C + (1-p)\log(t) + e(t) \quad (11)$$

However, the challenging question is how to define "shock" and 'aftershock'. In the present paper, we consider the most severe fall for each indice to be shock. We define an aftershock as a return, whose magnitude exceeds two returns standard deviations. The standard deviation is computed on the last 252-trading days before the date of shock. On the other hand, the 252-trading day window after the shock is reserved for computing the number of aftershocks. It should be noted that for some cases, we use the second largest shock instead of the largest one as the after-shock window has fewer observations. The results reported in Table 4 show that the sectoral and global indices have experienced a major shock during different dates. The largest one-day drop in percentage terms is observed in "SFAF" (30.38%, 26/05/2005) and the smallest one-day fall is observed in MASI (5.017%, 19/05/2006). The relative shock measure, daily percent loss divided by the sample standard deviation, indicates that these major shocks lie within the range of  $3.32\sigma$  ("Mines" 17/03/2005) and  $25.32\sigma$  ("SFAF", 26/05/2005).

Having determined the dates of two major shocks in each indice, an aftershock is defined as daily absolute return greater than  $2\sigma$  immediately after the major shock. The number of cumulative aftershocks  $N(t)$  for  $t=252$  is computed. This serves to determine an estimated of  $p$  for each indice for all aftershocks, and for negative aftershocks. Table 4 shows that the values of estimated exponent  $p$  differ among indices. They are found to be within the range of 0.0296 to 0.5136 (0.0116 to 0.5054) when considering all aftershocks (negative aftershocks). Overall, the values of  $p$  are relatively small than the one found in (Masset, 2011) for the case of emerging and mature Market.

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**Table 7.** *Omori coefficients*

Index	Date	Loss	SD	Real shock	p	
					Using Total number of aftershocks	Using Number of negative aftershocks
AGRO	30/06/2004	0,0958	0,01	9,58	0,0513	0,0212
ASSUR	28/06/2018	0,08436	0,014	6,03	0,4875	0,5045
	13/04/2017	0,06046	0,014	4,32	0,211	0,1027
BANK	24/10/2008	0,05773	0,01	5,77	0,0296	0,0826
BMC	23/03/2017	0,07374	0,013	5,67	0,0424	0,053
BOISS	26/06/2009	0,1072	0,015	7,15	0,2697	0,012
CHIM	06/06/2003	0,15132	0,021	7,21	0,2538	0,2866
DISTR	13/04/2017	0,06556	0,012	5,46	0,2398	0,1289
EEE	30/03/2016	0,10512	0,023	4,57	0,204	0,1499
	30/03/2018	0,07306	0,015	4,87	0,0773	0,1561
IMMOB	30/03/2016	0,06693	0,015	4,46	0,1834	0,023
	LH	26/09/2017	0,10517	0,024	4,38	0,0493
MADX	19/05/2006	0,05094	0,008	6,37	0,25	0,4319
MASI	19/05/2006	0,05017	0,007	7,17	0,2282	0,4319
LSI	12/09/2008	0,06365	0,017	3,74	0,2073	0,176
	10/07/2018	0,06718	0,017	3,95	0,5367	0,6769
MINES	17/03/2005	0,06161	0,017	3,62	0,0093	0,3135
	PG	20/06/2002	0,09074	0,016	5,67	0,0765
PHARM	22/12/2014	0,06125	0,013	4,71	0,5136	0,3243
SAC	26/12/2016	0,10518	0,02	5,26	0,265	0,2577
SFAF	26/05/2005	0,30385	0,012	25,32	0,0716	0,0284
SPH	01/07/2014	0,1072	0,017	6,31	0,1158	0,0303
SP	15/11/2017	0,37475	0,03	12,49	0,3807	0,1687
	06/03/2015	0,10555	0,03	3,52	0,3022	0,3792
TCOM	21/05/2010	0,10504	0,011	9,55	0,3192	0,0716
TRANS	19/08/2013	0,1309	0,018	7,27	0,2058	0,2899

**Notes:** The table shows the largest drop in each stock market index and the second largest fall in "ASSUR", "IMMOB", "MINES" and "SP". Standard deviation and real shock are also presented in the table. The last two columns show the estimated values of parameter p in Eq. (11). P is computed at first when considering all aftershocks whether positive or negative, and second only for the negative shocks.



