Turkish Economic Review
www.kspjournals.org
Volume 3 September 2016 Issue 3

A Note on Construction of a Composite Index by Optimization of Shapley Value Shares of the Constituent Variables

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Abstract. This paper proposes a method to construct composite index, which is a linear combination of several variables, by deriving weights on the criterion of Shapley value (from cooperative game theory) that a constituent variable has in making the composite index. In practice it is found oftentimes that the most common method of principal component analysis has a tendency to ignore (or poorly weigh) those constituent variables that do not have strong correlation with the sister variables. This elitist nature of PCA forces a compromise upon the analyst's desire and need to incorporate those weakly correlated (but theoretically and practically important) variables into the composite index. In that case, one must construct a composite index that is more inclusive in nature. The Shapley value based composite index meets that requirement.
Keywords. Shapley value, Composite index, Principal Component Analysis, Inclusive indices, Global optimization.
JEL. C43, C63, C71.

1. Introduction

A large body of literature is available on the methods to construct a composite index, a linear, weighted, combination of a host of indicator variables, which are its constituents. Perhaps, the credit for devising a method of the principal component analysis to reduce the dimensionality of multivariate data goes to Pearson (1901) and Hotelling (1933a; 1933b) and its first application to construction of a composite index may be attributed to Adelman & Morris (1967) followed by Chatttopadhyay & Pal (1972). Booysen (2002) provides a discussion of application of composite index for quantifying socio-economic development. Some other notable contributions include Salzman (2003), OECD (2003), Nardo et al. (2005), Munda & Nardo (2005) and Saltelli (2007). On the methodological side, Somarriba and Pena (2009) and Montero et al. (2010) used weights based on Pena distance and partial R^2 rather than those based on the leading eigenvalue and the associated eigenvector as done in the principal component analysis. Mishra (2007a; 2007b; 2009) advocated for deriving weights by maximizing absolute or minimum non-Euclidean norm (unlike the principal component analysis that maximizes the Euclidean norm) of correlation coefficients between the composite index and its constituent variables to make the composite index more inclusive and less prone to outliers.

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2. Weight assignment for making linear combination

A composite index is Z = Xw, where X are the constituent variables, m in number with n replicates/observations, w is the weight vector with m elements and the combination Z is an array of n elements. The methodological issue lies in how w is determined. The principal component analysis obtains w such that the Euclidean norm of correlation between Z and X is maximized. That amounts to maximization of $\sum_{j=1}^{m} r^2(Z, x_j) = \sum_{j=1}^{m} r^2(Xw, x_j)$ with w being the decision variables. Instead, Mishra (2007a, 2007b and 2009) proposed to maximize $\sum_{j=1}^{m} |r(\mathrm{Xw}, x_j)|$ which is the absolute norm or maximize the minimum norm, $\min_j |(r(\mathrm{Xw}, x_j)|$. The principal component weights have a tendency to undermine those constituent variables that are poorly correlated with the sister variables. The weights based on absolute norm are relatively more accommodative or inclusive to such poorly correlated variable while the min norm weights are most inclusive in nature. When Pena's method is used for determining weights, $w_1 = 1$ and $w_j = (1 - R^2_{j|j-1,j-2,...,1}); j = 2, m$ where $R^2_{j|j-1,j-2,...,1}$ is the coefficient of determination while x_j is regressed on all other x_k for which the index $k < j$ until $k = 1$. Thus, $w_j = 1 - R^2(x_j, \bigcup_{k=1}^{j-1} x_k); \mathrm{j} = 2, \mathrm{m}$.

3. The objective of this paper

The objective of this paper is to propose for working out of weights, *w*, in $Z = Xw$ such that the Euclidean norm of $s(Z, x_j); \mathrm{j} = 1, \mathrm{m}$ is minimized. Here $s(Z, x_j)$ is the Shapley value of x_j in explaining Z. Minimization of the Euclidean norm of $s(Z, x_j); \mathrm{j} = 1, \mathrm{m}$ is proposed in order to ensure the maximum possible participation of all constituent variables in making Z. The weights, which are the decision variables, are constrained to be non-negative. These criteria ensure that the composite index is constructed by assigning weights to the constituent variables such that the contribution of each constituent variable, measured in terms of Shapley value, is non-negative, and most equitable

4. Algorithm

We set up a minimization problem $f(w) = \sum_{j=1}^{m} s^2(Xw, x_j)$ with $w_i = 1/m \; \forall i$ to initialize. With this initial w we work out $f(w)$. An appropriate algorithm to find out $s(Z, x_j) \forall j$ is available (Lipovetsky, 2006; Mishra, 2016). Then, in a non-negative domain, we suitably search for w, evaluating $f(w)$ at every move until minimal $f(w)$ is obtained.