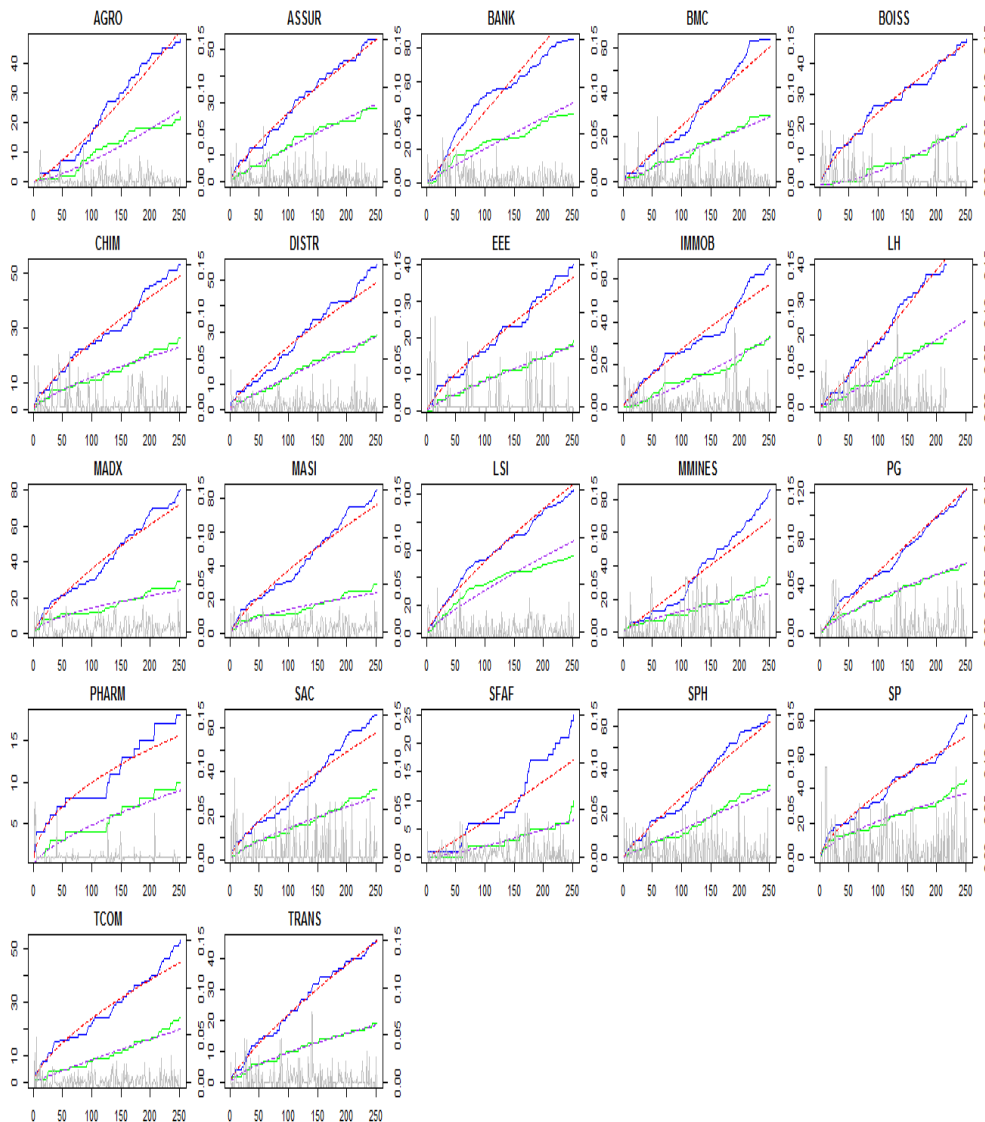


Figure 9. Cumulative number of after shocks

Notes: Absolute return (grey line) cumulative number of negative aftershocks (green line) and fit obtained based on Omori law

## 6. Conclusion

This paper has analyzed some important stylized facts in volatility returns of sectoral and global indices of Moroccan stock market. In fact, the main objective of this article is to characterize the behavior of volatility, in order to design an adequate model for forecasting this latent variable in the Moroccan context. The results unequivocally indicate that the markets indices share most of the stylized facts of traditional asset classes. In fact, we have been able to confirm that the conditional volatility reverts to its long-run value (unconditional volatility). However, the necessary speed of return depends on the sector. On the other hand, autocorrelation function of absolute returns shows a significant positive autocorrelation, suggesting the existence of volatility clustering. This fact is confirmed also by visual inspection of return Figures. More importantly, the results show that there is evidence of the asymmetry and leverage effect for the stock returns

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of some sectors. Hence, the volatility is higher after negative shocks than after positive shocks of the same magnitude. However, the level is statistically significant only in few cases: Insurance, Pharmaceutical Industry, Holding Companies and Telecommunications sectors.

As far as long memory is concerned, results show a strong evidence of long memory in the volatility dynamics of the Moroccan stock market. However, we may suspect the existence of anti-persistent processes in the volatility of PHARM and SPH and BOISS sectors, as at least two estimators of the fractional integration parameter are negative. Still, a challenging question is to assess whether the identified long memory is real or spurious. Furthermore, the partition functions show evidence of multifractality evidence for almost all the indices. As regards to extreme events, very large fluctuations are much more likely in the stock market than what normal distribution predicts. Moreover, the values of Omori Law Exponents are relatively smaller than the one found in Masset (2011) for the case of emerging and mature Market. This is further indication that Moroccan Stock market is relatively stable.

We believe that the main contribution of this paper is to investigate a variety of stylized fact in Moroccan stock market. Furthermore, our approach differs from Masset (2011) in the sense that we focus on sectoral indices rather than a panel of country indices. This study can be extended in many ways, for instance by comparing stylized facts in Moroccan family to non-family stock returns and volatility.

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