5. Implementation

For the purpose of demonstration, we implement our proposed algorithm on the data provided by Sarker et al. (2006), reproduced in the appendix (Table-A1). The data pertain to human development indicators; life expectancy (LE), education (ED), per capita income (PCI) and a measure of equality (EQ) for 125 countries.

After setting up the $f(w)$, optimization has been done by the Host-Parasite Co-Evolutionary algorithm, which is a biologically-inspired algorithm for global optimization (Mishra, 2013). Shapley values have been computed by the computer program in Mishra (2016).

Table 1. *Particulars of Shapley-value based Composite Index and Other Statistics*

Particulars	LE	ED	PCI	EQ
Weights (w)	0.20992004	0.52962171	0.49131553	0.65881975
Shapley value (s)	0.25022641	0.24951509	0.25051323	0.24974528
Beta values (β)	0.140	0.353	0.328	0.440
Correlation with Z	0.864	0.826	0.838	0.711
Correlation with PCA Score	0.924	0.870	0.890	0.568

The composite index (Z) has been reported in Table-A2 in the Appendix. If we regress Z on LE, ED, PCI and EQ, i.e. $Z = \beta_1 LE + \beta_2 ED + \beta_3 PCI + \beta_4 EQ + u$, we cannot retrieve weights due to multicollinearity among the regressors, although R^2=1. However, we note (Table-1) that Shapley values are almost equal which indicates that all the constituent variables have contributed almost equally in making Z. It may also be noted that weights assigned to different variables in making Z are not even close to being equal. Correlation coefficients of Z with different constituent variables are quite high. If the composite index were constructed by the principal component analysis (PCA Score), its correlation would have been more in favour of LE, ED and PCI and less in favour of EQ. Shapley value based composite index has highlighted the contribution of EQ in the composite (human development) index.

6. Correlation of various composite indices among themselves and the constituent variables

In Table-2 we present the coefficients of correlation among different alternative composite indices and the constituent variables. We have considered four different alternative composite indices, HDI_2 (the leading principal component score, that maximizes the sum of squared correlation coefficients or the squared Euclidean norm between the composite index and the constituent variables), HDI_1 (based on maximization of absolute norm of correlation coefficients between the composite index and the constituent variables), HDIPena (that is derived by applying the Pena-distance based weights to different constituent variables in accordance with partial R^2) and HDISap (based on the criterion that the composite index should be constructed by assigning weights to the constituent variables such that the contribution of each constituent variable, measured in terms of Shapley value, is positive, or at least non-negative, and most equitable).

We observe that HDI_2 is closest to HDI_1 (having the highest correlation between them), followed by HDIPena and HDISap. It also maximized the sum of squared correlation between itself and the constituent variable (SS_COR), as it has been derived to have that property. HDI_1 maximizes the sum of the magnitude of correlation coefficients between itself and the S_COR into constituent variables (S_COR, as has been designed to do so) and has stronger correlation with HDISap (than HDIPena). HDIPena is closer to HDI_2 on the principle of SS_COR, but farther from it on the principle of S_COR. However, HDISap is closer to HDI_1 on the S_COR criterion, but farther from it on the SS_COR criterion. What emerges is that HDISap is a more inclusive composite index than HDIPena. In another sense, HDI_2 and HDIPena are more elitist composite indices (prone to maximize representation regardless of best possible representation of the variables having lesser explanatory capability), while HDI_1 and HDISap are more inclusive (caring for the representation of those variables that have lesser explanatory capability). This outcome is expected because HDI_1 and HDISap have been designed to be more inclusive.

Table 2. *Correlation of various composite indices among themselves and the constituent variables*

	HDI_2	HDI_1	HDIPena	HDISap	LE	ED	PCI	EQ	SS_COR	S_COR
HDI_2	1	0.996	0.978	0.978	0.924	0.870	0.890	0.568	2.7254	3.252
HDI_1	0.996	1	0.990	0.992	0.915	0.844	0.866	0.640	2.7091	3.265
HDIPena	0.978	0.990	1	0.982	0.937	0.779	0.811	0.704	2.6381	3.231
SDISap	0.978	0.992	0.982	1	0.864	0.826	0.838	0.711	2.6365	3.239
LE	0.924	0.915	0.937	0.864	1	0.729	0.764	0.492	-	-
ED	0.870	0.844	0.779	0.826	0.729	1	0.750	0.283	-	-
PCI	0.890	0.866	0.811	0.838	0.764	0.750	1	0.313	-	-
EQ	0.568	0.640	0.704	0.711	0.492	0.283	0.313	1	-	-

7. Concluding remarks

The use of Shapley value criterion to construct composite index adds to the methodology of representing indicator variables by a single composite index. The index so derived is inclusive rather than elitist in nature. In practice it is found oftentimes that the most common method of principal component analysis has a tendency to ignore (or poorly weigh) those constituent variables that do not have strong correlation with the sister variables. This elitist nature of PCA forces a compromise upon the analyst's desire and need to incorporate those weakly correlated (but theoretically and practically important) variables into the composite index. In that case, one must construct a composite index that is more inclusive in nature. The Shapley value based composite index meets that requirement.

Computation of Shapley value is inherently combinatorial in nature and it becomes increasingly demanding (computational power and time) when the number of variables under analysis increases beyond 15 or so. The method proposed here partakes of this difficulty. This difficulty may, to some extent, be overcome by grouping the constituent variables into several clusters (coalitions that might or might not be hierarchical, but they are structured). By the way, it may be mentioned that the practice of grouping and construction of a composite index at two steps or stages is prevalent even among those who use the PCA for constructing a composite index (e.g. Chattopadhyay & Pal, 1972; Dreher in KOF, 2012). Evidently, as pointed out by Mishra (2012), this procedure is suboptimal on account of ignoring the correlation among the constituent variables across the groups.

Grouping of constituent variables into clusters (or coalitions, so to say) may require the use of Owen value (Owen, 1977) which is a generalization of Shapley value for games with coalition structure having super-additive properties. Super-additivity may arise on account of cooperation among the coalitions (or their members across the coalitions). In this regard, some work on cooperative games with coalition structure (such as Huettner (2010), Calvoy & Gutiérrezz (2011), Liben-Nowell et al. (2012), etc.) may be helpful. Therefore, it requires further research.

Appendix

Table A1. *Human Development Indicators (From Sarker et al., 2006)*

Country	LE	ED	PCI	EQ	Country	LE	ED	PCI	EQ
Norway	0.90	0.99	0.99	0.96	Turkey	0.76	0.80	0.69	0.66
Sweden	0.92	0.99	0.93	0.98	Azerbaijan	0.78	0.88	0.58	0.73
Canada	0.90	0.98	0.95	0.81	Jordan	0.76	0.86	0.62	0.74
Netherlands	0.89	0.99	0.95	0.82	Tunisia	0.79	0.74	0.70	0.66
Australia	0.90	0.99	0.94	0.76	China	0.76	0.83	0.64	0.56
Belgium	0.90	0.99	0.94	0.98	Georgia	0.81	0.89	0.52	0.73
United_States	0.87	0.97	0.98	0.64	Dominican_Republi	0.70	0.82	0.70	0.50
Japan	0.94	0.94	0.93	0.98	Sri_Lanka	0.79	0.83	0.60	0.78
Luxembourg	0.89	0.91	1.00	0.86	Ecuador	0.76	0.85	0.60	0.58
Ireland	0.86	0.96	0.98	0.75	Iran_Islamic_Rep.	0.75	0.74	0.70	0.60
Switzerland	0.90	0.95	0.95	0.81	El_Salvador	0.76	0.75	0.65	0.38
Austria	0.89	0.96	0.95	0.87	Guyana	0.64	0.89	0.63	0.59
United_Kingdom	0.88	0.99	0.93	0.74	Uzbekistan	0.74	0.91	0.47	0.94
Finland	0.88	0.99	0.93	0.94	Algeria	0.74	0.69	0.68	0.76
Denmark	0.86	0.98	0.96	0.99	Kyrgyzstan	0.72	0.92	0.46	0.89
France	0.90	0.96	0.93	0.81	Indonesia	0.69	0.80	0.58	0.78
New_Zealand	0.89	0.99	0.90	0.74	Viet_Nam	0.73	0.82	0.52	0.74
Germany	0.89	0.95	0.94	0.91	Moldova_Rep_of	0.73	0.87	0.45	0.74
Spain	0.90	0.97	0.90	0.82	Bolivia	0.64	0.86	0.53	0.56
Italy	0.89	0.93	0.93	0.74	Honduras	0.73	0.74	0.54	0.34
Israel	0.90	0.94	0.88	0.76	Tajikistan	0.73	0.90	0.38	0.77
Singapore	0.88	0.91	0.92	0.61	Nicaragua	0.74	0.73	0.54	0.34
Greece	0.89	0.95	0.87	0.76	Mongolia	0.64	0.89	0.47	0.57
Hong_Kong_China	0.91	0.86	0.93	0.59	South_Africa	0.40	0.83	0.77	0.25
Portugal	0.85	0.97	0.87	0.69	Egypt	0.73	0.62	0.61	0.78
Slovenia	0.85	0.96	0.87	0.91	Guatemala	0.68	0.65	0.62	0.48
Korea_Rep_of	0.84	0.97	0.86	0.84	Morocco	0.72	0.53	0.61	0.67
Czech_Republic	0.84	0.92	0.84	0.97	Namibia	0.34	0.79	0.69	0.00
Argentina	0.82	0.96	0.78	0.40	India	0.64	0.59	0.55	0.82
Estonia	0.78	0.98	0.80	0.72	Botswana	0.27	0.76	0.73	0.17
Poland	0.81	0.96	0.78	0.84	Ghana	0.55	0.65	0.51	0.87
Hungary	0.78	0.95	0.82	1.00	Cambodia	0.54	0.66	0.50	0.65
Slovakia	0.81	0.91	0.81	0.96	Papua_New_Guine	0.54	0.57	0.52	0.43
Lithuania	0.79	0.96	0.77	0.83	Lao_People's_Dem.	0.49	0.64	0.47	0.72
Chile	0.85	0.90	0.77	0.30	Swaziland	0.18	0.74	0.64	0.21
Uruguay	0.84	0.94	0.73	0.56	Bangladesh	0.60	0.45	0.47	0.83
Costa_Rica	0.88	0.87	0.75	0.52	Nepal	0.58	0.50	0.44	0.73
Croatia	0.82	0.90	0.77	0.89	Cameroon	0.36	0.64	0.50	0.56
Latvia	0.76	0.95	0.75	0.82	Pakistan	0.60	0.40	0.49	0.81
Mexico	0.81	0.85	0.75	0.35	Lesotho	0.19	0.76	0.53	0.17
Trinidad_and_Tobag	0.77	0.87	0.76	0.65	Uganda	0.34	0.70	0.44	0.60
Bulgaria	0.77	0.91	0.71	0.83	Zimbabwe	0.15	0.79	0.53	0.30
Malaysia	0.80	0.83	0.75	0.46	Kenya	0.34	0.74	0.39	0.56
Russian_Federation	0.69	0.95	0.74	0.54	Yemen	0.58	0.50	0.36	0.80
Macedonia_TFYR	0.81	0.87	0.70	0.91	Madagascar	0.47	0.60	0.33	0.50
Panama	0.83	0.86	0.69	0.31	Nigeria	0.44	0.59	0.36	0.43
Belarus	0.75	0.95	0.67	0.86	Mauritania	0.45	0.42	0.52	0.68
Albania	0.81	0.89	0.65	0.91	Gambia	0.48	0.40	0.47	0.70
Bosnia_and_Herzeg	0.82	0.84	0.68	0.95	Senegal	0.46	0.39	0.46	0.63
Venezuela	0.81	0.86	0.67	0.47	Guinea	0.40	0.37	0.51	0.65
Romania	0.76	0.88	0.70	0.87	Tanzania_U_Rep.	0.31	0.62	0.29	0.70
Ukraine	0.74	0.94	0.65	0.89	Cote_d_Ivoire	0.27	0.47	0.45	0.55
Saint_Lucia	0.79	0.88	0.66	0.60	Zambia	0.13	0.68	0.36	0.39
Brazil	0.72	0.88	0.73	0.25	Malawi	0.21	0.66	0.29	0.44
Colombia	0.78	0.84	0.69	0.29	Central_African_Re	0.25	0.43	0.41	0.21
Thailand	0.74	0.86	0.71	0.59	Ethiopia	0.34	0.39	0.34	0.87
Kazakhstan	0.69	0.93	0.68	0.84	Mozambique	0.22	0.45	0.39	0.67
Jamaica	0.84	0.83	0.61	0.70	Guinea-Bissau	0.34	0.39	0.33	0.51
Armenia	0.79	0.90	0.57	0.70	Burundi	0.26	0.45	0.31	0.80
Philippines	0.75	0.89	0.62	0.53	Mali	0.39	0.21	0.37	0.44
Turkmenistan	0.70	0.93	0.63	0.64	Burkina_Faso	0.35	0.16	0.40	0.49
Paraguay	0.76	0.85	0.64	0.30	Niger	0.35	0.18	0.35	0.44
Peru	0.74	0.86	0.65	0.45	Note:	From Sarker et al (2006)			

Table A2. *Different Types of Composit Indices of Human Development*

Country	HDI2	HDI1	HDIPen a	HDISA P	Country	HDI2	HDI1	HDIPen a	HDISA P
Norway	0.96	0.96	0.9959	1.6914	Turkey	0.73	0.73	0.6578	0.1298
Sweden	0.95	0.95	1.0000	1.6426	Azerbaijan	0.74	0.74	0.6890	0.2414
Canada	0.92	0.91	0.9182	1.2984	Jordan	0.74	0.74	0.6876	0.2812
Netherlands	0.92	0.92	0.9190	1.3302	Tunisia	0.73	0.72	0.6637	0.0588
Australia	0.91	0.90	0.8964	1.1972	China	0.71	0.70	0.6084	-0.1062
Belgium	0.95	0.95	0.9910	1.6465	Georgia	0.73	0.73	0.6928	0.1763
United_States	0.89	0.87	0.8332	0.9648	Dominican_Republic	0.70	0.68	0.5613	-0.1835
Japan	0.94	0.95	0.9997	1.5658	Sri_Lanka	0.74	0.75	0.7098	0.2944
Luxembourg	0.92	0.92	0.9305	1.3537	Ecuador	0.71	0.70	0.6110	-0.0979
Ireland	0.90	0.89	0.8722	1.1649	Iran_Islamic_Rep_of	0.71	0.70	0.6144	-0.0902
Switzerland	0.91	0.90	0.9110	1.2444	El_Salvador	0.66	0.64	0.5145	-0.6006
Austria	0.92	0.92	0.9333	1.3784	Guyana	0.70	0.69	0.5626	-0.0333
United_Kingdom	0.90	0.89	0.8733	1.1259	Uzbekistan	0.74	0.76	0.7332	0.5093
Finland	0.93	0.94	0.9593	1.5344	Algeria	0.71	0.72	0.6599	0.1057
Denmark	0.94	0.95	0.9748	1.6568	Kyrgyzstan	0.73	0.74	0.6996	0.3948
France	0.91	0.90	0.9080	1.2281	Indonesia	0.70	0.71	0.6384	0.1401
New_Zealand	0.90	0.88	0.8711	1.0811	Viet_Nam	0.70	0.70	0.6333	0.0179
Germany	0.92	0.92	0.9454	1.4249	Moldova_Rep_of	0.69	0.69	0.6263	-0.0121
Spain	0.91	0.90	0.9065	1.2151	Bolivia	0.66	0.65	0.5154	-0.3201
Italy	0.89	0.88	0.8648	1.0246	Honduras	0.61	0.59	0.4475	-0.9087
Israel	0.88	0.87	0.8681	1.0043	Tajikistan	0.68	0.69	0.6274	-0.0169
Singapore	0.86	0.83	0.7955	0.6993	Nicaragua	0.61	0.59	0.4510	-0.9201
Greece	0.88	0.87	0.8619	0.9985	Mongolia	0.65	0.64	0.5106	-0.3485
Hong_Kong_China	0.85	0.83	0.7952	0.6054	South_Africa	0.60	0.57	0.2990	-0.7543
Portugal	0.86	0.85	0.8132	0.8651	Egypt	0.67	0.68	0.6268	-0.1060
Slovenia	0.90	0.90	0.9053	1.2965	Guatemala	0.62	0.61	0.4784	-0.6807
Korea_Rep_of	0.88	0.88	0.8691	1.1477	Morocco	0.63	0.63	0.5520	-0.4993
Czech_Republic	0.88	0.89	0.9075	1.2890	Namibia	0.51	0.47	0.1250	-1.5137
Argentina	0.78	0.75	0.6441	0.0807	India	0.63	0.65	0.5677	-0.2407
Estonia	0.83	0.82	0.7684	0.7781	Botswana	0.52	0.50	0.1606	-1.1982
Poland	0.85	0.85	0.8274	0.9728	Ghana	0.62	0.64	0.5399	-0.1586
Hungary	0.87	0.89	0.8870	1.3303	Cambodia	0.58	0.59	0.4391	-0.6137
Slovakia	0.86	0.87	0.8751	1.1793	Papua_New_Guinea	0.52	0.52	0.3283	-1.1907
Lithuania	0.84	0.84	0.8086	0.9220	Lao_People's_Dem	0.56	0.58	0.4269	-0.5912
Chile	0.75	0.71	0.6016	-0.2288	Swaziland	0.47	0.45	0.0957	-1.3662
Uruguay	0.79	0.77	0.7062	0.2990	Bangladesh	0.56	0.58	0.4932	-0.6358
Costa_Rica	0.78	0.76	0.7011	0.1520	Nepal	0.54	0.56	0.4423	-0.8147
Croatia	0.84	0.84	0.8376	0.9564	Cameroon	0.51	0.52	0.2899	-0.9525
Latvia	0.82	0.82	0.7789	0.8294	Pakistan	0.55	0.57	0.4780	-0.7324
Mexico	0.73	0.70	0.5821	-0.2774	Lesotho	0.44	0.42	0.0594	-1.5939
Trinidad_and_Tobago	0.77	0.76	0.6951	0.3620	Uganda	0.51	0.52	0.2935	-0.8789
Bulgaria	0.80	0.80	0.7686	0.7159	Zimbabwe	0.46	0.45	0.0990	-1.3009
Malaysia	0.74	0.71	0.6187	-0.0954	Kenya	0.50	0.51	0.2724	-0.9743
Russian_Federation	0.75	0.74	0.6147	0.1942	Yemen	0.53	0.55	0.4507	-0.8089
Macedonia_TFYR	0.81	0.82	0.8141	0.8166	Madagascar	0.47	0.47	0.2730	-1.3658
Panama	0.71	0.68	0.5628	-0.4308	Nigeria	0.46	0.46	0.2311	-1.4951
Belarus	0.80	0.81	0.7685	0.7674	Mauritania	0.50	0.52	0.3469	-1.0096
Albania	0.80	0.81	0.8054	0.7669	Gambia	0.49	0.51	0.3548	-1.0706
Bosnia_and_Herzego	0.81	0.82	0.8246	0.8166	Senegal	0.47	0.48	0.3078	-1.2619
Venezuela	0.73	0.71	0.6144	-0.1515	Guinea	0.46	0.48	0.2899	-1.2110
Romania	0.79	0.80	0.7700	0.7198	Tanzania_U_Rep_of	0.45	0.48	0.2590	-1.0956
Ukraine	0.79	0.80	0.7677	0.7697	Cote_d_Ivoire	0.42	0.44	0.1784	-1.4240
Saint_Lucia	0.75	0.73	0.6607	0.1196	Zambia	0.39	0.39	0.0534	-1.6197
Brazil	0.69	0.65	0.4881	-0.5214	Malawi	0.39	0.40	0.0981	-1.6207
Colombia	0.69	0.66	0.5201	-0.5407	Central_African_Re	0.34	0.33	0.0000	-2.2722
Thailand	0.74	0.73	0.6358	0.1159	Ethiopia	0.44	0.48	0.3080	-1.0568
Kazakhstan	0.78	0.78	0.7226	0.6680	Mozambique	0.41	0.43	0.1795	-1.3509
Jamaica	0.75	0.74	0.7075	0.1812	Guinea-Bissau	0.38	0.39	0.1505	-1.8092
Armenia	0.74	0.74	0.6841	0.2056	Burundi	0.42	0.45	0.2372	-1.1961
Philippines	0.71	0.70	0.5987	-0.1003	Mali	0.34	0.35	0.1174	-2.1745
Turkmenistan	0.73	0.73	0.6289	0.1804	Burkina_Faso	0.34	0.35	0.1115	-2.1374
Paraguay	0.67	0.64	0.5015	-0.6012	Niger	0.32	0.33	0.0813	-2.2892
Peru	0.70	0.68	0.5593	-0.2729					

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